Likelihood Estimation and Order Statistics based on Frechat distribution under Type-I Censoring

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In this paper, we have investigated some interesting properties of Frechat failure model. The model has two parameters (one scale and one shape). The reliability function, hazard rate and moments of the distribution have been derived under Type-I (time) censoring scheme. The maximum likelihood estimators of the parameters and Fisher Information matrix have been derived. Furthermore, order statistics have been derived.

Keywords: Frechat distribution, Order statistics, Type-I Censoring, Fisher Information matrix.

1. Introduction

In statistical analysis theory, the concept of Extreme value is quite indispensable to extract the Bayesian information. Most often, the extreme or peak data are represented by the extreme value distribution (Jenkinson, 1955). The density function is given by

$$F(y,\eta,\lambda,\theta) = \begin{cases} e^{-\{1+\theta(y-\lambda)/\eta\}_+^{-1/\theta}}, & \theta \neq 0 \\ e^{-\exp\{-(y-\lambda)/\eta\}}, & \theta = 0 \end{cases}$$

Some of the distributions such as Gumbel, Weibull, and Frechet distributions are the different forms of generalized extreme value distribution. In 1920s, French mathematician Maurice Rene Frechet proposed the Frechet distribution as a value distribution (which is also known as the extreme value distribution of type-II). Later on, (Kotz and Nadarajah, 2000) elaborated this distribution both theoretically and applicability in different fields of research such as natural calamities, reliability theory, censoring, optimization, routing problems, logistics management, inventory control, record values and so on. The Frechat failure modelis also known as Inverse Weibull distribution (Khan et al., 2004). The inverse Rayleigh distribution and inverse exponential distribution are a particular form of the Frechat

distribution.(Ramos et al., 2020)tackled with the problem of estimating the parameters of the Frechet failure model from both frequentist and Bayesian points of view. (Harlow, 2002) proposed some important and useful applications of Frechat failure model.

Several authors have studied different aspects of inferential procedures for the Frechat failure model. The maximum likelihood estimators and the ordinary least-square estimators for this model are discussed by (Calabria and Pulcini, 1989) and (Erto, 1989), respectively. In the occurrence of cure fraction, the maximum likelihood estimators of this model presented by (Ramos et al., 2017). A comparison of maximum likelihood estimators, least-square estimators, weighted least square estimators, and the method of moments presented by (Loganathan and Uma, 2017). The result of this comparison is that weighted least square estimators provided comparable results. (Salman and Amer, 2003) dealt within the context of order statistics for Frechat distribution. While,(Maswadah, 2003) dealt within the context of generalized order statistics for Frechat distribution. (Calabria and Pulcini, 1994) and (Kundu and Howlader, 2010) provided the Bayes estimators with subjective or informative priors such as flat priors for Frechat distribution. (Bernardo, 2005) disagreed that apply of simple proper flat priors presumed to be non-informative often conceal main unwarranted assumptions which may simply control, or even overthrow the statistical analysis and should be powerfully discouraged. Based on Jeffreys and reference priors, a study presented by (Abbas and Tang, 2005) for Frechet distribution. (Ramos et al., 2019) provided a study on Frechat distribution under estimation and its application. They considered the problem of estimating the parameters of this distribution from both statistical inference and Bayesian points of view.

This Frechat distribution plays an important role in reliability theory also because of its simple forms of the probability density function and cumulative density function. The lifetime data is censored when the exact failure time of any item or product is unknown. There are many types of censoring, such as left, right, interval, Type-I, Type-II, hybrid, progressive, progressive Type-I, and progressive Type-II censoring, etc. Here we consider only the Type-I censoring scheme. The Type-I and Type-II censoring schemes are the most common and popular schemes in reliability theory. The only difficulty in both Type-I and Type-II censoring schemes is that we cannot withdraw live items during the test.The Type-I (time) censoring occurs when a life experiment end after a fixed amount of time.

Many pieces of literature related to Frechat distribution and different censoring schemes are available. For example, (Mubarak, 2011) presented a study on Frechat distribution using a progressive Type-II censoring scheme. At the same time, the number of test units removed at each failure time has a Binomial distribution. (Maithi and Kayal, 2019) presented the reliability individuality for a generalized Frechet distribution under progressive Type-II censoring scheme. In the presence of the Type-II censoring scheme, the Classical and Bayesian studies presented by (Goyal et al., 2019). They proposed a novel group of distribution using the conception of Exponentiated distribution function that presented a more suitable

model to the baseline model. The authors also propose a novel lifetime distribution with special types of hazard rates such as decreasing, increasing and bathtub. (Kang and Han, 2015) dealt with the graphical techniques for the goodness of fit test in the Frechat distribution using multiply Type-II Censoring scheme. They derived the approximate maximum likelihood estimators of the scale parameter and the shape parameter inverse using multiply Type-II censoring for Frechat distribution. (Vishwakarma et al., 2018) presented Bayesian estimation for Frechat distribution using a progressive Type-II censoring scheme when removals follow Beta-binomial probability law. They obtained the maximum likelihood and Bayes estimators under the progressive censoring scheme to estimate the unknown parameters. (Joarder et al., 2011) presented the statistical inferences of the unknown parameters of Weibull distribution using the Type-I censoring scheme. (Kazemi and Azizpour, 2017) presented the statistical inferences of the Frechat distribution when the lifetime of data are Type-I hybrid censored. (Xu and Gui, 2019) discussed entropy estimations for Frechat distribution using adaptive Type-II progressive hybrid censoring schemes. They constructed the estimations of entropy derived using the maximum likelihood estimation method and Bayes estimation method. (Shahab et al., 2015) dealt with the optimal design of step-stress partially accelerated life test using a progressive Type-II censoring scheme for Frechat distribution with random removals. (Bai et al., 1992) dealt with an optimum simple ramp test-accelerated life test with two special linearly increasing stresses for the Weibull distribution using Type-I censoring.(Joarder et al., 2011) presented a study on Type-I censoring scheme using the Weibull failure model.(Jia et al., 2018) tackled with a inference on Weibull failure model under multiply Type-I censoring scheme. The multiply Type-I censoring is the common type of Type-I censoring and represents that all the test items are ended at unlike times. (Mohie El-Din and Abu-Moussa, 2018) proposed a statistical inference for the Gompertz failure model based on multiply Type-I censored data. The author determined a censoring time point for each item in the life time test.(Alam et al., 2020) dealt with a study on Burr Type-X failure model using Type-II censoring in the presence of age-replacement policy and accelerated life tests. The current work can be extended by using different lifetime distributions with different censoring schemes. Also, the Bayesian approach can be used in the extension of current work.

The paper is organized as follows. The model description is given in section II. The likelihood estimation and information matrix are presented in section III. In section IV, the moments and order statistics are developed. Finally, the conclusions are made in section V.

2. Model Description

Let the random variable X follows Frechet distribution then its probability density function and cumulative density function are given as

The Probability density function (pdf) of the model is given as

$$f(x,\alpha,\beta) = \alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}}, \quad x > 0, \alpha > 0, \beta > 0 \tag{1}$$

Where α and β are scale and shape parameters, respectively.

The Cumulative density function (cdf) of the model is given as

$$F(x,\alpha,\beta) = e^{-\alpha x^{-\beta}}, \quad x > 0, \alpha > 0, \beta > 0 \tag{2}$$

The Reliability function of the model is given as

$$R(x,\alpha,\beta) = 1 - e^{-\alpha x^{-\beta}} \tag{3}$$

The Hazard function of the model is given as

$$h(x,\alpha,\beta) = \frac{\alpha\beta x^{-(\beta+1)} e^{-\alpha x^{\beta}}}{1 - e^{-\alpha x^{-\beta}}} \tag{4}$$

This failure model is also known as Inverse Weibull distribution (IWD). The hazard function is always unimodal while the probability density function depends on the shape parameter as it is unimodal or decreasing. This distribution is a max stable distribution and the negative of a random variable (r.v.) having a Frechet distribution is a min stable distribution. (Liu, 1997) provided details in his research if the product consists of various components and each component has a similar failure time distribution, and the product falls when the powerless component fails, then the Weibull failure be a suitable model of that failure mode (Nelson, 1982). Similarly, the Frechat failure model will be suitable for modeling when electrical or mechanical parts lying in the life testing experimentation.
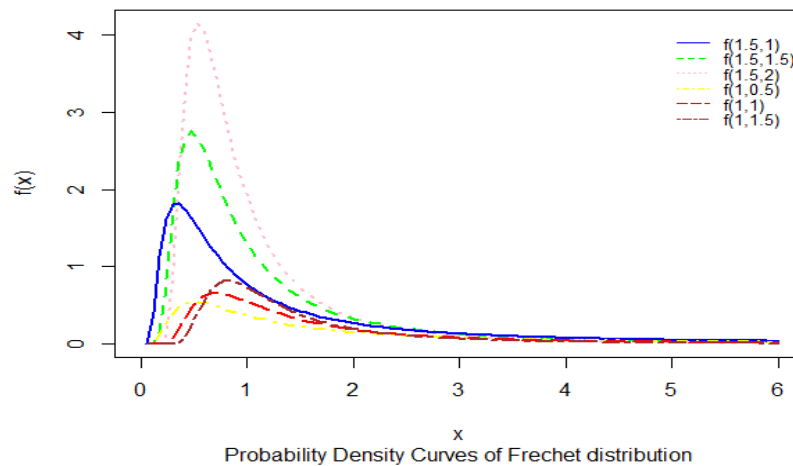

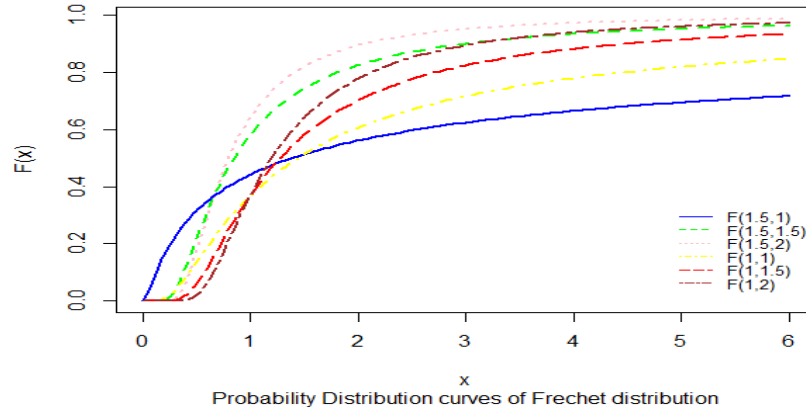**Figure 1:** Probability density curve of Frechat distribution

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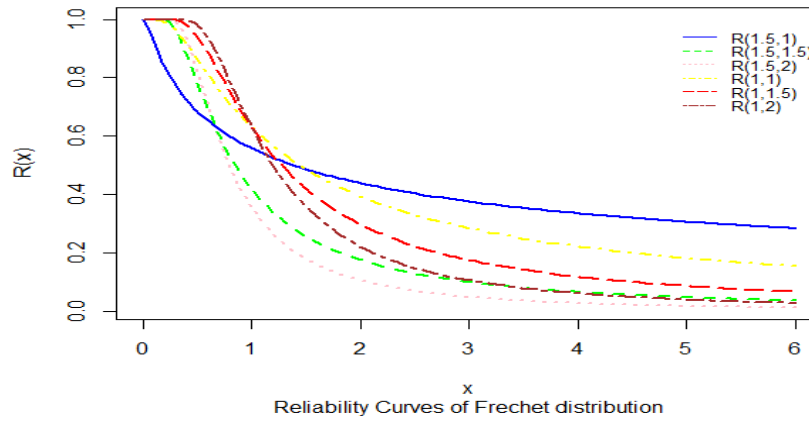**Figure 2:** Probability distribution curve of Frechat distribution

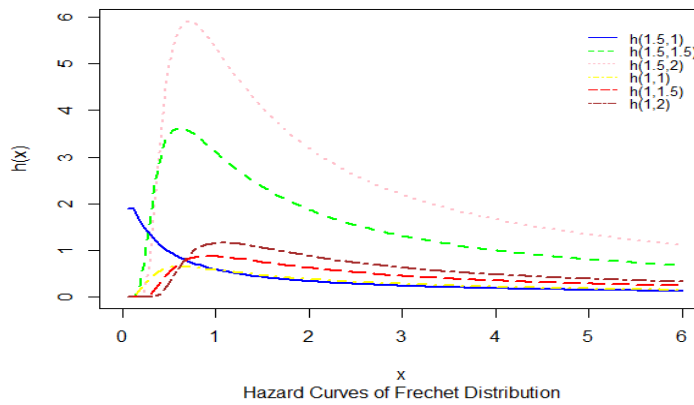**Figure 3:** Reliability curve of Frechat distribution

Figure 4: Hazard curve of Frechat distribution

This study is based on constant stress and Type-I censoring scheme. We have considered the Stress $V_j, j = 1,2,..., k$ which affects the shape parameter of the used distribution, β_j through the following equation (5) called the power rule model.

$$\beta_j = CV_j^{-p}; \quad C > 0, p > 0, \quad j = 1, 2, ..., k \tag{5}$$

where C and p are the proportionality constant; and power of the applied stress respectively.

3. Likelihood Estimation and Information Matrix

In this section, the maximum likelihood (ML) estimation method is used. The reliability practitioner used this method because it is very robust and provides estimates of the parameters with excellent properties. At the stress level V_j, the authors constructed the likelihood function of an observation x (time to failure) and at each stress level V_j, n_j units put on the test.

Therefore, $N = \sum_{i=1}^{k} n_j$ is the total number of units in the test. When a Type-I (time) censoring scheme is adopted at each stress level, the experiment ends once the censoring time "x_0" is reached. It is assumed that $r_j (\leq n_j)$ units are observed at the jth stress level before the test is terminated and $(n_j - r_j)$ units still carry on till the end of the test. In this situation, the likelihood function of the experiment for Frechat failure model is taking the following form

$$L(\alpha_j, C, \beta) = \prod_{j=1}^{k} \frac{n_j}{(n_j - r_j)!} \left[\prod_{i=1}^{r_j} f(x_{ij}; \beta_j, C, \alpha) \right] [1 - F(x_0)]^{n_j - r_j} \tag{6}$$

where x_0 is the time of cessation of the test.

The log-likelihood function is obtained by taking the logarithm of the above equation (6) and given as

$$\ln L = Q + \sum_{j=1}^{k} r_j (\ln \alpha + \ln C - p \ln V_j) + \sum_{i=1}^{r_j} \sum_{j=1}^{k} (1 - CV_j^{-p}) \ln x_{ij} + \sum_{i=1}^{r_j} \sum_{j=1}^{k} (-\alpha x_{ij} - CV_j^{-p})$$

$$+ \sum_{j=1}^{k} (n_j - r_j) \ln(1 - e^{-\alpha x_0 - CV_j^{-p}}) \tag{7}$$

where Q is the constant.

The ML estimates of α, C and p can be estimated from the following three equations.

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{j=1}^{k} \frac{r_j}{\alpha} + \sum_{i=1}^{r_j} \sum_{j=1}^{k} x_{ij} + \sum_{j=1}^{k} (n_j - r_j) \frac{x_0 e^{-\alpha x_0 - CV_j^{-p}}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} = 0$$

$$\frac{\partial \ln L}{\partial p} = -\sum_{j=1}^{k} r_j \ln V_j + \sum_{i=1}^{r_j} \sum_{j=1}^{k} \ln x_{ij} CpV_j^{-p-1} + \sum_{i=1}^{r_j} \sum_{j=1}^{k} CpV_j^{-p-1} - \sum_{j=1}^{k} (n_j - r_j) \frac{e^{-\alpha x_0 - CV_j^{-p}} CV_j^{-p-1}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} = 0$$

$$\frac{\partial \ln L}{\partial C} = \sum_{j=1}^{k} \frac{r_j}{C} - \sum_{i=1}^{r_j} \sum_{j=1}^{k} \ln x_{ij} V_j^{-p} - \sum_{i=1}^{r_j} \sum_{j=1}^{k} V_j^{-p} + \sum_{j=1}^{k} (n_j - r_j) \frac{e^{-\alpha x_0 - CV_j^{-p}} V_j^{-p}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} = 0$$

It looks impossible to solve the above three non-linear equations manually. Therefore, an iterative technique (Newton-Raphson) can be used to get MLEs of parameters.

In mathematical statistics, the Fisher information or Information matrix is a technique of evaluating the amount of information that an observable random variable carries about an unknown parameter of a distribution. Simply, Information matrix is the variance of the score, or the expected value of the observed information.

The Information matrix for Frechat failure model under Type-I censoring is given as

$$F = \begin{vmatrix} \dfrac{-\partial^2 \ln L}{\partial \alpha^2} & \dfrac{-\partial^2 \ln L}{\partial \alpha \partial C} & \dfrac{-\partial^2 \ln L}{\partial \alpha \partial p} \\[3mm] \dfrac{-\partial^2 \ln L}{\partial C \partial \alpha} & \dfrac{-\partial^2 \ln L}{\partial C^2} & \dfrac{-\partial^2 \ln L}{\partial C \partial p} \\[3mm] \dfrac{-\partial^2 \ln L}{\partial p \partial \alpha} & \dfrac{-\partial^2 \ln L}{\partial p \partial C} & \dfrac{-\partial^2 \ln L}{\partial p^2} \end{vmatrix} \qquad (8)$$

The elements of the matrix F are obtained by second partial derivatives of log-likelihood function with respect to parameters α, p and C. Consequently, the elements are expressed by the following equations

$$-\frac{\partial^2 \ln L}{\partial \alpha^2} = \sum_{j=1}^{k} \frac{r_j}{\alpha^2} + \sum_{j=1}^{k} (n_j - r_j) x_0 \frac{e^{-\alpha x_0 - CV_j^{-p}}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} \left[x_0 + \frac{x_0 e^{-\alpha x_0 - CV_j^{-p}}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} \right]$$

$$-\frac{\partial^2 \ln L}{\partial C^2} = \sum_{j=1}^{k} \frac{r_j}{C^2} - \sum_{j=1}^{k} (n_j - r_j) CpV_j^{-p-1} \frac{e^{-\alpha x_0 - CV_j^{-p}} V_j^{-p}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} \left[1 - \frac{e^{-\alpha x_0 - CV_j^{-p}}}{1 - e^{-\alpha x_0 - CV_j^{-p}}} \right]$$

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$$\frac{\partial^2 \ln L}{\partial p^2} = -\sum_{i=1}^{r_j}\sum_{j=1}^{k}\ln x_{ij}CpV_j^{-p-1}(p^{-1}-\ln V_j) - \sum_{i=1}^{r_j}\sum_{j=1}^{k}CpV_j^{-p-1}(p^{-1}-\ln V_j)$$

$$+\sum_{j=1}^{k}(n_j-r_j)\frac{e^{-\alpha x_0-CV_j^{-p}}CV_j^{-p}}{1-e^{-\alpha x_0-CV_j^{-p}}}\left[-\ln V_j + pCV_j^{-p-1} - \frac{e^{-\alpha x_0-CV_j^{-p}}CV_j^{-p}\ln V_j}{1-e^{-\alpha x_0-CV_j^{-p}}}\right]$$

$$-\frac{\partial^2 \ln L}{\partial\alpha\partial C} = -\frac{\partial^2 \ln L}{\partial C\partial\alpha}$$

$$-\frac{\partial^2 \ln L}{\partial\alpha\partial C} = -\sum_{j=1}^{k}(n_j-r_j)\frac{x_0 e^{-\alpha x_0-CV_j^{-p}}}{1-e^{-\alpha x_0-CV_j^{-p}}}\left[V_j^{-p} - \frac{e^{-\alpha x_0-CV_j^{-p}}V_j^{-p}}{1-e^{-\alpha x_0-CV_j^{-p}}}\right]$$

$$-\frac{\partial^2 \ln L}{\partial\alpha\partial p} = -\frac{\partial^2 \ln L}{\partial p\partial\alpha}$$

$$-\frac{\partial^2 \ln L}{\partial\alpha\partial p} = -\sum_{j=1}^{k}(n_j-r_j)\frac{x_0 e^{-\alpha x_0-CV_j^{-p}}}{1-e^{-\alpha x_0-CV_j^{-p}}}\left[CV_j^{-p} + CV_j^{-p}\ln V_j\frac{e^{-\alpha x_0-CV_j^{-p}}}{1-e^{-\alpha x_0-CV_j^{-p}}}\right]$$

$$-\frac{\partial^2 \ln L}{\partial C\partial p} = -\frac{\partial^2 \ln L}{\partial p\partial C}$$

$$-\frac{\partial^2 \ln L}{\partial C\partial p} = \sum_{i=1}^{r_j}\sum_{j=1}^{k}\ln x_{ij}V_j^{-p}\ln V_j + \sum_{i=1}^{r_j}\sum_{j=1}^{k}V_j^{-p}\ln V_j$$

$$-\sum_{j=1}^{k}(n_j-r_j)\frac{e^{-\alpha x_0-CV_j^{-p}}V_j^{-p}}{1-e^{-\alpha x_0-CV_j^{-p}}}\left[-CV_j^{-p}\ln V_j - \ln V_j - \frac{e^{-\alpha x_0-CV_j^{-p}}V_j^{-p}C\ln V_j}{1-e^{-\alpha x_0-CV_j^{-p}}}\right]$$

Now, the variance-covariance matrix is the inverse of the Fisher Information matrix and given as $\Sigma = F^{-1}$ (9)

$$\Sigma = \begin{bmatrix} \dfrac{-\partial^2 \ln L}{\partial\alpha^2} & \dfrac{-\partial^2 \ln L}{\partial\alpha\partial C} & \dfrac{-\partial^2 \ln L}{\partial\alpha\partial p} \\ \dfrac{-\partial^2 \ln L}{\partial C\partial\alpha} & \dfrac{-\partial^2 \ln L}{\partial C^2} & \dfrac{-\partial^2 \ln L}{\partial C\partial p} \\ \dfrac{-\partial^2 \ln L}{\partial p\partial\alpha} & \dfrac{-\partial^2 \ln L}{\partial p\partial C} & \dfrac{-\partial^2 \ln L}{\partial p^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{C}) & ACov(\hat{\alpha}\hat{p}) \\ ACov(\hat{C}\hat{\alpha}) & AVar(\hat{C}) & ACov(\hat{C}\hat{p}) \\ ACov(\hat{p}\hat{\alpha}) & ACov(\hat{p}\hat{C}) & AVar(\hat{p}) \end{bmatrix}$$

$AVar$ is asymptotic variance and $ACov$ is asymptotic covariance.

The $100(1-\gamma)\%$ approximated two-sided limits of confidence for parameters α, C and p are given as

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$$\hat{\alpha} \pm Z_{\gamma/2} \sqrt[-1]{I_{11}^{-1}(\hat{\alpha}, \hat{C}, \hat{p})} \, , \; \hat{C} \pm Z_{\gamma/2} \sqrt[-1]{I_{22}^{-1}(\hat{\alpha}, \hat{C}, \hat{p})} \; \text{and} \; \hat{p} \pm Z_{\gamma/2} \sqrt[-1]{I_{33}^{-1}(\hat{\alpha}, \hat{C}, \hat{p})}$$

4. Moments and Order Statistics

The r^{th}ordered moment $E(X^r \mid \alpha, \beta)$ with parameters α and β for Frechat distribution are given as

$$E(X^r \mid \alpha, \beta) = \int_0^\infty x^r f(x \mid \alpha, \beta) dx$$

$$E(X^r \mid \alpha, \beta) = \alpha^{r/\beta} \, \Gamma\!\left(1 - \frac{r}{\beta}\right)$$

For $r = 1, 2$

$$E(X \mid \alpha, \beta) = \alpha^{1/\beta} \, \Gamma\!\left(1 - \frac{1}{\beta}\right), \;\; E(X^2 \mid \alpha, \beta) = \alpha^{2/\beta} \, \Gamma\!\left(1 - \frac{2}{\beta}\right)$$

Therefore, Variance is given as

$$Var(X \mid \alpha, \beta) = E(X^2 \mid \alpha, \beta) - \left(E(X \mid \alpha, \beta)\right)^2$$

$$Var(X \mid \alpha, \beta) = \alpha^{2/\beta} \, \Gamma\!\left(1 - \frac{2}{\beta}\right) - \left(\alpha^{1/\beta} \, \Gamma\!\left(1 - \frac{1}{\beta}\right)\right)^2$$

where $r \in N$ and $\Gamma\theta = \int_0^\infty e^{-y} y^{\theta-1} dy$ is the gamma function. Note that $E(T^r \mid \alpha, \beta)$ does not have a finite value $\beta > r$. By equating the first two theoretical moments with the sample moments, one can obtain the moment estimators (MEs) for the Frechet distribution.

Let X have Frechat distribution, the moment generating function of X is obtained as

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x) dx \,. \text{where} \; f(x) = \alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}}$$

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} \alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}} \, dx$$

$$M_X(t) = \sum_{j=1}^\infty \frac{t^j}{j!} \left(\frac{1}{\alpha}\right)^{j/\beta} \Gamma(1 - j)$$

The characteristics function is obtained as

$$\phi(t) = E(e^{itx}) = \int_0^\infty e^{itx} \alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}} \, dx$$

$$\phi(t) = \sum_{j=1}^\infty \frac{(it)^j}{j!} \left(\frac{1}{\alpha}\right)^{j/\beta} \Gamma(1-j)$$

Let $X_1, X_2, ..., X_n$ be the random samples with order statistics $X_{(1)}, X_{(2)}, ..., X_{(n)}$ with probability density function $f(x)$ and cumulative density function $F(x)$ from a continuous population, then the pdf of rth order statistics is given as

$$f_k(x) = n\binom{n-1}{k-1} f(x)(F(x))^{k-1}(1-F(x))^{n-k}$$

$$f_k(x) = n\binom{n-1}{k-1} \alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}} (e^{-\alpha x^{-\beta}})^{k-1}(1-e^{-\alpha x^{-\beta}})^{n-k}$$

The pdf and cdf of the minimum are given as follows:

$$f_1(x) = nf(x)(1-F(x))^{n-1}$$

$$f_1(x) = n\alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}} (1-e^{-\alpha x^{-\beta}})^{n-1}$$

$$F_1(x) = 1 - (1-F(x))^{n-1}$$

$$F_1(x) = 1 - (1-e^{-\alpha x^{-\beta}})^{n-1}$$

The pdf and cdf of the maximum are given as follows:

$$f_n(x) = nf(x)(F(x))^{n-1}$$

$$f_n(x) = n\alpha\beta x^{-(\beta+1)} e^{-\alpha x^{-\beta}} (e^{-\alpha x^{-\beta}})^{n-1}$$

$$F_n(x) = (F(x))^n$$

$$F_n(x) = (e^{-\alpha x^{-\beta}})^n$$

5. Conclusion

In this paper, we have derived survival function, failure rate and moments of the Frechat distribution. The likelihood and log-likelihood functions have been examined. In addition, Fisher Information matrix, variance-covariance matrix, moments and order statistics have been derived. This research work can be extended for different type of censoring schemes with various lifetime distributions.

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