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The paper develops Bayesian estimators for the parameters and reliability measures of Nakagami distribution. Some new priors are introduced for the shape parameter of Nakagami distribution keeping in view the range of the parameter. Monte Carlo simulation study is conducted to compute the Bayes estimates along with their expected loss functions and corresponding Highest posterior density (HPD) credible intervals. Real life data set is also given for illustration purpose.

Keywords: Nakagami distribution, Bayesian estimators, HPD Credible intervals, Priors and Loss Functions.

1. Introduction

The Nakagami distribution was suggested by Nakagami (1960) for modelling the fading of radio signals and can be measured as a flexible lifetime distribution. It has many applications in the general area of communications engineering and also has been applied effectively in numerous fields. Kim and Latchman (2009) analysed the multimedia (MPEG-2 frame) data traffic over networks using Nakagami distribution. The Nakagami distribution is also used in biomedical fields, such as to model the time to existence of tumours and the presence of lung cancer. It has the applications in medical field, predicting readings to model the ultrasounds mainly in Echo (heart efficiency test). Shankar et al. (2005) and Tsui et al. (2006) used this distribution to model ultrasound data in medical imaging studies. Recently, Carcole and Sato (2009) have presented the utility of the Nakagami distribution for the modelling high-frequency seismogram envelopes. The distribution can be considered to model failure times of a variety of products (and electrical components) such as vacuum tubes, ball bearing and electrical insulation. This distribution is broadly applied in reliability theory and reliability engineering as well. Kumar et al. (2017) have discussed Nakagami distribution as a reliability model under progressive censoring.

In Non-Bayesian set up three different estimators for the shape parameter of the Nakagami distribution have been compared by Abdi and Kaveh (2000) using Monte Carlo simulation study.

Cheng and Beaulieu (2001) also derived maximum likelihood estimation of the shape parameter of the Nakagami distribution. Schwartz et al. (2013) have suggested improved maximum likelihood estimation of the parameter of Nakagami distribution and they have discussed some distributional properties also.

Ahmed et al. (2016) have discussed classical and Bayesian approach in estimation of scale parameter of Nakagami distribution wherein the shape parameter is assumed to be fixed. However in the Bayesian set up, the estimation of both the shape and scale parameter of Nakagami distribution still awaits the attention of the researchers. Bayesian estimation of shape parameter is also as much significant as of scale parameter which is taken up in the present work.

In the present work considering Nakagami distribution as a life time model, the Bayes estimators of both the shape and scale parameters are derived. Considering both shape and scale parameters to be unknown, Bayes estimators for the reliability function and hazard rate are also derived. Taking into consideration the specific nature of the range of shape parameter for Nakagami distribution, some new priors (both informative and non-informative) are introduced for the Bayesian estimation. Bayesian estimation is carried out for symmetric and asymmetric loss functions. Using simulation technique, highest posterior density (HPD) credible intervals for the Bayes estimates of the parameters, reliability function and hazard rate are also obtained.

The scheme of the paper is as follows: after Introduction, an outline related to the model with distributional properties and likelihood function is provided. Some concepts related to priors, loss functions and HPD credible intervals used in the study are described in the next section on Bayesian Estimation. The posterior distributions and estimation procedure is also discussed in that Section. Bayesian estimators are derived using informative and non-informative priors under squared error loss function (SELF) and generalised entropy loss function (GELF) in the same section. Under section of Simulation Study, simulation is carried out to find the Bayes estimates under different configurations of sample sizes. Highest posterior density (HPD) Credible intervals along with the width of intervals are also provided in the same section. The next section deals with a real life example followed by the brief discussion of the results while conclusion is given in the last section.

2. Model

The cumulative distribution function (cdf) of Nakagami distribution with shape parameter α and scale parameter β is given by

$$(x) = \frac{1}{\Gamma\alpha}\Gamma\left(\frac{\alpha}{\beta}x^2, \alpha\right) \quad ; \quad x > 0, \alpha \geq 0.5, \beta > 0,$$

where $\Gamma b = \int_0^\infty t^{b-1} e^{t} dt$ is the incomplete gamma function.

The corresponding probability density function (pdf) is

$$(x) = \frac{2}{\Gamma\alpha}\left(\frac{\alpha}{\beta}\right)^{\alpha} x^{2\alpha-1}\, exp\left(-\frac{\alpha}{\beta}x^2\right) \quad ; \quad x > 0, \alpha \geq 0.5, \beta > 0,$$

where $\Gamma b = \int_0^\infty t^{b-1}e^{t}dt$ is the complete gamma function. The plot of probability density function is shown in Figure 1 for different values of α and β.

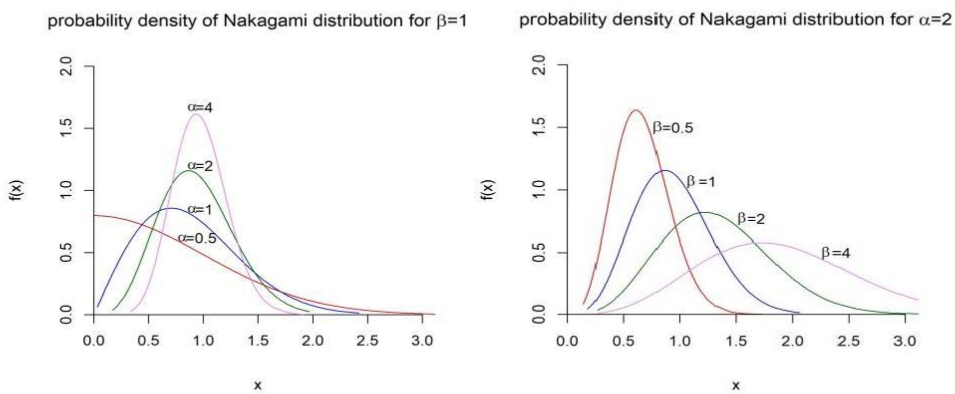

Figure 1. Probability density of Nakagami distribution

The reliability function of Nakagami distribution is given by

$$(x) = [X > x] = 1 - \frac{1}{\Gamma\alpha}\Gamma\left(\frac{\alpha}{\beta}x^2, \alpha\right) \quad ; \quad x > 0, \alpha \geq 0.5, \beta > 0,$$

and it is plotted in Figure 2 for different values of α and β.

The hazard rate of Nakagami distribution is given by

$$h(x) = \frac{f(x)}{R(x)} = \frac{\frac{2}{\Gamma\alpha}\left(\frac{\alpha}{\beta}\right)^{\alpha} x^{2\alpha-1}\, exp\left(-\frac{\alpha}{\beta}x^2\right)}{1-\frac{1}{\Gamma\alpha}\Gamma\left(\frac{\alpha}{\beta}x^2, \alpha\right)} \quad ; \quad x > 0, \alpha \geq 0.5, \beta > 0,$$

and it is plotted in Figure 3 for different values of α and β.

If α = 0.50, Nakagami distribution coincides with Half Normal distribution and if α = 1.0, Nakagami distribution is similar to Rayleigh distribution. If Y follows Gamma(k, β) then $\sqrt{Y}$ follows Nakagami distribution with shape parameter k and scale parameter $k\beta$. Moreover, if 2α is integer value and Z follows the Chi-Square distribution with 2α degree of freedom then $\sqrt{\frac{\beta}{2\alpha}Z}$ follows Nakagami distribution with shape parameter α and scale parameter β.

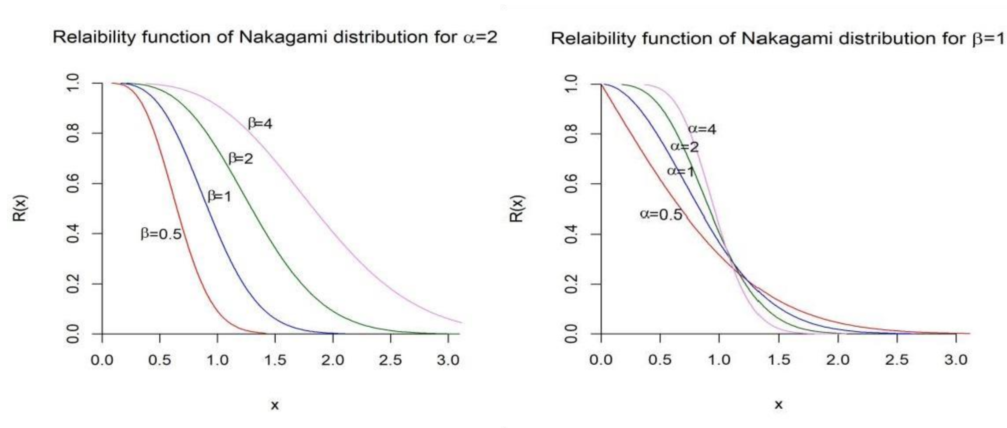

Figure 2. Reliability function of Nakagami distribution

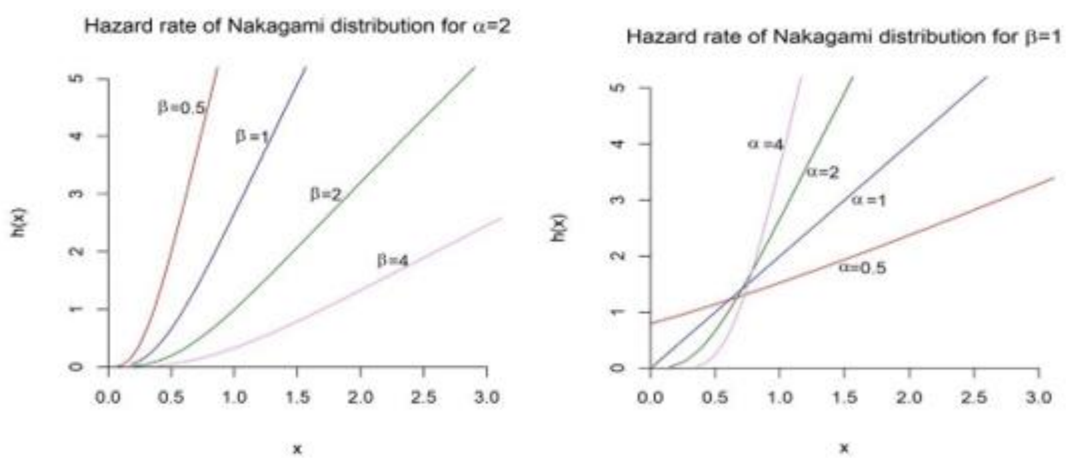

Figure 3. Hazard rate of Nakagami distribution

2.1 Likelihood Function

Let $(x1, x2, \dots, xn)$ be a random sample follows Nakagami distribution with shape parameter α and scale parameter β. The likelihood function is given by

$$L(\underline{x};\alpha,\beta) = \left(\frac{2}{\Gamma\alpha}\right)^{n} \left(\frac{\alpha}{\beta}\right)^{n\alpha} \prod_{i=1}^{n} x_i^{2\alpha-1}\, exp\left(-\frac{\alpha}{\beta}\sum_{i=1}^{n} x_i^2\right) \quad ; x_i > 0, \alpha \geq 0.5, \beta > 0, \tag{1}$$

where $\underline{x} = (x1, x2, \dots, xn)$.

2.2 Bayesian Estimation

In this section, Bayes estimators for shape and scale parameters, reliability function and hazard rate are derived using both informative and non-informative priors. Two types of loss functions have been used, one is symmetric loss function (squared error loss function) and the other one is asymmetric loss function (generalised entropy loss function). A brief outline of loss functions, priors and HPD credible intervals is given below:

2.3 Squared Error Loss Function (SELF)

The squared error loss function is a symmetric loss function and it is defined as $L(\hat{\xi}, \xi) \propto (\hat{\xi} - \xi)^2$ where $\hat{\xi}$ is the Bayes estimator of the unknown parameter ξ. The Bayes estimator of ξ under SELF is $\hat{\xi}_s = (\xi|\underline{x})$ and expected loss function under SELF is $E[L(\hat{\xi}, \xi)] = E[(\hat{\xi}- \xi)^2|x]$, where the expectation is taken with respect to the posterior density.

2.4 Generalized Entropy Loss Function (GELF)

SELF provides equal weights to over estimation and under estimation. However, in many situations over estimation is more serious than under estimations and vice versa. So, another valuable asymmetric loss function viz. GELF is used here to overcome this problem. GELF is a generalization of the entropy loss function defined by Calabria and Pulcini (1996).

This asymmetric loss function is defined as $L(\xi,\hat{\xi}) \propto \left(\frac{\hat{\xi}}{\xi}\right)^b - b\ln\left(\frac{\hat{\xi}}{\xi}\right) - 1$ where $b \neq 0$. The constant b decides the shape of the loss function. If $b > 0$ then over estimation is more severe than under estimation and vice-versa. Bayes estimator of ξ under GELF is $\hat{\xi}_g = [E(\xi^{-b}|x)]^{(-1/b)}$ and expected loss function is $E[L(\xi,\hat{\xi})] = E\left[\left(\frac{\hat{\xi}}{\xi}\right)^b - b\ln\left(\frac{\hat{\xi}}{\xi}\right) - 1|x\right]$, where the expectation is taken with respect to the posterior density.

2.5 Informative Priors

A new informative prior known as two parameter exponential distribution for the shape parameter α is introduced.

$$g_1(\alpha) = \frac{1}{\lambda} exp\left(-\frac{\alpha-0.5}{\lambda}\right) \quad ; \; \alpha \geq 0.5, \lambda > 0.$$

Here the location parameter of two parameter exponential prior is assumed to be 0.5 keeping in view the range of the parameter α ($\geq$ 0.5) in the underlying Nakagami distribution.

For the scale parameter β the informative prior is taken as gamma prior

$$g_2(\beta) = \frac{\gamma^\theta}{\Gamma\theta} \beta^{-\theta-1} exp\left(-\frac{\gamma}{\beta}\right) \quad ; \; \beta > 0; \; \theta, \gamma > 0.$$

Assuming independence of both these priors, the joint prior distribution of α and β is

$$g_3(\alpha,\beta) = \frac{\gamma^\theta}{\lambda\Gamma\theta} \beta^{-\theta-1} exp\left(-\frac{\gamma}{\beta} - \frac{\alpha-0.5}{\lambda}\right) \quad ; \alpha \geq 0.5, \beta > 0; \theta, \lambda > 0, \gamma > 0. \tag{2}$$

2.6 Non-informative Priors

The non-informative prior distributions for the shape parameter α and scale parameter β are

$$g_4(\alpha) = \frac{1}{\alpha} \quad ; \alpha \geq 0.5.$$

$$g_5(\beta) = \frac{1}{\beta} \quad ; \beta > 0.$$

The joint prior distribution of α and β (assuming independence) is

$$g_6(\alpha, \beta) = \frac{1}{\beta\alpha} \quad ; \alpha \geq 0.5, \beta > 0. \tag{3}$$

2.7 Highest posterior density (HPD) Credible Intervals

Chen and Shao (1999) introduced the algorithm to find the HPD credible intervals. $100(1-\gamma)\%$ HPD credible interval is that $100(1-\gamma)\%$ credible interval which is having smallest width among all possible $100(1-\gamma)\%$ credible intervals.

Once the posterior sample is generated for αj ($j = 1,2, \ldots, (N - N0)$), then $\alpha(1) \leq \alpha(2) \leq \cdots \leq_{(N-N0)}$ denotes the ordered values of $\alpha 1, \alpha 2, \ldots, \alpha_{(N-N0)}$. The $100(1-\gamma)\%$ HPD interval for α is defined by $(\alpha(j), \alpha_{(j+[(1-\gamma)(N-N0)])})$, where j is chosen such that

$$\alpha_{(j+[(1-\gamma)(N-N_0)])} - \alpha_{(j)} = \min_{1\leq j\leq M}\left(\alpha_{(j+[(1-\gamma)(N-N_0)])} - \alpha_{(j)}\right), \qquad j = 1,2, \ldots, (N - N_0),$$

where $[x]$ denotes the greatest integer of x.

2.8 Posterior distribution of α and Q using informative priors

The joint posterior distribution of α and β is

$$\pi_1(\alpha, \beta) = \frac{(x; \alpha, \beta) g_3(\alpha, \beta)}{\int_0^{\infty}\int_{0.5}^{\infty} (\underline{x}; \alpha, \beta) \ {}_3(\alpha, \beta)\, d\alpha\, d\beta}$$

$$\mathrm{a} \left(\frac{1}{\Gamma\alpha}\right)^n \left(\frac{\alpha}{\beta}\right)^{n\alpha} \prod_{i=1}^{n} x_i^{2\alpha} \beta^{-\theta-1} \exp\left(-\sum_{i=1} \left(\frac{\alpha}{\beta} x_i^2\right) - \frac{\gamma}{\beta} - \frac{\alpha - 0.5}{\lambda}\right) \tag{4}$$

The full conditional distributions of α and β from equation (4) are respectively

$$\pi_1(\alpha|\beta, \underline{x}) \ \mathrm{a} \ \left(\frac{1}{\Gamma\alpha}\right)^n \left(\frac{\alpha}{\beta}\right)^{n\alpha} \exp\left(-\alpha \left(\frac{\sum_{i=1}^{n} x_i^2}{\beta} + \frac{1}{\lambda}\right)\right) \prod_{i=1}^{n} x_i^{2\alpha}, \tag{5}$$

$$\pi_1(\beta|\alpha, \underline{x}) \ \mathrm{a} \ \left(\frac{1}{\beta}\right)^{+\theta+1} \exp\left(-\frac{1}{\beta}\left(\alpha \sum_{i=1}^{n} x_i^2 + \gamma\right)\right). \tag{6}$$

2.9 Posterior distribution of α and Q using non-informative priors

The joint posterior distribution of α and β is

$$\pi_2(\alpha,\beta) = \frac{(x;\alpha,\beta)g_6(\alpha,\beta)}{\int_0^{\infty}\int_{0.5}^{\infty}(\underline{x};\alpha,\beta)\ (\alpha,\beta)_6\ d\alpha\ d\beta}$$

$$\text{a}\ \frac{1}{\beta\alpha}\left(\frac{1}{\Gamma\alpha}\right)^n\left(\frac{\alpha}{\beta}\right)^{n\alpha}\prod_{i=1} x_i^{2\alpha}\ exp\left(-\frac{\alpha}{\beta}\sum_{i=1}^{n}\ ^2\right) \tag{7}$$

The full conditional distributions of α and β from equation (7) are respectively

$$\pi_2(\alpha|\beta,\underline{x})\ \text{a}\ \frac{1}{\alpha}\left(\frac{1}{\Gamma\alpha}\right)^n\left(\frac{\alpha}{\beta}\right)^{n\alpha} exp\left(-\frac{\alpha}{\beta}\sum_{i=1}^{n} x_i^2\right)\prod_{i=1}^{n} x_i^2, \tag{8}$$

$$\pi_2(\beta|\alpha,\underline{x})\ \text{a}\ \left(\frac{1}{\beta}\right)^{n\alpha+1} exp\left(-\frac{\alpha}{\beta}\sum_{i=1}^{n}\ ^2\right). \tag{9}$$

2.10 Estimation Procedure

Since, the joint posterior distribution of α, β in equations (4) and (7) cannot be obtained in an explicit form, the Markov Chain Monte Carlo (MCMC) technique is implemented to obtain the Bayes estimates and corresponding HPD credible intervals of α, β, (x) and $h(x)$.

The full posterior conditional distributions of α and β in equations (5), (6), (8) and (9) are not in known form. Hence to generate the random samples from the population with the conditional posterior distributions given by (5), (6), (8) and (9), the Metropolis-Hasting (M-H) algorithm considering the normal distribution as proposal density is used.

The M–H algorithm was established by Metropolis et al. (1953) and later extended by Hastings (1970). Gibbs sampler creates a sequence of samples from the full conditional probability distributions. Since the full posterior conditional distribution of each parameter depends on the other parameter, the Gibbs sampler is also adopted here.

The MCMC technique of M-H algorithm using Gibbs sampler to generate samples from (5) and (6) is follows:

(i) Start with the initial values (0), $\beta\ (0)$.

(ii) Set $k = 1$.

(iii) Generate $\alpha^{(k)}$ from $\pi 1(\alpha_{(k-1)}|\beta_{(k-1)}, \underline{x})$ using M-H algorithm with Normal transition kernel $(\alpha^{(k-1)}, e_1)$, where standard deviation e_1 can be chosen on the basis of few trials.

(iv) Similar way, Generate $^{(k)}$ from $\pi 1(\beta_{(k-1)}|\alpha_{(k)}, x)$ using M-H algorithm with Normal transition kernel $N(\beta^{(k-1)}, e_2)$, where standard deviation e_2 can be chosen on the basis of few trials.

(v) Set $k = k + 1$.

(vi) Repeat step (iii) to (v) N times and obtain $\alpha_1, \alpha_2, \dots, \alpha_N$ and $\beta_1, \beta_2, \dots, \beta_N$.

Then after burn-in and thinning the chain we get the samples $\alpha_1, \alpha_2, \ldots, \alpha_s$ and $\beta_1, \beta_2, \ldots, \beta_s$ from

$\alpha_1, \alpha_2, \ldots, \alpha_N$ and $\beta_1, \beta_2, \ldots, \beta_N$, respectively where $s < N$. Now, the Bayes estimators of α, β, (x) and $h(x)$ under SELF are

$$\hat{\alpha}_s = \frac{1}{s}\sum_{j=1}^{s} ,$$

$$\hat{\beta}_s = \frac{1}{s}\sum_{j=1}^{s}\beta ,$$

$$\hat{R}_s(x) = \frac{1}{s}\sum_{j=1}^{s}\left(1-\frac{1}{\Gamma\alpha_j}\Gamma\left(\frac{\alpha_j}{\beta_j}x^2,\alpha_j\right)\right),$$

$$\hat{h}_s(x) = \frac{1}{s}\sum_{j=1}^{s}\left[\frac{\frac{2}{\Gamma\alpha_j}\left(\frac{\alpha_j}{\beta_j}\right)^{\alpha_j}x^{2\alpha_j-1}\left(-\frac{\alpha_j}{\beta_j}x^2\right)}{1-\frac{1}{\Gamma\alpha_j}\Gamma\left(\frac{\alpha_j}{\beta_j}x^2,\alpha_j\right)}\right].$$

The Bayes estimators of α, β, (x) and $h(x)$ under GELF are

$$\hat{\alpha}_g = \left[\frac{1}{s}\sum_{j=1}^{s}\alpha^{-b}\right]^{(-1/b)},$$

$$\hat{\beta}_g = \left[\frac{1}{s}\sum_{j=1}^{s}\beta^{-b}\right]^{(-1/b)},$$

$$\hat{R}_g(x) = \left[\frac{1}{s}\sum_{j=1}^{s}\left(1-\frac{1}{\Gamma\alpha_j}\Gamma\left(\frac{\alpha_j}{\beta_j}x^2,\alpha_j\right)\right)^{-b}\right]^{(-1/b)},$$

$$\hat{h}(x) = \left[\frac{1}{s}\sum_{j=1}^{s}\left(\frac{\frac{2}{\Gamma\alpha_j}\left(\frac{\alpha_j}{\beta_j}\right)^{\alpha_j}x^{2\alpha_j-1}\left(-\frac{\alpha_j}{\beta_j}x^2\right)}{1-\frac{1}{\Gamma\alpha_j}\Gamma\left(\frac{\alpha_j}{\beta_j}x^2,\alpha_j\right)}\right)^{-b}\right]^{(-1/b)}.$$

Same procedure is followed to generate the random samples from equations (8) and (9) as well to obtain the Bayes estimators of α, β, (x) and $h(x)$.

3. Simulation Study

In this section, Monte Carlo simulation study is conducted for Bayes estimates of α, β, (x) and $h(x)$ of Nakagami distribution as the estimators cannot be obtained in a simplified form theoretically. The Bayes estimates are derived for both informative as well as non-informative priors under SELF and GELF. This simulation study is based on 3,000 replications i.e. the procedure described in above section is replicated

3000 times. For GELF the value of b is taken 0.5 (for over estimation) and −0.5 (for under estimation).

For simulation, true value of the parameter β is considered as 0.5 with corresponding informative hyper-parameters γ = 1, θ = 3. The true values of the parameter α are considered as α = 0.75, 1.0, 1.5 with corresponding informative hyper-parameters λ = 0.25, 0.5, 1.0, respectively. Three cases of reliability are considered as small, medium and large values. As reliability is the probability that a manufacturer will operate without failure for a given period of time under some given operating conditions. So, to estimate reliability this time "t" has to be fixed and it can be interpreted as a specific value of the random variable X. For the Bayes estimates of (t) and $h(t)$, three different configuration of time t have been considered as t = 0.37, 0.58, 0.80. The true values of (t) and $h(t)$ for different configurations are given in Table 1.

Table 1. True values for α, t, (t) and $h(t)$ considered for simulation.

α	t	$R(t)$	$h(t)$
0.75	0.37	0.695480	1.575609
	0.58	0.4693646	2.167072
	0.80	0.2723344	2.781855
1	0.37	0.7604842	1.480000
	0.58	0.5102778	2.320000
	0.80	0.2780373	3.200000
1.5	0.37	0.8443419	1.260919
	0.58	0.5685965	2.528883
	0.80	0.2792676	3.939861

Three different combinations of sample sizes are considered (small, medium and large viz. 20, 30 and 50). For simulation study MCMC technique of M-H algorithm using Gibbs sampler is used and described in estimation procedure earlier. For which a chain of 15,000 observations is generated using M-H algorithm with Gibbs sampler. From cumulative mean plots, the burn-in period is 5,000 i.e. first 5,000 observations are discarded from 15,000 observations as burn–in period. The lag value is 20 from autocorrelation plot i.e. every 20th observation is considered from the remaining 10,000 observations. Trace plots, cumulative mean plots, autocorrelation plots and density plots are shown in next section for a real data set.

The Bayes estimates, expected loss functions and HPD credible intervals with their length are provided for α and β using informative and non-informative priors under SELF and GELF in Tables [2-4]. The Bayes estimates, expected loss function and HPD credible intervals with their length using informative and non-informative priors under SELF and GELF for reliability function (t) are provided in Tables [5-7]. The Bayes estimates, expected loss function and HPD credible intervals with their length are provided for hazard rate $h(t)$ using informative and non-informative priors under SELF and GELF in Tables [8-10].

Table 2. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for α and β under informative and non-informative priors for α = 0.75 and β = 0.5.

		n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior	α	20	0.745410 (0.002106)	0.745402 (0.000473)	0.745386 (0.000475)	(0.747454, 0.766859) 0.019405
		30	0.753895 (0.001517)	0.753886 (0.000334)	0.753869 (0.000331)	(0.735416, 0.754550) 0.019134
		50	0.751961 (0.000385)	0.751953 (0.000085)	0.751937 (0.000083)	(0.729910, 0.748635) 0.018725
	β	20	0.504615 (0.002130)	0.504590 (0.001042)	0.504542 (0.001024)	(0.492916, 0.520274) 0.027358
		30	0.504052 (0.001642)	0.504025 (0.000803)	0.503972 (0.000784)	(0.481229, 0.508457) 0.027228
		50	0.500743 (0.000055)	0.500720 (0.000026)	0.500673 (0.000023)	(0.487255, 0.512720) 0.025465
Non-Informative Prior	α	20	0.739749 (0.010509)	0.739741 (0.002377)	0.739725 (0.002373)	(0.743787, 0.763855) 0.020068
		30	0.756525 (0.004258)	0.756517 (0.000934)	0.756500 (0.000932)	(0.742470, 0.761979) 0.019509
		50	0.745088 (0.002412)	0.745080 (0.000542)	0.745064 (0.000544)	(0.735015, 0.754227) 0.019212
	β	20	0.506182 (0.003822)	0.506159 (0.001870)	0.506113 (0.001849)	(0.480130, 0.509629) 0.029499
		30	0.494571 (0.002947)	0.494542 (0.001508)	0.494485 (0.001535)	(0.489414, 0.516692) 0.027278
		50	0.494733 (0.002774)	0.494708 (0.001418)	0.494658 (0.001440)	(0.494805, 0.520971) 0.026166

Table 3. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for α and β under informative and non-informative priors for α = 1 and β = 0.5.

		n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior	α	20	0.994932 (0.002568)	0.994926 (0.000324)	0.994912 (0.000325)	(0.985149, 1.005016) 0.019867
		30	0.995902 (0.001679)	0.995896 (0.000212)	0.995884 (0.000213)	(0.987626, 1.007095) 0.019469
		50	0.998305 (0.000287)	0.998299 (0.000036)	0.998287 (0.000037)	(0.985634, 1.004057) 0.18423
	β	20	0.508216 (0.006750)	0.508190 (0.003291)	0.508140 (0.003269)	(0.494112, 0.521884) 0.027772
		30	0.494887 (0.002615)	0.494863 (0.001336)	0.494815 (0.001356)	(0.479785, 0.506974) 0.027189
		50	0.503142 (0.000987)	0.503118 (0.000483)	0.503070 (0.000469)	(0.489810, 0.516047) 0.026237
	α	20	0.992931 (0.004997)	0.992924 (0.000631)	0.992911 (0.000632)	(0.983251, 1.003616) 0.020365

		30	0.993085 (0.004782)	0.993078 (0.000604)	0.993065 (0.000605)	(0.982932, 1.003238) 0.020306
		50	1.006250 (0.003906)	1.006244 (0.000484)	1.006232 (0.000483)	(0.998383, 1.017534) 0.019151
	β	20	0.511643 (0.013556)	0.511620 (0.006573)	0.511575 (0.006572)	(0.494146, 0.522068) 0.027922
		30	0.508260 (0.006822)	0.508234 (0.003326)	0.508184 (0.003304)	(0.492631, 0.519776) 0.027145
		50	0.506443 (0.004152)	0.506418 (0.002029)	0.506368 (0.002006)	(0.499313, 0.525314) 0.026001

It is clear from simulation study (Table 2-10) that as sample size increases the expected loss function decreases. The expected loss function using informative prior is less than the expected loss function using non-informative priors. The length of 95% HPD credible intervals decreases as sample size increases for the Bayes estimates of α, β, (t) and $h(t)$ of Nakagami distribution. The length of 95% HPD credible intervals for the Bayes estimates of α, β, (t) and $h(t)$ using informative priors is smaller than using non-informative priors.

3.1 Real Data Example

A real data set from an accelerated life test of 59 conductors is considered here to illustrate the estimation procedure adopted in the previous sections. This data set given below originally reported by Schafft et al. (1987) and also mentioned in Lawless ((2003) p. 267). This data set represents the failure time (in hours) of 59 conductors of 400-micrometer length. All 59 specimens ran to failure and tested under the same temperature and current density.

6.545, 9.289, 7.543, 6.956, 6.492, 5.459, 8.120, 4.706, 8.687, 2.997, 8.591, 6.129, 11.038, 5.381, 6.958, 4.288, 6.522, 4.137, 7.459, 7.495, 6.573, 6.538, 5.589, 6.087, 5.807, 6.725, 8.532, 9.663, 6.369, 7.024, 8.336, 9.218, 7.945, 6.869, 6.352, 4.700, 6.948, 9.254, 5.009, 7.489, 7.398, 6.033, 10.092, 7.496, 4.531, 7.974, 8.799, 7.683, 7.224, 7.365, 6.923, 5.640, 5.434, 7.937, 6.515, 6.476, 6.071, 10.491, 5.923.

Table 4. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for α and β under informative and non-informative priors for α = 1.5 and β = 0.5.

Informative Prior		n	SELF	GELF		HPD interval withlength
				$b=-0.5$	$b=0.5$	
	α	20	1.496056 (0.001556)	1.496052 (0.000087)	1.496043 (0.000087)	(1.488404, 1.507393) 0.018989
		30	1.502938 (0.000863)	1.502934 (0.000048)	1.502925 (0.000047)	(1.487241, 1.506033) 0.018792
		50	1.497216 (0.000775)	1.499892 (0.000019)	1.499883 (0.000024)	(1.493551, 1.511886) 0.018335
	β	20	0.504231 (0.001790)	0.504209 (0.000877)	0.504164 (0.000861)	(0.505586, 0.533052) 0.027466

		30	0.503483 0.001213	0.503459 (0.000594)	0.503413 (0.000579)	(0.492054, 0.518206) 0.026152
		50	0.503226 (0.001040)	0.503201 (0.000509)	0.503152 (0.000494)	(0.489498, 0.515600) 0.026102
Non-Informative Prior	α	20	1.504927 (0.002427)	1.504922 (0.000134)	1.504913 (0.000134)	(1.493124, 1.513679) 0.020555
		30	1.503725 (0.001388)	1.503721 (0.000077)	1.503712 (0.000076)	(1.489696, 1.509590) 0.019894
		50	1.499896 (0.000001)	1.497212 (0.000043)	1.497204 (0.000044)	(1.495534, 1.515134) 0.01960
	β	20	0.519458 (0.037863)	0.519433 (0.018058)	0.519382 (0.018194)	(0.489164, 0.516761) 0.027597
		30	0.485468 (0.021119)	0.485442 (0.010967)	0.485392 (0.010936)	(0.480083, 0.507023) 0.02694
		50	0.493821 (0.003818)	0.493795 (0.001953)	0.493744 (0.001978)	(0.472079, 0.498640) 0.026561

The data set is shown to follow the Nakagami distribution as Kolmogorov-Smirnov (K-S) test statistic value is 0.062566 and the corresponding p-value is 0.9639. The empirical cdf and fitted cdf (x) evaluated for $\hat{\alpha}$ = 4.8336 and $\hat{\beta}$ = 51.2823 (maximum likelihood estimates of α and β). The expression for K-S test statistic $Dn = Supx|F(x) - F(x)|$ is used to compute the empirical cdf in which $Fn(x)$ represents the empirical cdf and $F(x)$ represents the distribution function for Nakagami distribution. In Figure 4, the plot of empirical cdf and fitted cdf does show that it follows Nakagami distribution.

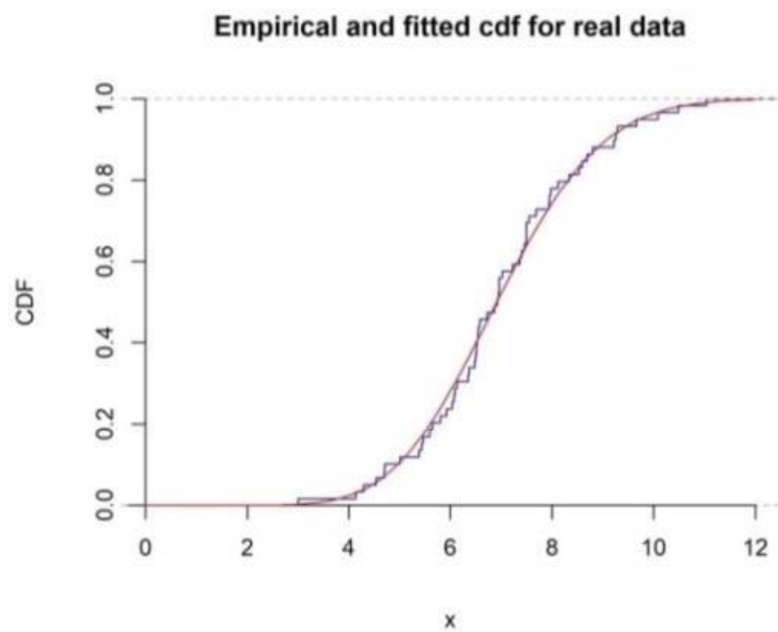

Figure 4: Empirical and fitted cdf of real data.

Using Akaike information criterion (AIC), Bayesian information criterion (BIC), quantile–quantile (Q–Q) plots and K–S test, Kumar et al. (2017) showed that this given data set best fit to Nakagami distribution for the estimated $\hat{\alpha}$ = 4.8336 and $\hat{\beta}$ = 51.2823.

As no prior information is available to this real data set, the estimation is done only for non- informative prior (Table 11) and the true value used for estimation are α = 4.8336, β = 51.2823, t = 7, (t) = 0.4786612 and $h(t)$ = 0.5120702.

Table 5. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for (t) under informative and non-informative priors for α = 0.75 and β = 0.5.

	t	n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior b	0.8	20	0.274899 (0.001642)	0.274881 (0.001081)	0.274845 (0.001054)	(0.265783, 0.282296) 0.016513
		30	0.274922 (0.002130)	0.274906 (0.001103)	0.274874 (0.001079)	(0.267934, 0.284421) 0.016487
		50	0.272830 (0.000055)	0.272815 (0.000039)	0.272783 (0.000034)	(0.265181, 0.280650) 0.015469
	0.58	20	0.474989 (0.009698)	0.474979 (0.001764)	0.475000 (0.001784)	(0.464350, 0.480189) 0.015839
		30	0.472118 (0.002333)	0.472109 (0.000424)	0.465989 (0.000651)	(0.457677, 0.473148) 0.015471
		50	0.465865 (0.000208)	0.467480 (0.000202)	0.467462 (0.000206)	(0.467271, 0.482081) 0.01481
	0.37	20	0.700367 (0.004447)	0.700285 (0.000592)	0.700278 (0.000592)	(0.686122, 0.698706) 0.012584
		30	0.697317 (0.001391)	0.697826 (0.000142)	0.697306 (0.000086)	(0.691485, 0.703256) 0.011771
		50	0.697830 (0.001124)	0.697314 (0.000087)	0.693154 (0.000140)	(0.695227, 0.706761) 0.011534
Non-Informative Prior	0.8	20	0.276286 (0.003822)	0.276270 (0.002568)	0.276240 (0.002540)	(0.259694, 0.277449) 0.017755
		30	0.268657 (0.002947)	0.268637 (0.002341)	0.268598 (0.002380)	(0.260463, 0.277078) 0.016615
		50	0.268939 (0.002774)	0.268923 (0.001991)	0.268889 (0.002022)	(0.269130, 0.285070) 0.015940
	0.58	20	0.475027 (0.011131)	0.475018 (0.001788)	0.474960 (0.001759)	(0.466078, 0.482650) 0.016572
		30	0.467489 (0.002385)	0.466008 (0.000645)	0.465838 (0.000710)	(0.457409, 0.473610) 0.016201
		50	0.466018 (0.001466)	0.465856 (0.000705)	0.472091 (0.000420)	(0.458834, 0.474900) 0.016066
	0.37	20	0.700289 (0.009774)	0.700363 (0.000611)	0.700356 (0.000611)	(0.694066, 0.706696) 0.012630
		30	0.693165 (0.003142)	0.691702 (0.000371)	0.691695 (0.000372)	(0.686605, 0.699531) 0.012926

		50	0.691706 (0.002311)	0.693161 (0.000139)	0.697819 (0.000141)	(0.691367, 0.703137) 0.011770

For this real data set the cumulative mean plots, trace plots, autocorrelation plots and density plots are presented in Figure 5, 6, 7 and 8 respectively. Cumulative mean plots (Figure 5) showing the convergence of chain and trace plots (Figure 6) represent the randomness of the observations. The lag value is 20 and it can be seen by autocorrelation plots (Figure 7). Figure 9, shows that after burn-in and thinning the chain, the autocorrelation is zero.

The results obtained in case of simulation do hold in case of real data as well and reported in Table 11.

Table 6. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for (t) under informative and non-informative priors for α = 1 and β = 0.5.

	t	n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior	0.8	20	0.283741 (0.006750)	0.283719 (0.005098)	0.283675 (0.005053)	(0.273807, 0.293427) 0.019620
		30	0.274314 (0.002615)	0.274292 (0.002305)	0.274248 (0.002349)	(0.263886, 0.283410) 0.019524
		50	0.280220 (0.000987)	0.280198 (0.000748)	0.280155 (0.000720)	(0.270748, 0.289205) 0.018457
	0.58	20	0.515072 (0.006750)	0.515061 (0.001086)	0.515037 (0.001079)	(0.503401, 0.524368) 0.020967
		30	0.512582 (0.002805)	0.506107 (0.000843)	0.506084 (0.000850)	(0.498143, 0.519024) 0.020881
		50	0.506119 (0.002615)	0.512571 (0.000251)	0.512548 (0.000247)	(0.504142, 0.517497) 0.013355
	0.37	20	0.762701 (0.006750)	0.762697 (0.000106)	0.762691 (0.000105)	(0.753600, 0.765507) 0.011907
		30	0.759714 (0.000996)	0.761881 (0.000042)	0.761875 (0.000042)	(0.756762, 0.768637) 0.011875
		50	0.761884 (0.000109)	0.759711 (0.000013)	0.759704 (0.000013)	(0.756522, 0.767545) 0.011023
Non-Informative Prior	0.8	20	0.283772 (0.006822)	0.283750 (0.005153)	0.283705 (0.005107)	(0.273807, 0.293469) 0.019662
		30	0.282468 (0.004152)	0.282445 (0.003085)	0.282401 (0.003039)	(0.263886, 0.283410) 0.019524
		50	0.274314 (0.002615)	0.274292 (0.002305)	0.274248 (0.002349)	(0.272712, 0.291876) 0.019164
	0.58	20	0.515101 (0.006822)	0.515090 (0.001099)	0.515066 (0.001092)	(0.498401, 0.518258) 0.019857
		30	0.513612 (0.004152)	0.506107 (0.000843)	0.506084 (0.000850)	(0.500213, 0.519084) 0.018871
		50	0.506119 (0.002615)	0.513601 (0.000526)	0.513578 (0.000520)	(0.504801, 0.523509) 0.018708

	0.37	20	0.762718 (0.006822)	0.757391 (0.000208)	0.757385 (0.000208)	(0.750502, 0.763807) 0.013305
		30	0.761572 (0.004152)	0.762715 (0.000107)	0.762708 (0.000107)	(0.755290, 0.767475) 0.012185
		50	0.757394 (0.002615)	0.761568 (0.000025)	0.761562 (0.000025)	(0.756762, 0.768852) 0.01209

Table 7. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for (t) under informative and non-informative priors for α = 1.5 and β = 0.5.

	t	n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior	0.8	20	0.282948 (0.001790)	0.282917 (0.002103)	0.282855 (0.002041)	(0.261583, 0.285406) 0.023823
		30	0.282316 (0.001213)	0.282284 (0.001440)	0.282219 (0.001384)	(0.272219, 0.295124) 0.022905
		50	0.282087 (0.001040)	0.282053 (0.001229)	0.281986 (0.001175)	(0.270053, 0.292873) 0.022820
	0.58	20	0.560622 (0.010685)	0.577121 (0.002761)	0.577092 (0.002756)	(0.545041, 0.567610) 0.022569
		30	0.506119 (0.002615)	0.506107 (0.000843)	0.506084 (0.000850)	(0.557989, 0.579841) 0.021852
		50	0.572903 (0.003050)	0.568566 (0.000000)	0.568535 (0.000000)	(0.495401, 0.514358) 0.018957
	0.37	20	0.840923 (0.005746)	0.840921 (0.000206)	0.840916 (0.000206)	(0.836927, 0.847657) 0.010730
		30	0.842577 (0.001903)	0.842574 (0.000055)	0.842570 (0.000055)	(0.835896, 0.846065) 0.010169
		50	0.843893 (0.000018)	0.843891 (0.000004)	0.843886 (0.000004)	(0.839309, 0.849847) 0.010538
Non-Informative Prior	0.8	20	0.296174 (0.037863)	0.296140 (0.042600)	0.296074 (0.043106)	(0.269682, 0.293869) 0.024187
		30	0.266347 (0.021119)	0.266310 (0.028438)	0.266237 (0.028317)	(0.254324, 0.278075) 0.023751
		50	0.273759 (0.003818)	0.273723 (0.005044)	0.273650 (0.005144)	(0.284265, 0.307915) 0.02365
	0.58	20	0.556208 (0.019015)	0.556193 (0.006103)	0.556164 (0.006087)	(0.548320, 0.572256) 0.023936
		30	0.577135 (0.008058)	0.560606 (0.002510)	0.560574 (0.002518)	(0.561598, 0.584774) 0.023176
		50	0.568581 (0.000012)	0.572887 (0.000706)	0.572857 (0.000697)	(0.565072, 0.587941) 0.022869
	0.37	20	0.838460 (0.024803)	0.838458 (0.000612)	0.838452 (0.000612)	(0.831499, 0.843636) 0.012137
		30	0.839474 (0.010181)	0.839472 (0.000419)	0.839467 (0.000419)	(0.833607, 0.844518) 0.010911
		50	0.845019 (0.000029)	0.845017 (0.000008)	0.845012 (0.000008)	(0.838313, 0.848872) 0.010559

Table 8. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for $h(t)$ under informative and non-informative priors for α = 0.75 and β = 0.5.

	t	n	SELF	GELF		HPD interval withlength
				$b=-0.5$	$b=0.5$	
Informative Prior	0.8	20	2.762898 (0.035934)	2.762787 (0.000592)	2.762565 (0.000605)	(2.697202, 2.842262) 0.14506
		30	2.768967 (0.016609)	2.768836 (0.000275)	2.768575 (0.000286)	(2.719489, 2.858689) 0.1392
		50	2.782031 (0.000003)	2.781918 (0.000026)	2.781691 (0.000028)	(2.677935, 2.811252) 0.133317
	0.58	20	2.130575 (0.133201)	2.130490 (0.003633)	2.130319 (0.003647)	(2.108044, 2.211809) 0.103765
		30	2.149123 (0.032215)	2.149045 (0.000873)	2.148890 (0.000886)	(2.099348, 2.197855) 0.098507
		50	2.163020 (0.001642)	2.162942 (0.000045)	2.162787 (0.000049)	(2.079878, 2.174016) 0.094138
	0.37	20	1.555917 (0.038780)	1.555869 (0.001991)	1.555773 (0.002002)	(1.514563, 1.588358) 0.073795
		30	1.565670 (0.009878)	1.565616 (0.000507)	1.565508 (0.000516)	(1.531029, 1.601425) 0.070396
		50	1.565909 (0.009409)	1.565857 (0.000482)	1.565752 (0.000492)	(1.522287, 1.587522) 0.065235
Non-Informative Prior	0.8	20	2.751339 (0.093118)	2.751226 (0.001535)	2.750997 (0.001552)	(2.714441, 2.863644) 0.149203
		30	2.800243 (0.033813)	2.800120 (0.000535	2.799875 (0.000522)	(2.726977, 2.87079) 0.143813
		50	2.791509 (0.009320)	2.791370 (0.000146)	2.791093 (0.000137)	(2.696656, 2.835517) 0.138861
	0.58	20	2.126676 (0.163179)	2.126602 (0.004456)	2.126454 (0.004461)	(2.079701, 2.184385) 0.104684
		30	2.190189 (0.053442)	2.190110 (0.001395)	2.189950 (0.001381)	(2.125141, 2.229171) 0.10403
		50	2.178032 (0.012012)	2.177949 (0.000313)	2.177784 (0.000304)	(2.140581, 2.238783) 0.098202
	0.37	20	1.549242 (0.069522)	1.549189 (0.003585)	1.549083 (0.003593)	(1.550736, 1.630686) 0.07995
		30	1.590780 (0.023015)	1.590721 (0.001137)	1.590603 (0.001123)	(1.553809, 1.626527) 0.072718
		50	1.590517 (0.022224)	1.590461 (0.001099)	1.590349 (0.001085)	(1.530875, 1.599531) 0.068656

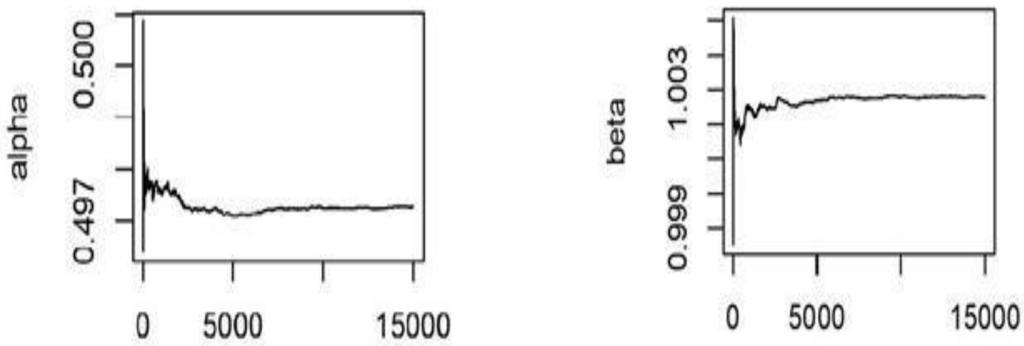

Figure 5: cumulative mean plots for α and β.

Table 9. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for $h(t)$ under informative and non-informative priors for α = 1 and β = 0.5.

	t	n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior	0.8	20	3.141026 (0.347799)	3.140867 (0.004362)	3.140551 (0.004382)	(3.138952, 3.316365) 0.177413
		30	3.227036 (0.073094)	3.226875 (0.000873)	3.226554 (0.000855)	(3.058582, 3.230635) 0.172053
		50	3.177937 (0.048678)	3.177783 (0.000607)	3.177475 (0.000623)	(3.086457, 3.252555) 0.166098
	0.58	20	2.280376 (0.157005)	2.280264 (0.003742)	2.280039 (0.003763)	(2.275061, 2.403857) 0.128796
		30	2.291658 (0.080327)	2.291545 (0.001908)	2.291319 (0.001930)	(2.228777, 2.352056) 0.123279
		50	2.342170 (0.049149)	2.342056 (0.001117)	2.341830 (0.001098)	(2.221892, 2.344517) 0.122625
	0.37	20	1.458509 (0.046185)	1.458436 (0.002699)	1.458290 (0.002723)	(1.448085, 1.529703) 0.081618
		30	1.488586 (0.007373)	1.488511 (0.000411)	1.488360 (0.000397)	(1.416286, 1.497232) 0.080946
		50	1.479817 (0.000003)	1.479746 (0.000002)	1.479603 (0.000001)	(1.436032, 1.513307) 0.077275
Non-Informative Prior	0.8	20	3.140757 (0.350968)	3.140598 (0.004402)	3.140280 (0.004422)	(3.138952, 3.316365) 0.177413
		30	3.149112 (0.258955)	3.148955 (0.003241)	3.148639 (0.003264)	(3.058582, 3.230635) 0.172053
		50	3.227036 (0.073094)	3.226875 (0.000873)	3.226554 (0.000855)	(3.058925, 3.229721) 0.170796
	0.58	20	2.280182 (0.158549)	2.280068 (0.003779)	2.279842 (0.003800)	(2.275061, 2.403857) 0.128796
		30	2.287389 (0.106349)	2.287276 (0.002528)	2.287052 (0.002551)	(2.220464, 2.344517) 0.124053
		50	2.342170 (0.049149)	2.342056 (0.001117)	2.341830 (0.001098)	(2.223004, 2.346756) 0.123752

	0.37	20	1.458385 (0.046720)	1.458311 (0.002731)	1.458164 (0.002755)	(1.441063, 1.523859) 0.082796
		30	1.497257 (0.029781)	1.497185 (0.001663)	1.497041 (0.001641)	(1.426286, 1.507232) 0.080946
		50	1.464375 (0.024413)	1.464302 (0.001424)	1.464156 (0.001446)	(1.424396, 1.504227) 0.079831

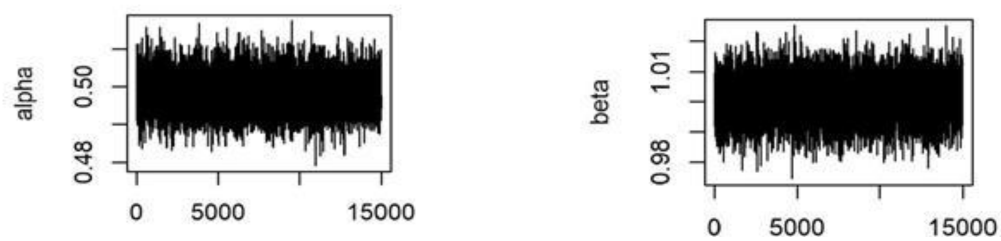

Figure 6: Trace plots for α and β.

Table 10. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for $h(t)$ under informative and non-informative priors for α = 1.5 and β = 0.5.

	t	n	SELF	GELF		HPD interval withlength
				$b = -0.5$	$b = 0.5$	
Informative Prior	0.8	20	3.903841 (0.129749)	3.903590 (0.001071)	3.903089 (0.001097)	(3.774306, 4.025139) 0.250833
		30	3.906516 (0.111188)	3.906275 (0.000918)	3.905792 (0.000942)	(3.798757, 4.037638) 0.238881
		50	3.907751 (0.103109)	3.907497 (0.000852)	3.906991 (0.000876)	(3.802842, 4.037486) 0.234644
	0.58	20	2.598192 0.480374	2.598001 (0.009048)	2.597619 (0.009030)	(2.529876, 2.703465) 0.173589
		30	2.342170 (0.049149)	2.342056 (0.001117)	2.341830 (0.001098)	(2.455191, 2.617218) 0.162027
		50	2.532789 (0.001526)	2.532603 (0.000027)	2.532231 0.000022	(2.275061, 2.403857) 0.128796
	0.37	20	1.288170 (0.074260)	1.288067 (0.005652)	1.287862 (0.005607)	(1.231416, 1.323158) 0.091742
		30	1.275943 (0.022572)	1.275834 (0.001725)	1.275616 (0.001682)	(1.218963, 1.305592) 0.086629
		50	1.263533 (0.000683)	1.263431 (0.000049)	1.263226 (0.000042)	(1.245201, 1.330900) 0.085699
Non-Informative Prior	0.8	20	3.776742 (2.660802)	3.776485 (0.022580)	3.775971 (0.022406)	(3.948129, 4.208666) 0.260537
		30	4.077191 (1.885938)	4.076905 (0.014531)	4.076334 (0.014577)	(3.880810, 4.137264) 0.256454
		50	4.003167 (0.400760)	4.002885 (0.003140)	4.002319 (0.003100)	(3.661368, 3.897844) 0.236476
	0.58	20	2.615141 (0.744037)	2.614959 (0.013926)	2.614597 (0.013965)	(2.504134, 2.684621) 0.180487

		30	2.478641 (0.252428)	2.478464 (0.005087)	2.478110 (0.005124)	(2.415732, 2.583783) 0.168051
		50	2.495102 (0.114114)	2.494915 (0.002291)	2.494539 (0.002332)	(2.391270, 2.558057) 0.166787
	0.37	20	1.313822 (0.279867)	1.313698 (0.020874)	1.313450 (0.020967)	(1.269096, 1.368335) 0.099239
		30	1.298327 (0.139930)	1.298222 (0.010573)	1.298012 (0.010558)	(1.252080, 1.344702) 0.092622
		50	1.257699 (0.001037)	1.257601 (0.000087)	1.257405 (0.000097)	(1.222596, 1.312749) 0.090153

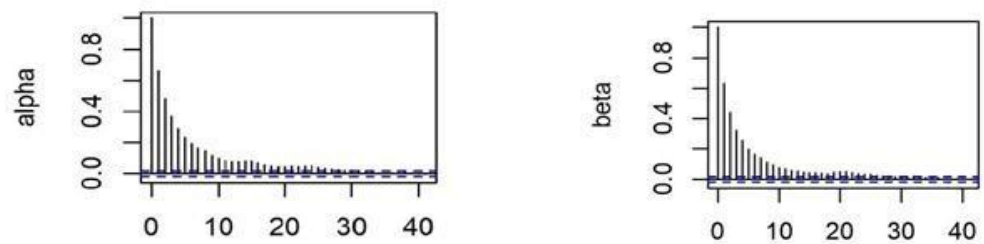

Figure 7: Autocorrelation plot of α and β.

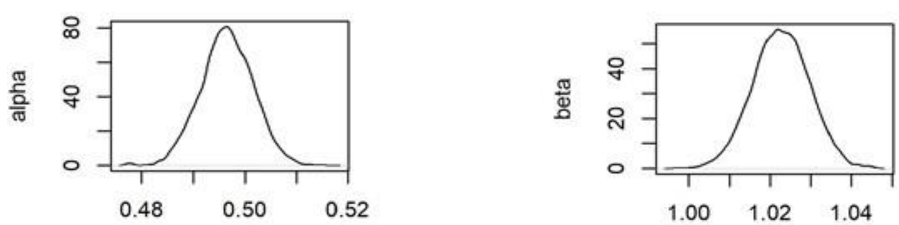

Figure 8: Density plots for α and β.

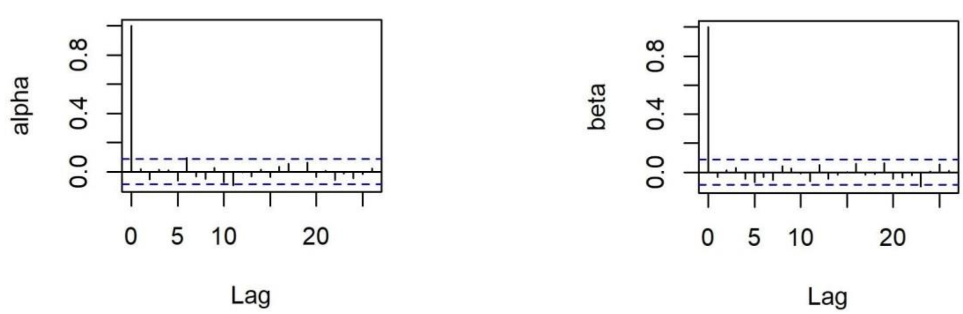

Figure 9: Autocorrelation plot of α and β after burn-in and thinning the chain.

Table 11. Bayes estimates, expected loss (within brackets) and HPD credible intervals (within brackets) with their length for α, β, (t) and $h(t)$ under non-informative priors with n = 59, t = 7 for real data.

SELF		GELF		HPD interval with length
		$b = -0.5$	$b = 0.5$	
α	4.8150	4.8206	4.8210	(4.809856, 4.839542)
	(0.000346)	(0.000091)	(0.000085)	0.029686
β	51.1856	51.29365	51.291021	(51.165941, 51.452761)
	(0.009351)	0.000061	0.000036	0.28682
$R(t)$	0.471462	0.476985	0.472672	(0.4688564, 0.4791064)
	(0.000052)	(0.000015)	(0.000019)	0.01025
$h(t)$	0.48569	0.495762	0.495961	(0.4493872, 0.514623)
	(0.000695)	(0.000132)	(0.000128)	0.0652358

4. Conclusion

As sample size increases the expected loss function decreases as can be seen by simulation study (Table 2-10). The expected loss function using informative prior is less than the expected loss function using non-informative priors. The length of 95% HPD credible intervals decreases as sample size increases for the Bayes estimates of α, β, (t) and $h(t)$ of Nakagami distribution. The length of 95% HPD credible intervals for the Bayes estimates of α, β, (t) and $h(t)$ using informative priors is smaller than using non-informative priors. A real life example is also considered and fitting of the data to the Nakagami distribution is given as well.

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