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Construction of Three Associate PBIB Designs using K - Maps

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A new series of s ($s \geq 3$) associate class Partially Balanced Incomplete Block (PBIB) designs have been constructed by applying hamming distance on Karnaugh maps. In this series, every character is expressed in binary set and calculate the weight, then hamming distance for each literal of k – map. We considers' different factors ($s \geq 3$) simultaneously and each factor has two levels, it means each factor is introduced in primal as well as dual form.
The correspondence among the factors with respect to their levels in all possible combinations is considered for the purpose of constructing new Partially Balanced Incomplete Block Designs. Association scheme, construction methodology, efficiencies of the series of designsalong with illustration of construction is also given in this paper.

Keywords: Partially balanced incomplete block designs, K – maps, Hamming distance, Group, Primal, and Dual.

1. Introduction

In experimental designs, the homogeneity is decreased among the experimental units in a block when block size increases and it reduces the efficiency of the given design. This problem can be solved by using partially balanced Incomplete block (PBIB) designs which are quiet useful in such type of situations and remain highly efficient. As we know every PBIB design has an association scheme.

In the available literature, Bose and Nair (1939) introduced the concept of PBIB design. Bose and Shimamoto (1952) introduced the classification and analysis of two associate cyclic designs. Then Tharthare (1963) and Ogasawara (1965) introduced right angular association scheme and Tm association scheme respectively for four and m associate class PBIB designs.

Similarly, the construction of rectangular designs, generating the elements of lattice by using binary relation, GD designs and combinatorial structure, Pseudo new modified Latin square type, rectangular and singular GD designs were introduced by Sinha et.al. (2002), Berry & Sigayret (2004), Sinha & Kageyama (2006), Garg (2010), Rezgui & Gheribi (2014) respectively.

Recently, Sharma & Garg (2017, 2018) introduced construction methodologies of new PBIB designs having higher associate classes by using some sets of initial blocks and Youden – m squares.

After reviewing the available literature, a series of PBIB designs having s($s \geq 3$) associate class association scheme has been generated by using K – maps including the concept of hamming distance by considering all possible binary combinations of 's' different factors according to their level under study. Newly constructed designs are quite efficient and useful from practical point of view.

1.1 Series of three associate PBIB designs

Construction of new PBIB designs is based on the theory of Karnaugh Map Method. As we know that K – map is a graphical tool to simplify a logic equation or to convert a truth table to its corresponding logic circuit.

The correspondence of a Boolean expression of variables under study by taking possible fundamental products with existing literals and then calculating the hamming distance of these product reflect a three associate class PBIB designs with parameters

$v = 2s$, $k = s$, $b = 2^s - 2$, $r = b/2$, $n_1 = s-1$, $n_2 = s-1$, $n_3 = 1$ with $\lambda_1 = 2^{s-2}$, $\lambda_2 = 2^{s-2} - 1$ and $\lambda_3 = 0$

where $s \geq 3$ with the following P matrices:

$$P_1 = \begin{bmatrix} 0 & s-2 & 0 \\ s-2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} s-2 & 0 & 1 \\ 0 & s-2 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & s-1 & 0 \\ s-1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. Construction Methodology

Consider a finite number of characters say 's'. Every character is first expressed in binary form. It means that a sequence of characters is defined by the set B = {1,0} as each character has two levels say 'primal & dual'. Draw a K – map for 's' variables (characters) and the possible fundamental products (sequences) with 's' literals are 2^s and also called order of group B^s.

According to coding theory, number of 1's in a sequence (say $x \in B^s$) is called the weight of x.

Let '+' be the operator defined as

+	0	1
0	0	1
1	1	0

For any two variables x, y ϵ B^s, the hamming distance d(x, y) = x + y which is equal to number of positions in which x and y differ.

Now, calculate the hamming distance (1, 2,…, s) for all possible '$2^{s'}$ fundamental products (sequences) and distances are

d(1,2 ,…,s) $\geq$ 0.

Delete those sequences for which the hamming distance d(1,2 ,…,s) = 0 which is only for sequence whose weight is either '0' or 's'.

The remaining living fundamental products (sequences) are 2^s -2 as one purely primal sequence and one purely dual sequence of Boolean expression has distance zero.

Treat 2^s -2 sequences of K map of 's' literals as blocks. Primal and dual form of each variable be treated as experimental units (treatments) in each block and are '2s' in counting, finite occurrence of variables both primal and dual in each block treated as block size and is 's' in each sequence. Each level of every variable reflects (2^s-2)/ 2 in K – map product cells give the replication of treatments.

In this way, the above correspondence with respect to their levels in the form of PBIB design, we say

a) Primal and dual of 's' variables be treated as treatments.

b) Non zero d (1, 2,…, s) sequences of K map be treated as blocks.

c) Number of literals in a sequence of K – map represent blocks size.

d) Number of times a variable repeated in all the living sequences at the same position represents the replication of a treatment.

Hence, we get a three associate class PBIB design with parameters

v = 2s, k = s, b = 2^s-2, r= b /2, n_1=s-1, n_2 = s-1 , n_3 =1, $\lambda_1 = 2^{s-2}$, $\lambda_2 = 2^{s-2} - 1$ and $\lambda_3 = 0$

where s $\geq$ 3.

2.1 Association Scheme

As above, parameters of PBIB designs are based on 's' (s $\geq$3).

Let x,y,z,… be the variables expressed in primal form with position defined as 1's and x', y' , z',… be the variables expressed in dual form with position defined as 0's treated as treatments in 2^s-2 blocks (sequences of k – map) for 's' variables and are '2s' in total.

Now, consider a particular variable say 'x' in primal form.

Treatment pairing of primal 'x' with the duals of other variables which occur 2^{s-2} times represents the 1st associates of treatment 'x' and are s-1 in number.

Treatment pairs of primal 'x' with the primal of other variables which occurs 2^{s-2} -1 times represents the 2nd associate of treatment 'x' and are s-1 in number.

Treatment pair of primal ‘x’ and its dual not denoted by ‘x’ occurs in any block is called 3rd associate of ‘x’ with $n_3 = 1$.

In other words, total number of treatments (v) are ‘2s’,‘s’ in primal &‘s’ in dual form in 2^s-2 blocks (b) with block size (k) is ‘s’.

Treatment pairs like (primal of one, dual of other) and vice- versa are first associates to each other.

Treatment pairs like (primal of one, primal of other) and (dual of one, dual of other) are second associates to each other.

Primal and dual of same treatment do not occur in any block and are third associates of each other.

2.2 Illustration

Consider K – map for four variables (s=4), each variable has two levels primal and dual indicated by 1’s and 0’s respectively. Suppose the four variables in primal form say x, y, z, t and in dual form say x', y', z' and t'. Karnaugh (K) – map corresponding to Boolean expression f(x, y, z, t) of four variables given below

	z t	z t'	z't'	z't
x y	1111	1110	1100	1101
x y'	1011	1010	1000	1001
x'y'	0011	0010	0000	0001
x'y	0111	0110	0100	0101

The 2^4= 16 possible fundamental products (sequences) with four literals are

xyzt	xyzt'	xyz't'	xyz't
xy'zt	xy'zt'	xy'z't'	xy'z't
x'y'zt	x'y'zt'	x'y'z't'	x'y'z't
x'yzt	x'yzt'	x'yz't'	x'yz't

Weight and Distance of some literals are calculated as

Sequence	operation	weight	Distance
xyzt	1+1+1+1	4	0
x'y'z't'	0+0+0+0	0	0
xyzt'	1+1+1+1	3	1
x'yz't	0+1+0+1	2	1

Clearly, d(xyzt) = d(x'y'z't') = 0 and delete these two sequences (product cells of K – map).

Numerically, we can represent

x	y	z	t'	x'	y'	z'	t'
1	2	3	4	5	6	7	8

and sequences are

(1,2,3,8)	(1,2,7,8)	(1,2,7,4)	(1,6,3,4)
(1,6,3,8)	(1,6,7,8)	(1,6,7,4)	(5,6,3,4)
(5,6,3,8)	(5,6,7,4)	(5,2,3,4)	(5,2,3,8)
(5,2,7,8)	(5,2,7,4)	**(1,2,3,4)**	**(5,6,7,8)**

As we delete the sequences (1,2,3,4) & (5,6,7,8) due to zero hamming distance and the remaining sequence represents a PBIB design with parameters

$v = 8, b = 14, k = 4, r = 7$

Select a particular treatment '1', which is a variable in primal form,

Treatment pairs (1,6) (1,7) and (1,8) occur 4 times in the generated blocks as well as treatments 6 ,7 and 8 which are dual form of the other variables are 1^{st} associates of '1'and $n_1 = 3$.

Similarly, treatment pairs (1, 2) (1, 3) and (1, 4) occur 3 times in the generated block and all are primal form of other variables are 2^{nd} associates of '1' with $n_2 = 3$.

Lastly, primal '1' and its dual '5' does not occur simultaneously in any block and is called 3^{rd} associate of '1'.

In this way, it represents a three associate PBIB design with parameters as

$v = 8, b = 14, k = 4, r = 7, \lambda_1 = 4 , \lambda_2 = 3 , \lambda_3 = 0 , n_1 = 4 , n_2 = 4$ and $n_3 = 1$.

2.3 P -Matrices are

$$P_1 = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Table- 1: Series of three associate class PBIB designs

s	k-map	v b r k	λ_i	n_i	E – factors
2	Three Variables	6 6 3 3	λ_1 =2 λ_2= 1 λ_2= 0	n_1 =2 n_2 =2 n_3 =1	E_1 = .851E_2 = .714 E_3 = .635 **E = .743**
3	Four Variables	8 14 7 4	λ_1 =4 λ_2= 3 λ_3 = 0	n_1 =3 n_2 =3 n_3 =1	E_1 = .875E_2 = .833E_3 = .745 **E = .836**
4	Five Variables	10 30 15 5	λ_1 =8 λ_2= 7 λ_3 = 0	n_1 =4 n_2 =4 n_3 =1	E_1 = .896E_2 = .880E_3 = .799 **E = .877**
5	Six Variables	12 62 31 6	λ_1 = 16 λ_2= 15 λ_3 = 0	n_1 =5 n_2 =5 n_3 =0	E_1 = .912E_2 = .906E_3 = .833 **E = .901**

3. Conclusion

In this paper, we have introduced a new association scheme and constructed three associate class PBIB designs by considering all possible fundamental products of 's' variables with binary representation between primal and dual form for each variable by drawing K – map and applying the concept of hamming distance. In newly constructed designs, number of blocks and rate of replication increases rapidly with finite number of treatments and block size, but the efficiencies of three kinds of comparisons as well as the average efficiency factor (A.E.F) of the constructed designs are at an appropriate level. Construction of new series of PBIB designs is based on the tool (K – map with elementary application of coding theory) of discrete structure, which has not been widely applied in available literature yet.

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