

11-1-2003

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Recommended Citation

Walker, David A. (2003) "JMASM9: Converting Kendall's Tau For Correlational Or Meta-Analytic Analyses," *Journal of Modern Applied Statistical Methods*: Vol. 2 : Iss. 2 , Article 26.

DOI: 10.22237/jmasm/1067646360

JMASM9: Converting Kendall's Tau For Correlational Or Meta-Analytic Analyses

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Expanding on past research, this study provides researchers with a detailed table for use in meta-analytic applications when engaged in assorted examinations of various r -related statistics, such as Kendall's tau (τ) and Cohen's d , that estimate the magnitude of experimental or observational effect. A program to convert from the lesser-used tau coefficient to other effect size indices when conducting correlational or meta-analytic analyses is presented.

Key words: Effect size, meta-analysis

Introduction

There is a heightened effort within the social and behavioral sciences to report effect sizes with research findings (APA, 2001; Henson & Smith, 2000; Knapp, 1998). Effect sizes show the strength and magnitude of a relationship and account for the total variance of an outcome. The American Psychological Association (APA) encouraged recently, "Always provide some effect size estimate when reporting a p value" (Wilkinson & The APA Task Force on Statistical Inference, 1999, p. 599). An analysis of effect sizes allows researchers to evaluate the importance of the result and not just the probability of the result (Kirk, 1996; Shaver, 1985).

Furthermore, effect sizes fall into two categories: d and r . The d group encompasses measures of effect size in terms of mean difference and standardized mean difference. Cohen (1988) defined the values of effect sizes for this group as small $d = .20$, medium $d = .50$, and large $d = .80$.

The r group can be considered as based on the correlation between treatment and result (Levin, 1994). For this group, "Effect size is generally reported as some proportion of the total variance accounted for by a given effect" (Stewart, 2000, p. 687). Cohen (1988) suggested that values of .01, .06, and .14 serve as indicators of small, medium, and large effect sizes for this group. However, it is at the discretion of the researcher to note the context in which small, medium, and large effects are being defined when using d and r related indices.

A review of the literature indicated that researchers have discussed the merits, or lack thereof, of employing correlation coefficients, such as Kendall's tau (τ), to assist in conducting meta-analytic studies or other forms of correlational and/or experimental analyses (Cooper & Hedges, 1994; Ferguson & Takane, 1989; Gibbons, 1985; Gilpin, 1993; Roberts & Kunst, 1990; Smithson, 2001; Wolf, 1987). Indeed, the reporting of tau in research studies has not been as prevalent as found with Spearman's rho (ρ) or Pearson's r . However, tau has been emphasized recently as a substitute for r in various research contexts. For instance, Rupinski and Dunlap (1996) looked at the accuracy of formulas for estimating r from tau.

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Methodology

Purpose

Tables for transforming correlation coefficients, such as Pearson's r to Spearman's rho, have been produced in the recent past (Gilpin, 1993; Strahan, 1982). Expanding upon Gilpin and Strahan's research, this study will provide researchers with a detailed table for use in meta-analytic applications when engaged in assorted examinations of various r -related statistics, such as Fisher's Z_r , and Cohen's d , that estimate the magnitude of experimental or observational effect. In addition, the table will be expanded to measure values in increments of .001 of a percent from .001 to 1.000, add more commonly utilized effect size variants of r not found in the original, and, most importantly, provide SPSS syntax to convert from the lesser-used tau coefficient to other effect size indices when conducting correlational or meta-analytic analyses.

Assumptions

This research study is not intended to be an exhaustive study of effect sizes, but serves as a prologue to impart contextual reference to the internal matrix table being presented. Also, it is presupposed that researchers understand that tau and rho apply distinct metrics, which means that they cannot be likened to one another due to a great difference between their absolute values (cf. Kendall, 1970; Strahan, 1982). As noted by researchers (Kendall, 1970; Gilpin, 1993), as the values of τ and ρ increase, their numerical similitude decreases greatly. The same trend holds for these two correlation coefficients' squared indices, where " τ^2 does not reflect at all adequately the proportion of shared variance..." (Strahan, 1982, p. 764).

A further assumption is that the table produced by means of this procedure reflects accurate values when a normal distribution is present, as well as a relatively large sample size. Also, the true values of the squared indices will be non-negative and the non-squared index values are symmetrical (i.e., $\tau < 0$), thus remaining the same numerically when negative.

Results

In the accompanying SPSS data set, we are given a value for tau ranging from .001 to 1.000. With a presented value of tau, the table that ensues can be created in total as an internal matrix via the SPSS syntax program provided in Appendix A or as individual, selected conditions by way of the subsequent formulas.

With a presented value of tau, we can calculate a Pearson's r using Kendall's formula (1970, p. 126).

$$r = \sin (.5 \pi \tau) \quad (1)$$

```
COMPUTE r=SIN (3.141592654*tau*.5).
EXECUTE.
```

Further, with a known value for τ and r , we can compute a Spearman's rho statistic using Gilpin's formula (1993, p. 91),

$$\rho = 3[\tau \sin^{-1}(r/2)]/\sin^{-1} r \quad (2)$$

```
COMPUTE p=3*tau*ARSIN(r/2)/ARSIN(r).
EXECUTE.
```

To compute a Fisher's Z_r statistic from a given value of r , derived from tau, we can apply the following formula (Rosenthal, 1994, p. 237).

$$Z_r = \frac{1}{2} \log_e [(1+r)/(1-r)] \quad (3)$$

```
COMPUTE z = .5*LN((1+r)/(1-r)).
EXECUTE.
```

To calculate a Cohen's d from a known r , derived from tau, the subsequent formula was employed (Rosenthal, 1994, p. 239). Note that for small to medium sample sizes, this formula will yield positively biased estimates. To correct for this, if presented with this situation, the expression should be multiplied by the factor $[(n-1)/n]^5$

$$d = 2r/[(1-r^2)^5] \quad (4)$$

```
COMPUTE d = 2*r/SQRT(1-r**2).
EXECUTE.
```

To calculate a Cohen's f statistic from a given d , derived from tau, the ensuing formula was utilized (Cohen, 1988, p.276).

$$f = \frac{1}{2}d \quad (5)$$

```
COMPUTE f = .5*d.
EXECUTE.
```

To determine the amount of variance accounted for with correlation coefficients, such as r , ρ , or τ , we square their value, which yields the extent of the effect in terms of "how much of the variability in the dependent variable(s) is associated with the variation in the independent variable(s)" (Snyder & Lawson, 1993, p. 338). A caution should be noted, however, that when using squared indices to determine effect size, a loss of directionality is an issue and also the power affiliated with these indices is often distorted when reporting research findings (Rosenthal, 1994).

With r -related squared indices, it should be mentioned that eta-squared (η^2) and r^2 are identical numerically and r^2 and f^2 are related monotonically. Cohen (1994) determined that η^2 was a population correlation ratio that could be "... computed on samples and its population value estimated therefrom" (p. 281). Effect size estimates of this order have been called epsilon-squared (ξ^2) and omega-squared (ω^2). Thus, this type of effect size tends to measure the proportion of variance in the population due to a particular effect. Cohen's formula for η^2 (1994, p. 281) can be used if r^2 is not preferred, where $f^2 = d^2/4$.

$$\eta^2 = \frac{f^2}{1 + f^2} \quad (6)$$

```
COMPUTE eta2 = f**2/(1+f**2).
EXECUTE.
```

Conclusion

Methodological appropriateness is a consequential area within research that should be nearly perfect. Concepts such as effect size need to be

addressed correctly in a research study to make reliable, justifiable decisions and have these decisions, based on the statistics of study, authenticated by others. Converting from the lesser-used tau coefficient to other effect size indices, the presented SPSS syntax program can create an internal matrix table and new data set to assist researchers in determining the size of an effect for commonly utilized r -related indices when engaging in correlational and meta-analytic analyses.

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Appendix

Notes: To produce an internal matrix output and a complete table as a new data set in SPSS, access the embedded Tau Data Set and then run Tau Syntax. Create the data set in SPSS with a variable (1 column by 1000 rows) containing the values .001 - 1.000 (.001), or contact the author to obtain copies of the data set and the syntax. An example of the tabled values appears on the following page.

```

compute r = SIN(3.141592654 * t * .5).
compute rs = 3 * t * ARSIN(r / 2) / ARSIN(r).
compute zr = .5 * LN((1 + r) / (1 - r)).
compute d = 2 * r / SQRT(1 - r ** 2).
compute f = d*.5.
compute r2 = r **2.
compute f2 = d**2/4.
compute eta2 = (f2) / (1 + f2).
execute.
* FINAL REPORTS *.
FORMAT r to eta2 (f9.4).
VARIABLE LABELS t 'Tau' /r 'Pearsons r' /rs 'Spearman's Rank' /zr 'Fishers
z' /d 'Cohens d' /f 'f (Related to d as an SD of Standardized Means when k
= 2 and n = n)' / r2 'R Square' / eta2 'Eta Square'.
REPORT FORMAT=LIST AUTOMATIC ALIGN(CENTER)
/VARIABLES=t r rs zr
/TITLE "Proportion of Variance-Accounted-For Effect Sizes: Measures of
Relationship".
REPORT FORMAT=LIST AUTOMATIC ALIGN(CENTER)
/VARIABLES=d f /TITLE "Standardized Mean Difference Effect Sizes".
REPORT FORMAT=LIST AUTOMATIC ALIGN(CENTER)
/VARIABLES= r2 eta2
/TITLE "Proportion of Variance-Accounted-For Effect Sizes: Squared
Indices".

```

An Example Of Tabled Values

tau	r	p	Zr	d	f	R2	eta2
0.001	0.0016	0.0015	0.0016	0.0031	0.0016	0.0000	0.0000
0.002	0.0031	0.0030	0.0031	0.0063	0.0031	0.0000	0.0000
0.003	0.0047	0.0045	0.0047	0.0094	0.0047	0.0000	0.0000
0.004	0.0063	0.0060	0.0063	0.0126	0.0063	0.0000	0.0000
0.005	0.0079	0.0075	0.0079	0.0157	0.0079	0.0001	0.0001
0.006	0.0094	0.0090	0.0094	0.0189	0.0094	0.0001	0.0001
0.007	0.0110	0.0105	0.0110	0.0220	0.0110	0.0001	0.0001
0.008	0.0126	0.0120	0.0126	0.0251	0.0126	0.0002	0.0002
0.009	0.0141	0.0135	0.0141	0.0283	0.0141	0.0002	0.0002
0.010	0.0157	0.0150	0.0157	0.0314	0.0157	0.0002	0.0002
0.011	0.0173	0.0165	0.0173	0.0346	0.0173	0.0003	0.0003
0.012	0.0188	0.0180	0.0189	0.0377	0.0189	0.0004	0.0004
0.013	0.0204	0.0195	0.0204	0.0408	0.0204	0.0004	0.0004
0.014	0.0220	0.0210	0.0220	0.0440	0.0220	0.0005	0.0005
0.015	0.0236	0.0225	0.0236	0.0471	0.0236	0.0006	0.0006
0.016	0.0251	0.0240	0.0251	0.0503	0.0251	0.0006	0.0006
0.017	0.0267	0.0255	0.0267	0.0534	0.0267	0.0007	0.0007
0.018	0.0283	0.0270	0.0283	0.0566	0.0283	0.0008	0.0008
0.019	0.0298	0.0285	0.0298	0.0597	0.0299	0.0009	0.0009
0.020	0.0314	0.0300	0.0314	0.0629	0.0314	0.0010	0.0010
0.021	0.0330	0.0315	0.0330	0.0660	0.0330	0.0011	0.0011
0.022	0.0346	0.0330	0.0346	0.0691	0.0346	0.0012	0.0012
0.023	0.0361	0.0345	0.0361	0.0723	0.0361	0.0013	0.0013
0.024	0.0377	0.0360	0.0377	0.0754	0.0377	0.0014	0.0014
0.025	0.0393	0.0375	0.0393	0.0786	0.0393	0.0015	0.0015
0.026	0.0408	0.0390	0.0409	0.0817	0.0409	0.0017	0.0017
0.027	0.0424	0.0405	0.0424	0.0849	0.0424	0.0018	0.0018
0.028	0.0440	0.0420	0.0440	0.0880	0.0440	0.0019	0.0019
0.029	0.0455	0.0435	0.0456	0.0912	0.0456	0.0021	0.0021
0.030	0.0471	0.0450	0.0471	0.0943	0.0472	0.0022	0.0022
0.031	0.0487	0.0465	0.0487	0.0975	0.0487	0.0024	0.0024
0.032	0.0502	0.0480	0.0503	0.1006	0.0503	0.0025	0.0025
0.033	0.0518	0.0495	0.0519	0.1038	0.0519	0.0027	0.0027
0.034	0.0534	0.0510	0.0534	0.1069	0.0535	0.0028	0.0028
0.035	0.0550	0.0525	0.0550	0.1101	0.0550	0.0030	0.0030
0.036	0.0565	0.0540	0.0566	0.1132	0.0566	0.0032	0.0032
0.037	0.0581	0.0555	0.0582	0.1164	0.0582	0.0034	0.0034
0.038	0.0597	0.0570	0.0597	0.1195	0.0598	0.0036	0.0036
0.039	0.0612	0.0585	0.0613	0.1227	0.0613	0.0037	0.0037
0.040	0.0628	0.0600	0.0629	0.1258	0.0629	0.0039	0.0039
0.041	0.0644	0.0615	0.0644	0.1290	0.0645	0.0041	0.0041
0.042	0.0659	0.0630	0.0660	0.1321	0.0661	0.0043	0.0043
0.043	0.0675	0.0645	0.0676	0.1353	0.0676	0.0046	0.0046
0.044	0.0691	0.0660	0.0692	0.1385	0.0692	0.0048	0.0048
0.045	0.0706	0.0675	0.0707	0.1416	0.0708	0.0050	0.0050
0.046	0.0722	0.0690	0.0723	0.1448	0.0724	0.0052	0.0052
0.047	0.0738	0.0705	0.0739	0.1479	0.0740	0.0054	0.0054
0.048	0.0753	0.0719	0.0755	0.1511	0.0755	0.0057	0.0057
0.049	0.0769	0.0734	0.0770	0.1542	0.0771	0.0059	0.0059
0.050	0.0785	0.0749	0.0786	0.1574	0.0787	0.0062	0.0062