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Present work has investigated the inferences in covariate autoregressive (C-AR) model when error term is non-Gaussian. Spherically symmetric behaviour is one of the non-Gaussian characteristic and may be seen in agricultural, economics and biological field. So, we have considered here the spherical symmetric error in C-AR model. Under Bayesian methodology, estimators of the model parameters are obtained by conditional posterior distribution. A Bayes factor is derived for testing the unit root hypothesis. A simulation study has conducted when error term follows t-distribution. In empirical study, we considered REER time series of SAARC countries to illustrate the applicability of the proposed model.

Keywords: AR Process, Covariate, Spherical Symmetric Distribution, Prior and Posterior distributoion.

Introduction

In present scenario, time series analysis is a challenging and demandingresearch area because of its versatile applicability in different fields. In reference to modelling of time series, generally,errors are assumed normally distributed but sometimes it not happened so far, instead of this non-Gaussian distribution is more useful. The spherical symmetric form is one of the non-Gaussian distribution, and it is becoming popular in the last few decades. A wide literature is available in reference to regression modelling when errorsarebelonging to a class of spherical symmetric distribution, Ullah and Zinde-Walsh (1985) studied the estimation and testing procedure in a regression model. Jammalamadakaet al. (1987) discussed Bayes predictive inference in regression models. Forthe inferential assumption of parameters, several authors have studied the behaver of a random variable from a spherically symmetric distribution,see Strawderman (1974), Berger (1975), Judge et al. (1985), etc. For testing a linear regression parameterof the model, Wang and Wells (2002) explored the hypothesis testunder the assumption that errors are spherical distribution. Xu and Yang (2013) discussed a positive-rule stein-type ridge estimator and some related competing estimators in consideration of spherically symmetric disturbances.In Bayesian approach, Panday (2015) developed the state-space model with non-normal disturbances and estimated the parameters using the Gibbs sample technique and derived the marginal posterior densities.De Kock and Eggers (2017) proposed Bayesian variable selection for linear parametrizations with normal iid observations based on spherically symmetric distribution.Rather than error distribution, observed series is also affectedby other associates variables. These associated variables may be partially or continuously influence the series depends upon the circumstances. Therefore, these associate series is appropriate for the study to increase the efficiency of the model.

There isa variety of literature exist to deal with covariate in the time series model.Hensen (1995)proposed a covariate augmented Dickey-Fuller (CADF) unit root test and obtained the asymptotic local power function of the CADF statistic.Recently Chang et al. (2017)developed bootstrap unit root tests with a covariate method to the CADF test to deal the time series with the nuisance parameter dependency and provided a valid basis for inference based on the CADF test.Also, Kumar et al. (2017, 2018)explored an autoregressive model with consideration of covariatevariables and extended to the panel data time series model. In this paper, we studied the C-AR time series model when error terms are spherically distributed under Bayesian approach. To carry out the Bayesian approach, model involving various parameters which is difficult to obtain theconditional posterior densities, in that case, Markov Chain Monte Carlo (MCMC) technique as Gibbs sampler is used. For the unit root test, Bayes factor is derived from posterior probability. A simulation study is carried outwhen the error is distributed multivariate t-distribution. The applicability of the model is also verified by the empirical study.

Model Description

Let us assume that $\{y_t; t=1,2,...,T\}$be a time series with intercept term ϕ.

$$y_t = \phi + u_t \tag{1}$$

The error term $u_t$follows AR(1) process associated with a stationary covariate $\{w_t\}$. Then, times series may be serially correlated to the covariate series, and $u_t$follows the model

$$u_t = \rho u_{t-1} + \sum_{j=-r+1}^{p} \lambda_j w_{t-j} + \varepsilon_t \tag{2}$$

Where ρ is an autoregressive coefficient and λ_j is a covariate coefficient. Utilizing equation (2) in (1), the model can be written as

$$y_t = \phi(1-\rho) + \rho y_{t-1} + \sum_{j=-r+1}^{p} \lambda_j w_{t-j} + \varepsilon_t \tag{3}$$

Generally, the researcher considers the error term is normally distributed but in real life situations, the structure of the series has some skewed nature. So, in this paper, we assume that errors $\{\varepsilon_t ; t = 1, 2, ..., T\}$ are distributed according to probability law which belongs to the class of spherically symmetric distribution.Considering the limitation of the study, we have considered the following probability density function of ε_t as

$$f(\varepsilon_t) = \int_0^{\infty} \frac{\tau^{1/2}}{(2\pi)^{1/2}\psi(\zeta)} \exp\left[-\frac{\tau}{2\psi^2(\zeta)}\varepsilon_t^{\ 2}\right] dG(\zeta) \tag{4}$$

where $\psi(\zeta)$ is a positive measurable function and $dG(\zeta)$ is a cumulative distribution function of ζ .

The main motive behind the present study is to test the unit root hypothesis and estimate the model parameters under consideration of spherically symmetric error. Under the unit root hypothesis, model (3) becomes

$$\Delta y = \sum_{j=-r+1}^{p} \lambda_j w_{t-j} + \varepsilon_t \tag{5}$$

Model (3) and (5) can be written in matrix notation as follows

$$Y = \rho Y_{-1} + (1-\rho) l_T \phi + W\Lambda + \varepsilon \tag{6}$$

$$\Delta Y = W\Lambda + \varepsilon \tag{7}$$

where

$$Y = (y_1 \ y_2 \ ... \ y_T)' ; \ Y_{-1} = (y_0 \ y_1 \ ... \ y_{T-1})'$$

$$\Delta Y = (\Delta y_1 \ \Delta y_2 \ ... \ \Delta y_T)' = Y - Y_{-1}; \quad \varepsilon = (\varepsilon_1 \ \varepsilon_2 \ ... \ \varepsilon_T)'$$

$$\Lambda = (\lambda_{-r+1} \ \lambda_{-r+2} \ ... \ \lambda_0 \ \lambda_1 \ ... \ \lambda_p)'; \qquad l_T = (1 \ 1 \ ... \ 1)'$$

$$W = \begin{pmatrix} w_r & w_{r-1} & \cdots & w_{1-p} \\ w_{r+1} & w_r & \cdots & w_{2-p} \\ \vdots & \vdots & \ddots & \vdots \\ w_{T+r-1} & w_{T+r-2} & \cdots & w_{T-p} \end{pmatrix}$$

Bayesian Framework

Bayesian approach contains not only recorded observations but alsohasadditional information regarding the parameters known as prior. There is an independent area of the researcher about the form and nature of prior. So, we have assumed that intercept and error variance follow conjugate normal and chi-square distribution, respectively.A uniform distribution is considered for autoregressive coefficient (ρ) and covariate coefficient. The joint prior distribution(Berger (2013)) of model parametersis

$$P(\Theta)=\frac{\tau^{\frac{v+1}{2}-1}}{(1-a)(2\pi)^{\frac{1}{2}}2^{\frac{v}{2}}\Gamma\left(\frac{v}{2}\right)}\exp\left[-\frac{\tau}{2}\left\{(\phi-\phi_0)^2+1\right\}\right] \tag{8}$$

The likelihood function forthe models (6) and (7) denoted by L1and L0 are respectively

$$L_1(\varepsilon)=\int_0^\infty\frac{\tau^{\frac{T}{2}}}{(2\pi)^{\frac{T}{2}}\psi^T(\zeta)}\exp\left[-\frac{\tau}{2\psi^2(\zeta)}\left(Y-\rho Y_{-1}-l_T(1-\rho)\phi-\Lambda W\right)'\left(Y-\rho Y_{-1}-l_T(1-\rho)\phi-\Lambda W\right)\right]dG(\zeta) \tag{9}$$

$$L_0(\varepsilon)=\int_0^\infty\frac{\tau^{\frac{T}{2}}}{(2\pi)^{\frac{T}{2}}\psi^T(\zeta)}\exp\left[-\frac{\tau}{2\psi^2(\zeta)}(\Delta Y-\Lambda W)'(\Delta Y-\Lambda W)\right]dG(\zeta) \tag{10}$$

First, we are interested totest the unit root hypothesis $H_0:\rho=1$against the alternative $H_1:\rho\in s;s=\{\rho:1>\rho>a;a>-1\}$. The unit root test is very important before drawing the inference in ARIMA methodology see Box and Jenkins (1976), Phillips and Ploberger(1994), Lubrano (1995), etc. For testing unit root hypothesis, we used the posterior probability of H_1 and H_0 to obtain the Bayes factor. Bayes factor is the ratio of posterior probability under null and alternative

hypothesis with equal prior probability. For more details, refer to Berger (2013) and Chen et al. (2016). The posterior probability under H_1 and $H_0$is

$$P(Y|H_1)=\int_a^1\int_0^\infty\frac{\Gamma\left(\frac{T+v-r-p}{2}\right)\left[\psi^2(\zeta)\right]^{\frac{r+p+1}{2}}\left|W'W\right|^{-\frac{1}{2}}\left|A(\rho,\psi(\zeta))\right|^{-\frac{1}{2}}}{(1-a)(2\pi)^{\frac{T-r-p}{2}}2^{\frac{v}{2}}\Gamma\left(\frac{v}{2}\right)\left[C(\rho,\psi(\zeta))\right]^{\frac{T+v-r-p}{2}}\psi^T(\zeta)}d\rho\, dG(\zeta) \tag{11}$$

$$P(Y \mid H_0) = \int_0^\infty \frac{\Gamma\left(\frac{T+v-r-p}{2}\right)\left[\psi^2(\zeta)\right]^{\frac{r+p}{2}} \left|W^{'}W\right|^{-\frac{1}{2}}}{(2\pi)^{\frac{T-r-p}{2}} 2^{\frac{v}{2}} \Gamma\left(\frac{v}{2}\right)\left[D(\psi(\zeta))\right]^{\frac{T+v-r-p}{2}} \psi^T(\zeta)} dG(\zeta) \tag{12}$$

where

$$\Sigma = I - W(W^{'}W)^{-1}W^{'}$$

$$A(\rho,\psi(\zeta)) = (1-\rho)^2 l_T^{'} \Sigma l_T + \psi^2(\zeta)$$

$$B(\rho,\psi(\zeta)) = (1-\rho)\ l_T^{'} \Sigma (Y - \rho Y_{-1}) + \phi_0 \psi^2(\zeta)$$

$$C(\rho,\psi(\zeta)) = \frac{1}{2\psi^2(\xi)}\left[(Y-\rho Y_{-1})^{'} \Sigma (Y-\rho Y_{-1}) - B(\rho,\psi(\zeta))^{'} A^{-1}(\rho,\psi(\zeta)) B(\rho,\psi(\zeta))\right] + \frac{1}{2}(1+\phi_0{}^2)$$

$$D(\psi(\zeta)) = \frac{1}{2\psi^2(\xi)}\left[(\Delta Y^{'} \Sigma \Delta Y)\right] + \frac{1}{2}$$

In Bayesian testing procedure, the linking of the respective model is obtained through Bayes factor (B_{10}) which is the ratio of posterior probability for the respective hypotheses. We used the Kass and Raftery (1995) Bayes factor for rejection or acceptance ofa hypothesis. Forestimation of parameters of model,derived the conditional posterior distribution ofϕ, Λ, ρ and τ which is as follows.

$$\hat{\phi} \sim N\left(Z_1 H_1^{-1}, \frac{1}{\tau\psi^2(\zeta)} H_1^{-1} \right) \tag{13}$$

$$\hat{\Lambda} \sim N\left(Z_2 H_2^{-1}, \frac{1}{\tau\psi^2(\zeta)} H_2^{-1} \right) \tag{14}$$

$$\hat{\rho} \sim TN\left(Z_3 H_3^{-1}, \frac{1}{\tau\psi^2(\zeta)} H_3^{-1}, l, 1 \right) \tag{15}$$

$$\hat{\tau} \sim Gamma\left(\frac{v+T+1}{2}, V \right) \tag{16}$$

where,

$$H_1 = l_T^{'} l_T (1-\rho)^2 + \psi^2(\zeta)$$

$$H_2 = W^{'} W$$

$$H_3 = \left(Y_{-1} - l_T \phi\right)^{'} \left(Y_{-1} - l_T \phi\right)$$

$$Z_1 = l_T^{'} (1-\rho)(Y - \rho Y_{-1} - \Lambda W) + \frac{1}{\psi^2(\zeta)} \phi_0$$

$$Z_2 = (Y - \rho Y_{-1} - l_T (1-\rho)\phi)$$

$$Z_3 = (Y_{-1} - l_T \phi)^{'} (Y - l_T \phi - \Lambda W)$$

$$V = \frac{1}{\psi^2(\zeta)} (Y - \rho Y_{-1} - l_T (1-\rho)\phi - \Lambda W)^{'} (Y - \rho Y_{-1} - l_T (1-\rho)\phi - \Lambda W) + \psi^2(\zeta) \left\{ (\phi - \phi_0)^2 + 1 \right\}$$

The conditional posterior distribution of all parameters is conditionally in standard distribution form. Hence, we can use Gibbs algorithm to simulate the posterior sample from equations (13) to (16). For better interpretation, different loss functions (Asymmetric Loss Functions (ALF), Squared Error Loss Functions (SELF) and Precautionary Loss Function (PLF)) are considered under Bayesian approach(for details referNorstrom (1996), Schroeder and Zieliński (2011)).

Simulation Study

A simulation technique is an imitation of the action of a practical process or system. It is widely useful to provide the inference of the proposed methodology when a real application is not available. In present scenario, this technique is well developed in statistical/mathematical softwareslike R, Python, Matlab, etc. which validates the parameters of the model from simulated samples. For the proposed model, we used R for generating the time series considering errors have spherically symmetric distribution. For simplicity, we assumed that the covariate series follows AR(1) process with an intercept 0.2 and autocorrelation coefficient 0.8.The distribution of ζ is assumed to be a positive random variable and follows chi-square distribution with n degree of freedom. Then, the error distribution is obtained as student t-distribution when $\psi(\zeta) = \left(\zeta / n\right)^{-\frac{1}{2}}$. Hence, the likelihood function of the proposed model is

$$L = \frac{\zeta^{\frac{T}{2}}\Gamma\left(\frac{T+\nu}{2}\right)}{(2\pi)^{\frac{T}{2}} n^{\frac{T}{2}} 2^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}\left[1+\frac{\zeta}{n}\left(Y-\rho Y_{-1}+(1-\rho)l_T\phi+W\Lambda\right)'\left(Y-\rho Y_{-1}+(1-\rho)l_T\phi+W\Lambda\right)\right]$$

The size of the series is taken as T=100. To initiate the process, the initial value of the series and covariate is y_0=6 and w_0=5, respectively. For estimation and testing the unit root hypothesis, true value of the parameters (P) areϕ =10, ʌ=4, ρ=(0.90, 0.92, 0.94, 0.98, 0.99)and hyperparameterν=3. For more generalized inference, Bayes factor is calculatedunder various degrees of freedom like 10, 20, 30, 40 and 50.Table 1 reports the unit root testing using Bayes factor (B_{10}) with different degrees of freedom as well as lower and higher values of ρ. From Table 1, we observe that as the degrees of freedom and ρ changes, the value of Bayes factor also changes. But all the results significantly rejected the unit root hypothesis because of the large values of Bayes factor.

Table 1: Testing with different degrees of freedom and ρ

DF	n=10		n=20		n=30		n=40		n=50	
P	$\hat{\rho}$	B_{10}	$\hat{\rho}$	B_{10}	$\hat{\rho}$	B_{10}	$\hat{\rho}$	B_{10}	$\hat{\rho}$	B_{10}
0.9	0.903	681.574	0.882	566.783	0.893	572.586	0.907	636.977	0.899	616.894
0.92	0.920	777.064	0.918	567.677	0.915	592.137	0.915	590.874	0.915	631.813
0.94	0.941	707.849	0.927	554.457	0.933	551.672	0.939	621.312	0.933	579.017
0.96	0.958	631.002	0.957	537.286	0.954	575.744	0.953	606.264	0.951	592.467
0.98	0.981	565.460	0.982	538.737	0.976	563.424	0.976	600.949	0.971	588.150
0.99	0.989	517.477	0.985	522.986	0.985	546.107	0.991	565.846	0.981	562.666

Tables 2-4 provide mean squared error (MSE) and absolute bias (ABS)of estimated values under different estimation methods. From these tables, we notice that Bayes estimators are having minimum MSE and ABS in comparison to MLE. When the degree of freedom is increased, MSE is approximately decreased in all estimators. Under each loss function, we observe that an almost similar magnitude in terms of MSE and ABS for ρ and ʌare recorded. But there is a difference in MSE and ABS for ϕ and τ. In this case, the absolute loss function gives better results. Similar results are also observed with the varying values of autoregressive coefficients. Tables 5-7 represent the confidence and HPD interval with the varying degree of freedom as well as ρ. It shows that width of the interval is small when Bayes technique is considered.

Table 2: Estimation of parameters with different loss functions at ρ=0.90

DF	P	MLE		SELF		ALF		PLF	
		MSE	ABS	MSE	ABS	MSE	ABS	MSE	ABS
n=10	P	1.86E-05	3.25E-03	9.52E-06	2.36E-03	9.25E-06	2.30E-03	9.52E-06	2.36E-03
	Φ	4.60E+00	1.57E+00	7.92E-02	2.00E-01	1.21E-01	2.64E-01	8.38E-02	2.16E-01
	ʌ	5.88E-03	6.10E-02	5.46E-03	5.89E-02	4.96E-03	5.54E-02	5.47E-03	5.90E-02
	T	5.47E+00	2.31E+00	4.02E+00	1.99E+00	4.06E+00	2.00E+00	3.96E+00	1.98E+00
n=20	P	1.79E-05	3.26E-03	9.12E-06	2.34E-03	8.80E-06	2.27E-03	9.11E-06	2.34E-03
	Φ	4.07E+00	1.51E+00	7.74E-02	2.02E-01	1.42E-01	2.96E-01	8.29E-02	2.20E-01
	ʌ	5.36E-03	5.67E-02	4.82E-03	5.47E-02	4.16E-03	5.01E-02	4.83E-03	5.48E-02
	T	1.78E+00	1.31E+00	9.33E-01	9.34E-01	9.49E-01	9.43E-01	9.00E-01	9.16E-01
n=30	P	1.42E-05	2.99E-03	7.30E-06	2.12E-03	7.04E-06	2.06E-03	7.30E-06	2.12E-03
	Φ	3.87E+00	1.48E+00	8.20E-02	2.11E-01	1.43E-01	2.97E-01	8.42E-02	2.20E-01
	ʌ	4.43E-03	5.25E-02	4.32E-03	5.18E-02	3.81E-03	4.76E-02	4.33E-03	5.19E-02
	T	7.29E-01	8.40E-01	3.03E-01	5.04E-01	3.11E-01	5.12E-01	2.85E-01	4.88E-01
n=40	P	1.45E-05	2.94E-03	7.21E-06	2.10E-03	6.93E-06	2.04E-03	7.21E-06	2.10E-03
	Φ	3.77E+00	1.48E+00	8.31E-02	2.11E-01	1.42E-01	2.97E-01	8.61E-02	2.21E-01
	ʌ	4.69E-03	5.35E-02	4.35E-03	5.22E-02	3.83E-03	4.78E-02	4.36E-03	5.23E-02
	T	3.70E-01	5.87E-01	1.16E-01	2.91E-01	1.19E-01	2.96E-01	1.07E-01	2.79E-01
n=50	P	1.64E-05	3.09E-03	7.83E-06	2.12E-03	7.53E-06	2.05E-03	7.83E-06	2.12E-03
	Φ	4.27E+00	1.56E+00	7.66E-02	1.99E-01	1.41E-01	2.92E-01	8.11E-02	2.14E-01
	ʌ	4.65E-03	5.37E-02	4.47E-03	5.25E-02	3.93E-03	4.83E-02	4.49E-03	5.26E-02
	T	1.91E-01	4.12E-01	6.56E-02	2.09E-01	6.60E-02	2.10E-01	6.34E-02	2.04E-01

Table 3: Estimation of parameters with different loss functions at ρ=0.95

DF	P	MLE		SELF		ALF		PLF	
		MSE	ABS	MSE	ABS	MSE	ABS	MSE	ABS
n=10	ρ	8.81E-06	2.22E-03	3.39E-06	1.36E-03	3.40E-06	1.36E-03	3.39E-06	1.36E-03
	ϕ	2.49E+01	3.39E+00	3.49E-02	1.42E-01	3.48E-02	1.41E-01	3.57E-02	1.45E-01
	ʌ	5.51E-03	5.75E-02	4.98E-03	5.55E-02	4.98E-03	5.55E-02	4.98E-03	5.55E-02
	T	4.99E+00	2.11E+00	3.61E+00	1.89E+00	3.64E+00	1.90E+00	3.56E+00	1.87E+00
n=20	ρ	7.47E-06	2.05E-03	2.78E-06	1.26E-03	2.78E-06	1.26E-03	2.78E-06	1.26E-03
	ϕ	2.19E+01	3.20E+00	2.97E-02	1.30E-01	2.97E-02	1.30E-01	3.03E-02	1.31E-01
	ʌ	5.36E-03	5.55E-02	4.17E-03	5.11E-02	4.17E-03	5.11E-02	4.17E-03	5.11E-02
	T	1.69E+00	1.24E+00	6.76E-01	7.90E-01	6.90E-01	8.00E-01	6.54E-01	7.76E-01
n=30	ρ	8.55E-06	2.18E-03	3.10E-06	1.34E-03	3.11E-06	1.34E-03	3.10E-06	1.34E-03
	ϕ	2.22E+01	3.33E+00	2.93E-02	1.29E-01	2.93E-02	1.29E-01	3.00E-02	1.32E-01
	ʌ	5.07E-03	5.46E-02	4.79E-03	5.37E-02	4.79E-03	5.36E-02	4.79E-03	5.37E-02
	T	7.29E-01	8.31E-01	1.78E-01	3.76E-01	1.85E-01	3.84E-01	1.69E-01	3.64E-01
n=40	ρ	7.82E-06	2.12E-03	2.73E-06	1.26E-03	2.73E-06	1.26E-03	2.73E-06	1.26E-03
	ϕ	2.56E+01	3.44E+00	2.79E-02	1.26E-01	2.81E-02	1.26E-01	2.87E-02	1.29E-01
	ʌ	4.42E-03	5.27E-02	4.13E-03	5.16E-02	4.13E-03	5.16E-02	4.13E-03	5.17E-02
	T	3.58E-01	5.75E-01	5.93E-02	2.00E-01	6.03E-02	2.03E-01	5.82E-02	1.97E-01
n=50	ρ	7.53E-06	2.06E-03	2.76E-06	1.25E-03	2.76E-06	1.25E-03	2.76E-06	1.25E-03
	ϕ	2.32E+01	3.37E+00	2.70E-02	1.27E-01	2.71E-02	1.26E-01	2.75E-02	1.29E-01
	ʌ	4.22E-03	5.08E-02	3.93E-03	4.96E-02	3.94E-03	4.96E-02	3.93E-03	4.96E-02

	T	1.89E-01	4.07E-01	5.80E-02	1.87E-01	5.56E-02	1.83E-01	6.20E-02	1.93E-01

Table 4: Estimation of parameters with different loss functions at ρ=0.99

DF	P	MLE		SELF		ALF		PLF	
		MSE	ABS	MSE	ABS	MSE	ABS	MSE	ABS
n=10	ρ	1.98E-06	7.93E-04	7.74E-07	5.87E-04	7.77E-07	5.87E-04	7.74E-07	5.87E-04
	ϕ	4.21E+02	6.20E+00	1.64E-03	3.10E-02	1.89E-03	3.32E-02	3.03E-03	4.38E-02
	ʌ	3.52E-03	3.68E-02	4.19E-03	5.10E-02	4.20E-03	5.11E-02	4.19E-03	5.10E-02
	T	2.69E+00	1.14E+00	3.57E+00	1.88E+00	3.61E+00	1.89E+00	3.52E+00	1.86E+00
n=20	ρ	1.51E-06	7.49E-04	7.05E-07	5.72E-04	7.07E-07	5.73E-04	7.05E-07	5.72E-04
	ϕ	1.93E+02	4.69E+00	1.46E-03	2.96E-02	1.71E-03	3.23E-02	2.27E-03	3.84E-02
	ʌ	3.45E-03	3.70E-02	3.87E-03	4.88E-02	3.88E-03	4.89E-02	3.87E-03	4.88E-02
	T	9.29E-01	6.95E-01	6.81E-01	7.97E-01	6.96E-01	8.07E-01	6.60E-01	7.83E-01
n=30	ρ	1.30E-06	7.06E-04	5.32E-07	5.39E-04	5.33E-07	5.40E-04	5.32E-07	5.39E-04
	ϕ	4.93E+02	5.25E+00	1.33E-03	2.88E-02	1.55E-03	3.05E-02	2.08E-03	3.61E-02
	ʌ	3.31E-03	3.78E-02	3.74E-03	4.92E-02	3.75E-03	4.93E-02	3.74E-03	4.92E-02
	T	4.30E-01	4.90E-01	1.65E-01	3.61E-01	1.72E-01	3.69E-01	1.57E-01	3.49E-01
n=40	ρ	1.51E-06	7.39E-04	6.48E-07	5.71E-04	6.50E-07	5.72E-04	6.48E-07	5.71E-04
	ϕ	2.59E+02	4.89E+00	1.38E-03	2.92E-02	1.52E-03	3.01E-02	2.16E-03	3.67E-02
	ʌ	2.84E-03	3.42E-02	3.20E-03	4.52E-02	3.21E-03	4.52E-02	3.20E-03	4.52E-02
	T	2.12E-01	3.35E-01	5.72E-02	1.94E-01	5.79E-02	1.96E-01	5.64E-02	1.92E-01
n=50	ρ	1.20E-06	6.84E-04	5.30E-07	5.36E-04	5.31E-07	5.37E-04	5.30E-07	5.36E-04
	ϕ	6.32E+02	4.47E+00	1.37E-03	2.81E-02	1.57E-03	3.03E-02	1.91E-03	3.48E-02
	ʌ	3.11E-03	3.61E-02	3.51E-03	4.75E-02	3.52E-03	4.76E-02	3.51E-03	4.75E-02
	T	1.18E-01	2.50E-01	6.71E-02	1.97E-01	6.44E-02	1.92E-01	7.18E-02	2.04E-01

Table 5: Confidence and HPD Interval of parameters at ρ= 0.90

DF	P	MLE	SELF	ALF	PLF
n=10	ρ	(0.8911, 0.9087)	(0.8944, 0.9065)	(0.8941, 0.9061)	(0.8944, 0.9065)
	ϕ	(6.0006, 14.7752)	(9.3704, 10.5567)	(9.2951, 10.6897)	(9.3866, 10.5810)
	ʌ	(3.8606, 4.1590)	(3.8678, 4.1555)	(3.8770, 4.1541)	(3.8703, 4.1575)
	τ	(-0.0935, 1.1597)	(0.7065, 1.5978)	(0.7142, 1.5988)	(0.7163, 1.6084)
n=20	ρ	(0.8916, 0.9083)	(0.8943, 0.9055)	(0.8943, 0.9054)	(0.8943, 0.9055)
	ϕ	(5.9957, 14.2195)	(9.4325, 10.5838)	(9.1931, 10.6915)	(9.4301, 10.5958)
	ʌ	(3.8465, 4.1403)	(3.8711, 4.1459)	(3.8692, 4.1310)	(3.8729, 4.1479)
	τ	(0.6459, 1.4311)	(0.8850, 1.7920)	(0.8841, 1.7873)	(0.9005, 1.8093)
n=30	ρ	(0.8929, 0.9075)	(0.8944, 0.9048)	(0.8946, 0.9049)	(0.8944, 0.9048)
	ϕ	(6.1213, 13.9601)	(9.3768, 10.5552)	(9.1919, 10.6507)	(9.3966, 10.5830)
	ʌ	(3.8671, 4.1345)	(3.8596, 4.1241	(3.8675, 4.1190)	(3.8592, 4.1236)
	τ	(0.7169, 1.2988)	(0.9306, 1.8283)	(0.9271, 1.8224)	(0.9460, 1.8455)
n=40	ρ	(0.8934, 0.9081)	(0.8947, 0.9053)	(0.8947, 0.9051)	(0.8947, 0.9053)
	ϕ	(5.8196, 13.2417)	(9.3792, 10.6057)	(9.2232, 10.6797)	(9.3890, 10.6137)
	ʌ	(3.8554, 4.1275)	(3.8596, 4.1205)	(3.8700, 4.1220)	(3.8615, 4.1230)

	τ	(0.7286, 1.3212)	(0.8947, 1.8484)	(0.8856, 1.8315)	(0.9127, 1.8672)
n=50	ρ	(0.8916, 0.9072)	(0.8944, 0.9056)	(0.8944, 0.9053)	(0.8945, 0.9056)
	ϕ	(6.4401, 14.4326)	(9.3219, 10.5006)	(9.3013, 10.7879)	(9.3337, 10.5242)
	ʌ	(3.8683, 4.1311)	(3.8781, 4.1392)	(3.8845, 4.1290)	(3.8748, 4.1379)
	τ	(0.7346, 1.2987)	(0.8916, 1.8167)	(0.9131, 1.8326)	(0.9165, 1.8421)

Table 6: Confidence and HPD Interval of parameters at ρ= 0.95

DF	P	MLE	SELF	ALF	PLF
n=10	ρ	(0.9433, 0.9548)	(0.9458, 0.9533)	(0.9458, 0.9534)	(0.9458, 0.9533)
	ϕ	(-0.1648, 20.2269)	(9.6106, 10.3750)	(9.6194, 10.3790)	And (9.6476, 10.4156)
	ʌ	(3.8541, 4.1469)	(3.8761, 4.1587)	(3.8751, 4.1574)	(3.8762, 4.1589)
	τ	(0.6000, 3.1623)	(0.8913, 1.7248)	(0.8784, 1.7101)	(0.9030, 1.7428)
n=20	ρ	(0.9445, 0.9555)	(0.9467, 0.9530)	(0.9466, 0.9530)	(0.9467, 0.9530)
	ϕ	(1.0599, 20.7525)	(9.6823, 10.3934)	(9.6470, 10.3599)	(9.6979, 10.4146)
	ʌ	(3.8589, 4.1375)	(3.8689, 4.1304)	(3.8684, 4.1304)	(3.8690, 4.1305)
	τ	(0.7162, 2.2361)	(1.0655, 1.9383)	(0.9679, 1.8446)	(1.0758, 1.9575)
n=30	ρ	(0.9436, 0.9554)	(0.9463, 0.9532)	(0.9463, 0.9532)	(0.9463, 0.9532)
	ϕ	(-0.9762, 18.6275)	(9.6148, 10.3260)	(9.6753, 10.3798)	(9.7090, 10.4194)
	ʌ	(3.8517, 4.1359)	(3.8538, 4.1247)	(3.8530, 4.1241)	(3.8540, 4.1247)
	τ	(0.6704, 1.3158)	(1.0659, 1.8927)	(1.0621, 1.8852)	(1.0848, 1.9170)
n=40	ρ	(0.9438, 0.9548)	(0.9467, 0.9533)	(0.9467, 0.9533)	(0.9467, 0.9533)
	ϕ	(0.2021, 20.1662)	(9.6849, 10.3413)	(9.6781, 10.3276)	(9.6982, 10.3449)
	ʌ	(3.8704, 4.1316)	(3.8798, 4.1251)	(3.8790, 4.1240)	(3.8799, 4.1257)
	τ	(0.7083, 1.3387)	(1.0782, 1.9353)	(1.0708, 1.9196)	(1.0885, 1.9549)
n=50	ρ	(0.9439, 0.9552)	(0.9467, 0.9531)	(0.9467, 0.9531)	(0.9467, 0.9531)
	ϕ	(-0.4056, 18.8985)	(9.6584, 10.3010)	(9.6541, 10.3112)	(9.6789, 10.3160)
	ʌ	(3.8792, 4.1360)	(3.8859, 4.1266)	(3.8862, 4.1278)	(3.8859, 4.1268)
	τ	(0.7122, 1.3193)	(1.1041, 1.9298)	(1.0925, 1.9123)	(1.1097, 1.9436)

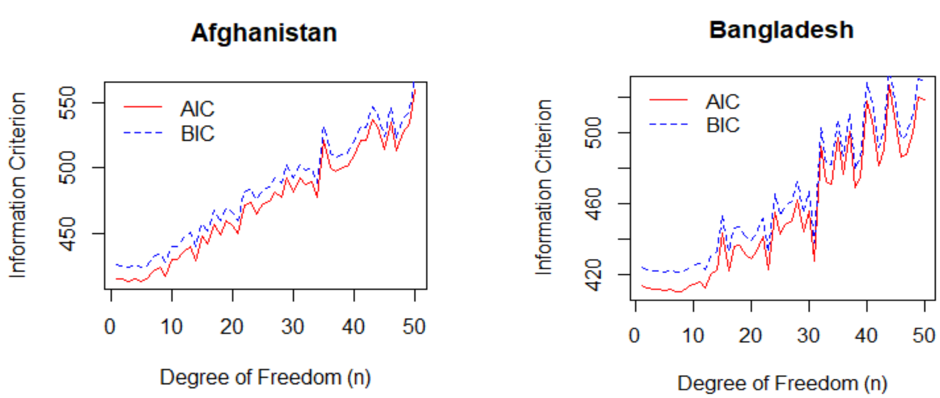
Table 7: Confidence and HPD Interval of parameters at ρ= 0.99

DF	P	MLE	SELF	ALF	PLF
n=10	ρ	(0.9875, 0.9926)	(0.9873, 0.9934)	(0.9883, 0.9944)	(0.9885, 0.9954)
	ϕ	(-25.4935, 50.7221)	(9.9119, 10.0669)	(9.9181, 10.0843)	(9.9519, 10.1148)
	ʌ	(3.8874, 4.1400)	(3.4578, 4.1449)	(3.8875, 4.1448)	(3.8878, 4.1559)
	τ	(0.6335, 3.1623)	(0.8619, 1.7142)	(0.8557, 1.7010)	(0.8731, 1.7303)
n=20	ρ	(0.9864, 0.9925)	(0.9874, 0.9915)	(0.9885, 0.9925)	(0.9875, 0.9945)
	ϕ	(-11.1359, 45.8047)	(9.9253, 10.0775)	(9.9168, 10.0803)	(9.9558, 10.1107)
	ʌ	(3.8666, 4.1169)	(3.8657, 4.1241)	(3.8666, 4.1143)	(3.8667, 4.1141)
	τ	(0.7844, 2.2361)	(1.0700, 1.8692)	(1.0652, 1.8576)	(1.0805, 1.8883)
n=30	ρ	(0.9871, 0.9919)	(0.9881, 0.9932)	(0.9883, 0.9941)	(0.9885, 0.9953)
	ϕ	(-15.5651, 44.1145)	(9.9293, 10.0722)	(9.9243, 10.0742)	(9.9521, 10.0974)
	ʌ	(3.8839, 4.1211)	(3.8841, 4.1311)	(3.8838, 4.1221)	(3.8840, 4.1233)
	τ	(0.7936, 1.8257)	(1.1236, 1.9828)	(1.1025, 1.9549)	(1.1341, 2.0015)
n=40	ρ	(0.9866, 0.9927)	(0.9872, 0.9943)	(0.9885, 0.9913)	(0.9884, 0.9943)

	ϕ	(-23.3936, 39.3858)	(9.9308, 10.0711)	(9.9293, 10.0781)	(9.9578, 10.1042)
	Λ	(3.8779, 4.1072)	(3.8507, 4.1022)	(3.8850, 4.1058)	(3.8807, 4.1213)
	τ	(0.7910, 1.5811)	(1.0909, 1.9627)	(1.0855, 1.9498)	(1.1017, 1.9806)
n=50	ρ	(0.9876, 0.9922)	(0.9876, 0.9914)	(0.9867, 0.9924)	(0.9887, 0.9955)
	ϕ	(-12.1334, 38.8478)	(9.9172, 10.0683)	(9.9279, 10.0882)	(9.9507, 10.1027)
	Λ	(3.8845, 4.1182)	(3.8846, 4.1127)	(3.8855, 4.1130)	(3.8846, 4.1227)
	τ	(0.7912, 1.4156)	(1.1041, 1.9819)	(1.1015, 1.9758)	(1.1142, 2.0013)

Empirical Study

In current trend, economy of a country is majorly measured by import and export of goods and commodities. This may be influence of the currency of a nation, mainly developing countries. India is one of the developing countries which also affect the portfolio currency issues. Hence, in this paper, we have used a real effective exchange rate (REER) for the South Asian Association for Regional Cooperation (SAARC) countries. Because REER contains information about the country currency which compare with other trading countries. SAARC is a regional organization of South Asia countries Afghanistan, Bangladesh, Bhutan, India, Nepal, Maldives, Pakistan and Sri Lanka. The series is recorded monthly from January 2009 to May 2017. For the proper analysis, we have considered all SAARC countries of REER series except Maldives due to unavailability of sufficient observations. Here, REER of India is considered as an observed time series and other series of SAARC countries as a covariate. First, we find out the appropriate degree of freedom for a particular covariate using information criterion. Figure 1 represents the AIC and BIC plot of SAARC countries with different degrees of freedom.



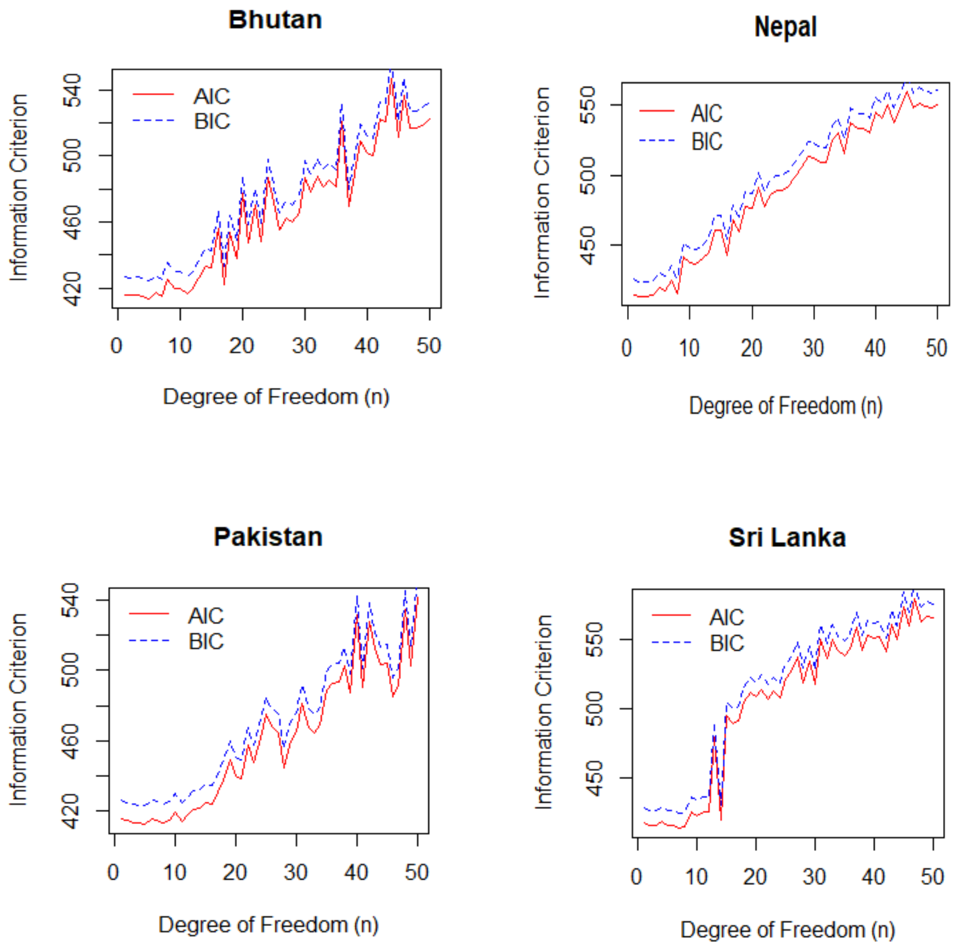


Figure 1: Information Criterion plot of SEERC countries

Observing Figure 1, we record the minimum AIC and BIC values of respective model in Table 8. Table 8 also observes that Bayes factor (B10) is comparatively high in each country when covariate series is included. Therefore, B10 strongly favours that the series rejects the null hypothesis, i.e., series is stationary. Once obtained the best suitable model with each covariate, estimation of parameters are carried out under different estimation methods where Indian REER is response series. The estimated values of MLE and Bayesian estimators with HPD interval are reported in Table 9.

Table 8: Selection of Best Suitable Model

DF	Countries	Min. AIC	Min. BIC	B_{10}
5	Afghanistan	413.2639	423.8029	135.5360
8	Bangladesh	410.2569	420.7958	213.2329
5	Bhutan	413.4703	424.0092	117.0011
3	Nepal	413.6300	424.1689	47.6660
5	Pakistan	412.5019	423.0408	143.0281
7	Sri Lanka	413.8527	424.3916	194.2551

Table 9: Estimates and Confidence Interval for Selected Models

DF	Countries	P	MLE	SELF	ALF	PLF	CI
5	Afghanistan	ρ	0.9510	0.9557	0.9588	0.9558	(0.9484, 0.9595)
		ϕ	48.5886	69.0581	73.5515	69.7209	(48.5886, 73.5515)
		λ	0.0282	0.0179	0.0150	0.0186	(0.0148, 0.0283)
		τ	0.3466	0.8892	1.0224	0.9725	(0.0762, 1.2999)
8	Bangladesh	ρ	0.9212	0.9296	0.9301	0.9296	(0.9163, 0.9372)
		ϕ	64.6240	79.1317	79.1281	79.1609	(76.2851, 81.5435)
		λ	0.0290	0.0183	0.0182	0.0184	(0.0156, 0.0222)
		τ	0.3573	0.1780	0.1769	0.1797	(0.1342, 0.2303)
5	Bhutan	ρ	0.9768	0.9753	0.9754	0.9753	(0.9744, 0.9766)
		ϕ	60.0033	80.8361	79.6659	80.9239	(79.6659, 90.4162)
		λ	0.0169	0.0115	0.0119	0.0116	(0.0087, 0.0119)
		τ	0.3439	0.2457	0.2458	0.2488	(0.1678, 0.3219)
3	Nepal	ρ	0.8543	0.8584	0.8593	0.8585	(0.8414, 0.8765)
		ϕ	30.5513	46.7379	44.6911	46.8590	(44.6911, 53.0181)
		λ	0.1051	0.0816	0.0830	0.0821	(0.0634, 0.0939)
		τ	0.3564	0.0640	0.0614	0.0659	(0.0413, 0.0901)
5	Pakistan	ρ	0.9388	0.9545	0.9566	0.9546	(0.9415, 0.9608)
		ϕ	58.5080	83.2600	84.0935	83.3036	(81.5354, 84.0935)
		λ	0.0298	0.0127	0.0119	0.0130	(0.0118, 0.0157)
		τ	0.3478	0.1744	0.1743	0.1766	(0.1238, 0.2283)
7	Sri Lanka	ρ	0.9451	0.9484	0.9491	0.9484	(0.9416, 0.9503)
		ϕ	62.6289	82.8046	83.1799	82.8225	(82.5800, 83.8421)
		λ	0.0225	0.0124	0.0123	0.0125	(0.0117, 0.0137)
		τ	0.3297	0.1569	0.1563	0.1586	(0.1124, 0.1992)

Conclusion

Present work investigated Bayesian inference of covariate autoregressive (C-AR) model under the condition that error is spherically symmetrically distributed. Under Bayesianmethodology, estimators of the model parameters are obtained by conditional posterior distribution. Further, the posterior probability is used for testing the unit root hypothesis. In the real application, we recorded that series is stationary for all SAARC countries. However, minimum AIC and BIC are recorded when degree of freedom of t-distribution is less than 10.This model may be further extended for panel and vector autoregressive model.

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