On Discriminant Analysis with some Bivariate Exponential Distributions

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Discriminant analysis has found relevance in many areas of science and engineering. Its purpose is to assign an observation $x$ to one of two or more groups on the basis of the value of $x$. Many problems of discriminant analysis have assumed that the underlying distribution is the multivariate normal in which $x$ is assumed to have means $\mu_i$ ($i=1,2,\ldots,g$) and variance-covariance matrix $\Sigma$ which is the same for all the groups. Discrete models, Bayesian techniques and the nonparametric density estimation procedure are also available for discriminant analysis but have only been studied within the context of multivariate normal situation. The interest here is the case where the underlying distribution (or data generating process) is not multivariate normal. None of such study is known to the authors at the moment.

**Keywords:** Bivariate Exponential, Bayesian techniques, Discriminant analysis.

Introduction

Discriminant analysis has found relevance in many areas of science and engineering. Its purpose is to assign an observation $x$ to one of two or more groups on the basis of the value of $x$. Many problems of discriminant analysis have assumed that the underlying distribution is the multivariate normal in which $x$ is assumed to have means $\mu_i$ ($i=1,2,\ldots,g$) and variance-covariance matrix $\Sigma$ which is the same for all the groups. Discrete models, Bayesian techniques and the nonparametric density estimation procedure are also available for discriminant analysis but have only been studied within the context of multivariate normal situation. The interest here is the case where the underlying distribution (or data generating process) is not multivariate normal. None of such study is known to the authors at the moment. Specifically, the bivariate exponential (BVE) distribution which has found wide application in reliability studies, competing risk, hydrology, rainfall and storm occurrence etc has not been considered in discriminant analysis. For example, on the basis of some suitably chosen vector of variables, rainfall occurrence can be classified as a particular type of various types, or failure of a system classified as either mild or severe. This study shall consider obtaining and applying allocation rule/region of classification when $(x_1,x_2) \in \mathbf{x}$ are correlated exponential random variables arising from the following distributions:
i. Freud (1961) bivariate exponential distribution with joint pdf given as:

\[ f(x_1, x_2) = \begin{cases} 
αβ' \exp\{-β' x_2 - (α + β - β')x_1\} & \text{if } x_1 < x_2 \\
αβ \exp\{-α' x_1 - (α + β - α')x_2\} & \text{if } x_2 < x_1 
\end{cases} \]

\( x_1 > 0, x_2 > 0, α > 0, β > 0, α' > 0, \text{ and } β' > 0 \)

ii. Marshal & Olkin (1967) bivariate exponential distribution with joint pdf given as:

\[ f(x_1, x_2) = \begin{cases} 
(θ_1(θ_2 + θ_3) exp\{−θ_1 x_1 - (θ_2 + θ_3)x_2\} & \text{if } x_1 < x_2 \\
θ_2(θ_1 + θ_3) exp\{−θ_2 x_2 - (θ_1 + θ_3)x_1\} & \text{if } x_2 < x_1 
\end{cases} \]

\( x_1 > 0, x_2 > 0, θ_1 > 0, θ_2 > 0, θ_3 > 0, \text{ and } θ = θ_1 + θ_2 + θ_3 \)

iii. Block & Basu (1974) bivariate exponential distribution with joint pdf given as:

\[ f(x_1, x_2) = \begin{cases} 
\frac{θ_1θ(θ_2 + θ_3)}{θ_1 + θ_2} \exp\{−θ_1 x_1 - (θ_2 + θ_3)x_2\} & \text{if } x_1 < x_2 \\
\frac{θ_2θ(θ_1 + θ_3)}{θ_1 + θ_2} \exp\{−θ_2 x_2 - (θ_1 + θ_3)x_1\} & \text{if } x_2 < x_1 
\end{cases} \]

\( x_1 > 0, x_2 > 0, θ_1 > 0, θ_2 > 0, θ_3 > 0, \text{ and } θ = θ_1 + θ_2 + θ_3 \)

The above listed distributions have been applied in real life situations. For example, see; Rai & Van (1984), Inaba & Shirahata (1986) and Gross & Lam (1981). Properties of various BVE distributions, including those considered in this study has been summarized in Balakrishnan & Lai (2009). In general, the bivariate exponential distributions assume that \( x_1 \) and \( x_2 \) are lifetimes of two independent exponential components. Dependence of \( x_1 \) and \( x_2 \) arises when a failure (or shock) occurs to either the \( x_1 \) component or the \( x_2 \) component or even to both components. The failure of one component changes the parameter of the other component by inducing an additional burden on the other component as in Block & Basu (1974) and Freud (1961).

### Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Π_i )</td>
<td>Population ( i ) (( i = 1, 2 ))</td>
</tr>
<tr>
<td>( f_i(x_1, x_2, ..., x_k) )</td>
<td>pdf of ( x = (x_1, x_2, ..., x_k) ) population ( i )</td>
</tr>
<tr>
<td>( x )</td>
<td>( k \times 1 ) observation</td>
</tr>
<tr>
<td>( α_i, α'_i, β_i, β'_i )</td>
<td>Parameters of ( f_i(x_1, x_2) ) for Freud distribution</td>
</tr>
<tr>
<td>( α_j, α'_j, β_j, β'_j )</td>
<td>Parameters of ( f_j(x_1, x_2) ) for Freud distribution</td>
</tr>
<tr>
<td>( θ_i, θ_{1i}, θ_{2i}, θ_{3i} )</td>
<td>Parameters of ( f_i(x_1, x_2) ) for Marshal/Olkin and Block/Basu distributions</td>
</tr>
<tr>
<td>( θ_j, θ_{1j}, θ_{2j}, θ_{3j} )</td>
<td>Parameters of ( f_j(x_1, x_2) ) for Marshal/Olkin and Block/Basu distributions</td>
</tr>
<tr>
<td>MOBVE</td>
<td>Marshal and Olkin Bivariate Exponential distribution</td>
</tr>
<tr>
<td>BBBVE</td>
<td>Block and Basu Bivariate Exponential distribution</td>
</tr>
<tr>
<td>FBEVE</td>
<td>Freud Bivariate Exponential distribution</td>
</tr>
</tbody>
</table>

### Methodology

**Definition:** Let \( f_i(x_1, x_2) \) and \( f_j(x_1, x_2) \) be the joint densities of observations coming from group \( Π_1 \) and group \( Π_2 \) respectively, the region of classification \( R \) according to Anderson (2003) is given as
$R: \frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} \geq c$

Where $c$ is a constant determined such that the critical region should have the required size. If the inequality in $R$ holds, the observation falls in region $R_1$ and it is classified as belonging to population $\Pi_1$, otherwise the observation is said to fall into $R_1$ region and consequently is classified a belonging to population $\Pi_2$. We now derive and present allocation rules for classifying a future observation to one of population $\Pi_1$ or population $\Pi_2$ given that the underlying distribution is any of Block & Basu (1974), Freud (1961) or Marshall & Olkin (1967).

**Freud BVE**

If $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are the joint densities for group $\Pi_1$ and group $\Pi_2$ respectively and $(x_1, x_2)$ follows the Freud BVE, then the best region of classification $R_1$ is

$$R_1: (x_2 - x_1)\lambda + (\gamma + \omega)x_1 \geq c_a \quad \text{if } x_1 < x_2$$

and

$$R_1: (x_1 - x_2)\nu + (\gamma + \omega)x_2 \geq c_b \quad \text{if } x_2 < x_1$$

Where $c_a = \log\left(\frac{\alpha_2\beta_1'}{\alpha_1\beta_2'}\right), c_b = \log\left(\frac{\alpha_2\beta_1'}{\alpha_1\beta_2'}\right), \lambda = \beta_2' - \beta_1', \nu = \alpha_2' - \alpha_1', \omega = \beta_2 - \beta_1$, and $\gamma = \alpha_2 - \alpha_1$.

**Marshall and Olkin BVE**

If $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are the joint densities for population $\Pi_1$ and population $\Pi_2$ respectively and $(x_1, x_2)$ follows the Marshall and Olkin BVE, then the best region of classification $R_1$ is

$$R_1: (\theta_{12} - \theta_{11})x_1 + (\lambda_2 - \lambda_1)x_2 \geq d_a \quad \text{if } x_1 < x_2$$

and

$$R_1: (\theta_{22} - \theta_{21})x_2 + (\delta_2 - \delta_1)x_1 \geq d_b \quad \text{if } x_2 < x_1$$

Where $d_a = \log\left(\frac{\beta_{22}\alpha_1}{\beta_{11}\alpha_2}\right), d_b = \log\left(\frac{\beta_{22}\delta_2}{\beta_{21}\delta_1}\right), \lambda_1 = \theta_{21} + \theta_{31}, \lambda_2 = \theta_{22} + \theta_{32}, \delta_1 = \theta_{11} + \theta_{31}, \delta_1 = \theta_{12} + \theta_{32}$.

**Block and Basu BVE**

If $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are the joint densities for population $\Pi_1$ and population $\Pi_2$ respectively and $(x_1, x_2)$ follows the Block and Basu BVE, then region of classification $R_1$ is

$$R_1: (\theta_{21} - \theta_{11})x_1 + (\varphi_2 - \varphi_1)x_2 \geq h_a \quad \text{if } x_1 < x_2$$

and

$$R_1: (\theta_{22} - \theta_{21})x_2 + (\psi_2 - \psi_1)x_1 \geq h_b \quad \text{if } x_2 < x_1$$
Where \( h_a = \log \left( \frac{\tau_2 \varphi \theta_1 \theta_1}{\tau_1 \varphi \theta_2 \theta_2} \right) \), \( h_b = \log \left( \frac{\tau_2 \varphi \theta_1 \theta_1}{\tau_1 \varphi \theta_2 \theta_2} \right) \).

\( \varphi_1 = \theta_{11} + \theta_{31}, \quad \psi_1 = \theta_{12} + \theta_{31}, \quad \tau_1 = \theta_{11} + \theta_{21}, \quad \tau_2 = \theta_{12} + \theta_{22}, \quad \theta_1 = \theta_{11} + \theta_{21} + \theta_{31}, \) and \( \theta_2 = \theta_{12} + \theta_{22} + \theta_{32} \)

For the three cases considered, any observation falling into \( R_1 \) is classified as belonging to population \( \Pi_1 \), otherwise it is classified as belonging to population \( \Pi_2 \). By replacing \( \geq \) with \( < \) in each of the \( R_1 \), we obtain the best region of classification \( R_2 \) satisfying the hypothesis that \( x = (x_1, x_2) \) belongs to population \( \Pi_2 \). The right-hand side of each of the regions of classification \( R_1 \) above represents the well-known discriminant function since in each case the right-hand side of \( R_1 \) is a function of the observation vector \( x = (x_1, x_2) \). By assuming that the two populations are equally likely and the cost of misclassification \((1/2) \) and \( c(2/1) \) are also equal, the right-hand side of the rules presented above will be purely a function of the parameters of the corresponding distribution.

**Probability of Misclassification**

Perhaps, we may wish to know the probabilities of misclassification before we draw our samples \( n_1 \) and \( n_2 \) for the purpose of determining the allocation rule. Let the probability of an observation coming from population \( \Pi_1 \) and \( \Pi_2 \) be \( p_1 \) and \( p_2 \) respectively \((p_1 + p_2 = 1)\). If \( x = (x_1, x_2) \) be a random observation then we wish to find the distribution of

\[
U = \begin{cases} 
(x_2 - x_1)\lambda + (\gamma + \omega)x_1 \geq c_a & \text{if } x_1 < x_2 \\
(x_1 - x_2)\nu + (\gamma + \omega)x_2 \geq c_b & \text{if } x_2 < x_1 
\end{cases}
\]

on the assumption that \( U \) is distributed according to the Freud bivariate exponential function. The exact distribution of \( U \) is very difficult to evaluate. Therefore, we resort to treating the asymptotic expansions of their probabilities as \( n_1 \) and \( n_2 \) increases. As such, the probability of wrongly classifying an observation originally from \( \Pi_1 \) as belonging to \( \Pi_2 \) is

\[
P(2|1) = \begin{cases} 
p_1 \Phi \left( \frac{c_a - E(U)}{\sqrt{\text{var}(U)}} \right), & \text{if } x_1 < x_2 \\
p_1 \Phi \left( \frac{c_b - E(U)}{\sqrt{V(U)}} \right), & \text{if } x_2 < x_1 
\end{cases} \tag{1}
\]

Also, the probability of wrongly classifying an observation originally from \( \Pi_2 \) as belonging to \( \Pi_1 \) is

\[
P(1|2) = \begin{cases} 
p_2 \Phi \left( \frac{c_a - E(U)}{\sqrt{\text{var}(U)}} \right), & \text{if } x_1 < x_2 \\
p_2 \Phi \left( \frac{c_b - E(U)}{\sqrt{V(U)}} \right), & \text{if } x_2 < x_1 
\end{cases} \tag{2}
\]

Where
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\[
E(U) = \begin{cases} 
\lambda \beta^2 (\beta \beta^1 - \alpha' \beta' \alpha) + (\gamma + \omega)(\alpha' + \beta), & \text{if } x_1 < x_2 \\
\lambda \left[ \frac{\beta \beta' - \alpha' \alpha}{\alpha' \beta' (\alpha + \beta)} \right] + (\gamma + \omega) \left[ \frac{\beta' + \alpha}{\beta' (\alpha + \beta)} \right], & \text{if } x_2 < x_1 
\end{cases}
\]

And

\[
V(U) = \begin{cases} 
\frac{1}{(\alpha + \beta)^2} \left[ \frac{\lambda^2 (\beta^2 - \alpha^2 \alpha^2)}{\alpha^2 \beta^2} \right] + (\gamma + \omega) \left[ \frac{\alpha^2 + 2 \alpha \beta + \beta^2}{\alpha^2} \right], & \text{if } x_1 < x_2 \\
\frac{1}{(\alpha + \beta)^2} \left[ \frac{\lambda^2 \beta^2 (\beta^2 - \alpha^2)}{\alpha^2 \beta^2} \right] + (\gamma + \omega) \left[ \frac{\beta^2 + 2 \alpha \beta + \alpha^2}{\beta^2} \right], & \text{if } x_2 < x_1 
\end{cases}
\]

In the same way, if

\[
U = \begin{cases} 
(\theta_{12} - \theta_{11})x_1 + (\lambda_2 - \lambda_1)x_2 \geq d_a & \text{if } x_1 < x_2 \\
(\theta_{22} - \theta_{21})x_2 + (\delta_2 - \delta_1)x_1 \geq d_b & \text{if } x_2 < x_1 
\end{cases}
\]
on the assumption that \(U\) is distributed according to the Marshal and Olkin bivariate exponential function, then \(P(2|1)\) and \(P(1|2)\) can be written as in (1) and (2) above by replacing \(c_a\) and \(c_b\) with \(d_a\) and \(d_b\) respectively with \(E(U)\) and \(V(U)\) given in the case of Marshal and Olkin as

\[
E(U) = \begin{cases} 
\frac{(\theta_{12} - \theta_{11})(\theta_2 + \theta_{12}) + (\lambda_2 - \lambda_1)(\theta_1 + \theta_{12})}{(\theta_1 + \theta_{12})(\theta_2 + \theta_{12})}, & \text{if } x_1 < x_2 \\
\frac{(\theta_{22} - \theta_{21})(\theta_2 + \theta_{12}) + (\delta_2 - \delta_1)(\theta_1 + \theta_{12})}{(\theta_1 + \theta_{12})(\theta_2 + \theta_{12})}, & \text{if } x_2 < x_1 
\end{cases}
\]

And

\[
V(U) = \begin{cases} 
\frac{(\theta_{12} - \theta_{11})^2(\theta_2 + \theta_{12})^2 + (\lambda_2 - \lambda_1)^2(\theta_1 + \theta_{12})^2}{(\theta_1 + \theta_{12})^2(\theta_2 + \theta_{12})^2}, & \text{if } x_1 < x_2 \\
\frac{(\theta_{22} - \theta_{21})^2(\theta_1 + \theta_{12})^2 + (\delta_2 - \delta_1)^2(\theta_2 + \theta_{12})^2}{(\theta_1 + \theta_{12})^2(\theta_2 + \theta_{12})^2}, & \text{if } x_2 < x_1 
\end{cases}
\]

Again, if

\[
U = \begin{cases} 
(\theta_{21} - \theta_{11})x_1 + (\varphi_2 - \varphi_1)x_2 \geq h_a & \text{if } x_1 < x_2 \\
(\theta_{22} - \theta_{21})x_2 + (\psi_2 - \psi_1)x_1 \geq h_b & \text{if } x_2 < x_1 
\end{cases}
\]
on the assumption that \(U\) is distributed according to the Block and Basu bivariate exponential function, then \(P(2|1)\) and \(P(1|2)\) can also be written as in (1) and (2) above by replacing \(c_a\) and \(c_b\) with \(h_a\) and \(h_b\) respectively with \(E(U)\) and \(V(U)\) given in the case of Block and Basu as
\[ E(U) \]
\[
= \left\{ \begin{array}{ll}
\left(\frac{\theta_{21} - \theta_{11}}{\theta_1 + \theta_{12}}\right) [1 + \frac{\theta_{12}\theta_2}{\theta_1 + \theta_2}] + \left(\frac{\varphi_2 - \varphi_1}{\theta_2 + \theta_{12}}\right) [1 + \frac{\theta_{12}\theta_1}{\theta_1 + \theta_2}], & \text{if } x_1 < x_2 \\
\left(\frac{\theta_{22} - \theta_{21}}{\theta_2 + \theta_{12}}\right) [1 + \frac{\theta_{12}\theta_1}{\theta_1 + \theta_2}] + \left(\frac{\psi_2 - \psi_1}{\theta_1 + \theta_{12}}\right) [1 + \frac{\theta_{12}\theta_2}{\theta_1 + \theta_2}], & \text{if } x_2 < x_1
\end{array} \right.
\]

And
\[ V(U) \]
\[
= \left\{ \begin{array}{ll}
\left(\frac{\theta_{12}\theta_2(\theta_{21} - \theta_{11})^2}{\theta_1 + \theta_{12}}\right) [1 + \frac{2\theta_1 + \theta_{12}\theta_2}{\theta_2 + \theta_{12}}] + \theta_{12}\theta_1(\varphi_2 - \varphi_1)^2 [1 + \frac{2\theta_2\theta + \theta_{12}\theta_1}{\theta_2 + \theta_{12}}], & \text{if } x_1 < x_2 \\
\left(\frac{\theta_{12}\theta_2(\theta_{22} - \theta_{21})^2}{\theta_2 + \theta_{12}}\right) [1 + \frac{2\theta_1 + \theta_{12}\theta_1}{\theta_2 + \theta_{12}}] + \theta_{12}\theta_2(\psi_2 - \psi_1)^2 [1 + \frac{2\theta_1 + \theta_{12}\theta_2}{\theta_2 + \theta_{12}}], & \text{if } x_2 < x_1
\end{array} \right.
\]

**Application**

In order to examine the applicability of the derived allocation rules, random data were generated for \( f(.) \) under various samples sizes where \( f(.) \) is the bivariate exponential density function. Apparent error rate was reported for each of MOBVE, BBBVE and FBVE for both \( x_1 < x_2 \) and \( x_2 < x_1 \) as seen in Table 1.

**Table 1: Apparent error rate for the three models using simulated data**

<table>
<thead>
<tr>
<th>Sample sizes</th>
<th>MOBVE</th>
<th>BBBVE</th>
<th>FBVE</th>
<th>MOBVE</th>
<th>BBBVE</th>
<th>FBVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.3500</td>
<td>0.4500</td>
<td>0.4000</td>
<td>0.4000</td>
<td>0.4500</td>
<td>0.4000</td>
</tr>
<tr>
<td>20</td>
<td>0.5250</td>
<td>0.5250</td>
<td>0.4750</td>
<td>0.5750</td>
<td>0.5750</td>
<td>0.4750</td>
</tr>
<tr>
<td>50</td>
<td>0.5400</td>
<td>0.5400</td>
<td>0.5200</td>
<td>0.5400</td>
<td>0.5200</td>
<td>0.4500</td>
</tr>
<tr>
<td>100</td>
<td>0.4600</td>
<td>0.4550</td>
<td>0.4250</td>
<td>0.5500</td>
<td>0.5300</td>
<td>0.3800</td>
</tr>
<tr>
<td>250</td>
<td>0.4220</td>
<td>0.4460</td>
<td>0.4120</td>
<td>0.4720</td>
<td>0.4720</td>
<td>0.3780</td>
</tr>
<tr>
<td>500</td>
<td>0.4720</td>
<td>0.4860</td>
<td>0.4520</td>
<td>0.4950</td>
<td>0.5100</td>
<td>0.3980</td>
</tr>
<tr>
<td>750</td>
<td>0.4600</td>
<td>0.4713</td>
<td>0.4453</td>
<td>0.5033</td>
<td>0.5220</td>
<td>0.3807</td>
</tr>
<tr>
<td>1000</td>
<td>0.4605</td>
<td>0.4680</td>
<td>0.4450</td>
<td>0.5020</td>
<td>0.5130</td>
<td>0.3765</td>
</tr>
<tr>
<td>1500</td>
<td>0.4473</td>
<td>0.4580</td>
<td>0.4343</td>
<td>0.4893</td>
<td>0.5023</td>
<td>0.3727</td>
</tr>
<tr>
<td>2000</td>
<td>0.4525</td>
<td>0.4680</td>
<td>0.4450</td>
<td>0.4875</td>
<td>0.5035</td>
<td>0.3813</td>
</tr>
</tbody>
</table>

Apparent error rate as presented in Table 1 indicates the applicability of the allocation rules presented above. MOBVE and BBBVE generally produced less error rate when \( x_1 < x_2 \) than when \( x_2 < x_1 \) while the FBVE produced less error rate when \( x_2 < x_1 \). In the normal distribution function, error rate mostly reduces (or stabilizes) with increase in sample size, however, such was not the case in the bivariate exponential functions studied here. Only few larger sample sizes produced less error rate.

**Illustrative examples**

The allocation rules presented above were used to examine two sets of real-life data. The first set consists of \( n=60 \) observations representing the number of days between observed successive lowest temperature, denoted as \( x_1 \) and number of days between observed successive highest precipitation denoted by \( x_2 \) recorded in two different
meteorological stations of Nigeria Institute for Oil Palm Research, Edo State. Bivariate exponential distribution has been applied and reported to be a good fit distribution for studying weather data. The grouping variable are the two different stations (both in same state of Nigeria) of the Institute. The data can be made available on request.

The second set is a summarized extract of reported observations representing time (in days) from exposure to illness onset denoted here as \( x_1 \), and, the number of days from illness onset to confirmation of infection status denoted by \( x_2 \) of the novel coronavirus (covid-19) of \( n=126 \) individuals in Wuhan Chinaas at January 2020. Only individuals with complete recorded information was included in the analysis. The data is available at [http://www.mdpi.com/2077-0383/9/2/538/s1](http://www.mdpi.com/2077-0383/9/2/538/s1). The grouping variable are; \( \Pi_1 \): Wuhan Residents (those who Lives Works and Studies in Wuhan) and \( \Pi_2 \): Other Residents (those who travelled to Wuhan).

Since both \( x_1 \) and \( x_2 \) in the two sets are from same sampling unit, there are obvious correlation between the two variables. Given that these data follows the bivariate exponential distribution and the allocation rules above holds, the essence is that, in the first set of data, an observation could be classified as either belonging to the main station or the sub-station; and in the second set, an individual case of covid-19 could be classified as either a Wuhan resident or other resident. Results obtained are reported in Table 2.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>( x_1 &lt; x_2 )</th>
<th>( x_2 &lt; x_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOBVE</td>
<td>BBBVE</td>
<td>FBVE</td>
</tr>
<tr>
<td>Set I (( n=60 ))</td>
<td>0.4688</td>
<td>0.5000</td>
</tr>
<tr>
<td>Set II (( n=126 ))</td>
<td>0.4400</td>
<td>0.4800</td>
</tr>
</tbody>
</table>

Reported apparent error rates for the real-life data do not differ markedly from those reported for the simulated data. Whereas, each of the originating exponential functions considered above may have different forms, their applicability in any appropriate real-life situation is not out of place. The choice of which to use would depend on availability and reported performance. As in this study, the choice of which allocation rule to use could be enhanced by considering their respective apparent error rates.

**Conclusion**

Discriminant analysis in the presence of violation of normality assumption of the distributions of the study variables has been discussed. This becomes necessary because such scenario often arise in many real-life situations as seen in the two illustrative examples above. This study has provided a framework for discriminant analysis when the assumption of normality is violated and the resulting data follows a bivariate exponential function.
References


