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Multivariate repeated measures (MRM) data, in which multiple outcomes are repeatedly measured at two or more occasions, are commonly collected in several disciplines including medicine, ecology, and environmental sciences, where investigators seek to understand changes in multiple correlated outcomes over time or different occasions1-6. Multivariate repeated measures data are particularly useful for studying evolutions in subjects' responses over time on multiple characteristics7.For example, Fieuws and Verbeke1reported data on a cohort of patients who having undergone kidney transplant, were longitudinally monitored at irregularly spaced intervals over a 10 year period. The repeated collection of multiple biochemical and physiological markers, which constitute multivariate repeated measures data, were used to predict 10-year success of graft. Multivariate repeated measures data are inherently challenging to analyze because they are typically characterized by non-Gaussian distributions, and high-dimensional data8, 9. Classical classification and prediction models developed for data collected in a cross-sectional study are not appropriate to address the complexities observed in multivariate repeated measures data 8, 9 .

Keywords: Multivariate repeated measures (MRM), biochemical and physiological markers.

1. Introduction

The stress strength relation is undoubtedly flexible relation to different areas of natural phenomena and human venture. It is an effective way in reliability analysis for measuring the system performance. Stress Strength Reliability (SSR) may be defined as an estimation of reliability of the system, in terms of a random variables "Y" which represents stress by the unit and "X" denoting the strength available in a unit available to resist the applied shock. It is the probability that the system will serve the purpose appropriately until the strength transcends stress i.e. if $R = (Y > X)$, the system collapse. Various attempts have been made to study the generalizations of the models and their application with respect to SSR. The notion of this idea was given by Bimbaum (1956) which was further developed by Bimbaum and Mc Carty (1958). The estimation of SSR for single component model have been considered by various researchers for several lifetime models. Johnson (1988) described SSR thoughtfully – as the probability that a system functions well in an effective environment, when a random stress Y is given to a component having the strength X, specifically Y is taken to be the highest value due to the "critical stress". The estimation procedure for SSR for power lindley distribution was obtained by Ghitany et al. (2013). Bhattacharyya and Johnson (1974) noticed that, in real world set-ups, the performance of a whole system does not only depend on single unit but depends on more than one unit which possess their own strength and considerably formulate SSR for multicomponent model. SSR for multicomponent models have been analyzed by many authors. The survival analysis in multicomponent model for Burr model have been studied by Panday and Bohran (1985). Rao (2017) et al. laid stress on generalized weibull model for obtaining profit analysis in terms of multicomponent model. Recently, Pandit and Joshi (2018) obtained SSR for generalized pareto Distribution. Also, Bashir et al. (2019) studied survival analysis for exponentiated inverse power lindley model and measured its system performance. Further, Sanku Dey and Fernando (2019) determined SSR of bathtub shape for multicomponent system.

Suppose a system under consideration consists of k iid units having strengths $X_1, X_2, \ldots, X_k$ and each unit is exposed to the random shock having magnitude Y. The system is observed to be active, if atleast $'s'$ $(s \leq k)$ out of $'k'$ components sustains the shock. If $Y, X, X_2, \ldots, X_k$ be independent random samples, $G(y)$ be cdf of Y and $F(X)$ be the common cdf of $X_1, X_2, \ldots, X_k$. The SSR for multicomponent model as proposed by Bhattacharyya and Johnson (1974) is given as

$$R_{s,k} = p[atleast\ s\ of (X_1, X_2 \ldots X_k) exceeds\ Y]$$

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} \left(1 - F(y)\right)^{i} [F(y)]^{k-i}\, dG(y) \tag{1}$$

Statistical distributions are very useful in unfolding the actual day to day phenomena. In the study of lifetime data, the Log-Logistic (LL) distribution is extensively put in use in practice and is an alternate to log-normal distribution, as it exhibits a hazard rate function which first shows an increasing trend, reaches to peak after some time and then gradually declines. It has a wide application in the field of survival analysis, actuarial sciences. Aryal (2013) transmuted the log logistic model and established its statistical properties and confirmed its flexibility in terms of survival analysis. Further, Tahir et al. (2014) demonstrated its use in reliability and life-testing experiments in censored data. Recently a new generalized odd LL family of distributions has been discussed by Hossein et al. (2017). Adeyinka and Olapade (2019) generalized LL Distribution by Transmuted technique and demonstrated the flexibility of the model in statistical data analysis by proving that the distribution obtained by the said technique has a better goodness of fit as compared to base distribution.

The technique of generalizing models is well known practice in which the parameters are added to the base distribution to generate the more flexible distribution. Various techniques have been used to obtain new statistical models. Some old practices are created on the notion of mixing the two models or by inducting some parameters to the base model. Adding single parameters to the existing distribution were first started by Azzalini (1985), Gupta et al. (1998). Marshall and okhlin (1997) developed a common technique and generate another family of life models, in terms of reliability function. Subsequently, adding two or more parameters to the base distribution was put forward by Eugene et al. (2002), Cordeiro and de-Castro (2011), and Alexander et al. (2012). In course of time a new generalized approach for generating flexible distributions was put forth by Alzaatreh et al. (2013) and gave the (T-X) method. The CDF and the corresponding pdf, if it exists, of the T-X family of distributions is given as:

$$G(x) = \int_{a}^{W(F(x))} r(t)dt$$

Here, r(t) is the pdf of continuous random variable T. F(x) is the CDF of a random variable X and W is an increasing function.

$$g(x) = \left\{\frac{d}{dx} W\left(F(x)\right)\right\} r\left(W\left(F(x)\right)\right)$$

Aljarrah et al. (2014) proposed the function *W*(*F*(*x*)) as the quantile function of a random variable *Y* and defined the *T-R*{*Y*} family.

The CDF and the corresponding pdf, if it exists, of the *T-X{Y}* family of distributions using quantile function Q_Y is given as:

$$G(x) = \int_{a}^{Q_Y(F(x))} r(t)dt \tag{2}$$

$$g(x) = \frac{f(x)}{p\{Q_Y(F(x))\}} r\{Q_Y(F(x))\} \tag{3}$$

The pdf of T-X{log-logistic} family as given by Aljarrah et al. (2014) is:

$$g(x) = \frac{\left(\frac{\alpha}{\beta}\right) f(x)}{F^{\frac{\beta-1}{\beta}}(x)\left(1-F(x)\right)^{\frac{\beta+1}{\beta}}} r\left\{\alpha\left(\frac{F(x)}{1-F(x)}\right)^{\frac{1}{\beta}}\right\} \tag{4}$$

In this article our motive is to enhance the flexibility of log logistic distribution using T-X{Y} family approach given by Aljarrah et al. (2014). The new model proposed is named as Rayleigh-Exponential {log-logistic} R-E(LL) distribution. The new model exhibits the complex shape of hazard rate function and outshines various existing model in terms of survival analysis and stress strength reliability. Various reliability measures of the proposed model have been obtained. The SSR for single and multicomponent ($R_{s,k}$) of the system has been established. Further, MLE of the parameters, asymptotic distribution and confidence interval (CI) of $R_{s,k}$ are derived.

A simulation study has been performed to access the behaviour of $R_{s,k}$, using MonteCarlo technique. Further comparison of the proposed model with existing models has been given in terms of SSR. Finally the conclusions are specified.

2. Rayleigh-Exponential (Log Logistic) Distribution

On keeping $\alpha = \beta = 1$ in the pdf on T-X{log logistic} family and let r be the pdf of Rayleigh distribution, $r(t) = \frac{2t}{\gamma^2}\, exp\left\{-\left(\frac{t}{\gamma}\right)^2\right\}$. Further F be the cdf of exponential model, $F(x) = 1 - exp\{-\theta x\}$, the pdf and cdf of R-E(LL) distribution by using 2, 3 and 4 is given as:

$$g(x) = \frac{2\theta\left(1-e^{-\theta x}\right)}{\gamma^2 e^{-2\theta x}}\, exp\left\{-\left(\left(\frac{1}{\gamma}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\} \qquad \gamma,\ \theta > 0, \quad x > 0 \qquad (5)$$

$$G(x) = 1 - exp\left\{-\left(\left(\frac{1}{\gamma}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\} \qquad \gamma,\ \theta > 0, \quad x > 0 \qquad (6)$$

3. Reliability Measures

In this section, the reliability measures of R-E(LL) has been obtained.

Survival Function: It is the probability that the object will work satisfactorily beyond any definite time. The survival function $R(x)$ of R-E(LL) distribution is given as:

$$R(x) = exp\left\{-\left(\left(\frac{1}{\gamma}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\}$$

Hazard Rate Function: It defines the probability that the component will fail in time $\{x, x + dx\}$ given that the component did not fail before in $[0, x]$. The hazard rate function $h(x)$ of R-E(LL) distribution is given as

$$h(x) = \frac{2\theta\left(1-e^{-\theta x}\right)}{\gamma^2 e^{-2\theta x}}$$

Reverse Hazard Rate: In reliability field, it is the probability of failure of the component in time interval $[x - dx, x]$ given that the failure had occurred at time x. It is the ratio of pdf and cdf . The reverse hazard rate function $\mu(x)$ of R-E(LL) distribution is given as:

$$\mu(x) = \frac{\frac{2\theta\left(1-e^{-\theta x}\right)}{\gamma^2 e^{-2\theta x}}\, exp\left\{-\left(\left(\frac{1}{\gamma}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\}}{1 - exp\left\{-\left(\left(\frac{1}{\gamma}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\}}$$

Cumulative Hazard Function: It is the probability of failure at time x given that the component survives until time x. It is given as

$$H(x) = \int_0^x h(x)dx$$

The function can also be expressed as:

$$H(x) = -\ln R(x)$$

Using above relation, the cumulative hazard function $H(x)$ of R-E(LL) distribution is given as:

$$H(x) = \left(\left(\frac{1}{\gamma}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2$$

Fig.1. pdf plot for R-E(LL)(γ,θ)

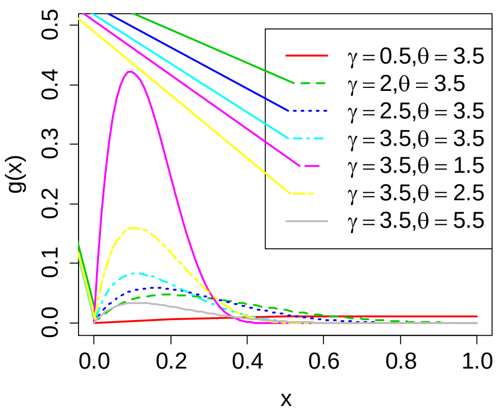


Fig.2. hazard plot for R-E(LL)(γ,θ)

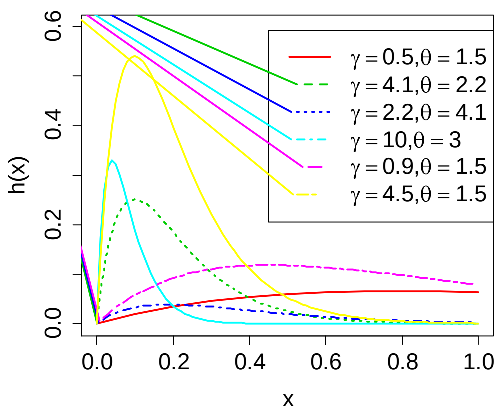
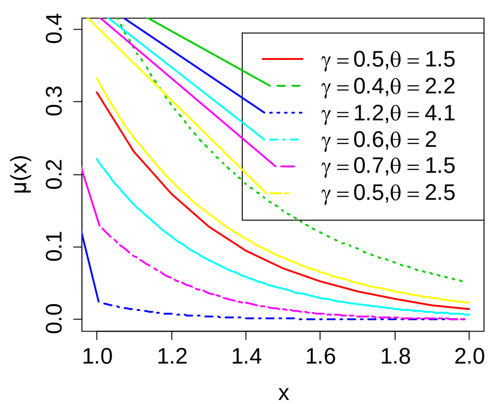


Fig.3. Reverse hazard plot for R-E(LL)(γ,θ)



4. SSR of Single Component Model

If X~ R − E(LL) with parameters (θ, γ_1) and Y~ R − E(LL) with parameters (θ, γ_2) ,then the SSR for single component model is given as

$$R = P(Y < X)$$
$$= \int_0^\infty G_Y(x) f(x)\, dx$$
$$= \int_0^\infty f(x, \theta, \gamma_1) F(x, \theta, \gamma_2)\, dx$$

$$R = \int_0^\infty \frac{2\theta(1-e^{-\theta x})}{\gamma_1^2 e^{-2\theta x}}\, exp\left\{-\left(\left(\frac{1}{\gamma_1}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\} \times \left(1 - exp\left\{-\left(\left(\frac{1}{\gamma_2}\right)\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2\right\}\right) dx$$

After solving the above integral, we get

$$R = \frac{\gamma^2}{\gamma^2+1} \qquad \text{where} \qquad \gamma = \frac{\gamma_1}{\gamma_2}$$

5. SSR for Multicomponent Model

Let X and Y be two independent random variables following R-E(LL) with parameters (θ, γ_1) and (θ, γ_2) respectively. The multicomponent SSR for R-E(LL) using (1) is given as:

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_0^\infty \left[e^{-\left(\frac{1}{\gamma_1}\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2}\right]^i \times \left[1 - e^{-\left(\frac{1}{\gamma_1}\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2}\right]^{k-i} \times \frac{2\theta(1-e^{-\theta x})\, e^{-\left(\frac{1}{\gamma_2}\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2}}{\gamma_2^2 e^{-\theta x}}\, dx$$

$$R_{s,k} = \gamma^2 \sum_{i=s}^{k} \binom{k}{i} \int_0^1 (1-z)^{k-i} z^{i+\gamma^2-1}\, dz$$

Where, $z = e^{-\left(\frac{1}{\gamma_1}\left(\frac{1-e^{-\theta x}}{e^{-\theta x}}\right)\right)^2}$ *and* $\gamma = \frac{\gamma_2}{\gamma_1}$

$$R_{s,k} = \gamma^2 \sum_{i=s}^{k} \binom{k}{i} \beta[i+\gamma^2, k-i+1]$$

$$R_{s,k} = \gamma^2 \sum_{i=1}^{k} \frac{k!(i+\gamma^2-1)}{i!\,(k+\gamma^2)} \qquad (7)$$

The probability in (7) is the expression for SSR in a multicomponent model.

6. Maximum Likelihood Estimator (MLE) Of $\mathbf{R_{s,k}}$

Let X~ R-E(LL) (θ, γ_1) of size n and Y~ R-E(LL) (θ, γ_2) of size m then, the log-likelihood function based on two independent random variables is obtained as:

$$l(\theta,\gamma_1,\gamma_2) = \sum_{i=1}^{n} ln[f(x_i)] + \sum_{j=1}^{m} ln\left[f\left(y_j\right)\right]$$

$$l(\theta,\gamma_1,\gamma_2) = nlog2 + nlog\theta - 2nlog\gamma_1 + \sum_{i=1}^{n} log\left(1-e^{-\theta x_i}\right) - 2\theta\sum_{i=1}^{n} x_i - \frac{1}{\gamma_1^2}\sum_{i=1}^{n}\left(\frac{1-e^{-\theta x_i}}{e^{-\theta x_i}}\right)^2$$

$$+mlog2 + mlog\theta - 2mlog\gamma_2 + \sum_{j=1}^{m} log\left(1-e^{-\theta y_j}\right) - 2\theta\sum_{j=1}^{m} y_j - \frac{1}{\gamma_2^2}\sum_{j=1}^{m}\left(\frac{1-e^{-\theta y_j}}{e^{-\theta y_j}}\right)^2$$

The MLE of $\theta, \gamma_1, \gamma_2$ are the solutions of the following equations:

$$\frac{\partial l}{\partial \gamma_1} = 0 => \frac{-2n}{\gamma_1} + \frac{2}{\gamma_1^3}\sum_{i=1}^{n}\left(\frac{1-e^{-\theta x_i}}{e^{-\theta x_i}}\right)^2 = 0 \qquad (8)$$

$$\frac{\partial l}{\partial \gamma_2} = 0 => \frac{-2m}{\gamma_1} + \frac{2}{\gamma_2^3}\sum_{j=1}^{m}\left(\frac{1-e^{-\theta y_j}}{e^{-\theta y_j}}\right)^2 = 0 \qquad (9)$$

$$\frac{\partial l}{\partial \theta} = 0 => \frac{n}{\theta} + \sum_{i=1}^{n} x_i\left(\frac{e^{-\theta x_i}}{1-e^{-\theta x_i}}\right) - 2\sum_{i=1}^{n} x_i - \frac{2}{\gamma_1^2}\sum_{i=1}^{n} x_i\left(\frac{1-e^{-\theta x_i}}{e^{-2\theta x_i}}\right) + \frac{m}{\theta} + \sum_{j=1}^{m} y_j\left(\frac{e^{-\theta y_j}}{1-e^{-\theta y_j}}\right) - 2\sum_{i=1}^{m} y_j - \frac{2}{\gamma_2^2}\sum_{j=1}^{m} y_j\left(\frac{1-e^{-\theta y_j}}{e^{-2\theta y_j}}\right) = 0 \qquad (10)$$

Simplifying 8 and 9, we get

$$\hat{\gamma}_1 = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1-e^{-\theta x_i}}{e^{-\theta x_i}}\right)^2\right)} \qquad (11)$$

$$\hat{\gamma}_2 = \sqrt{\left(\frac{1}{m}\sum_{j=1}^{m}\left(\frac{1-e^{-\theta y_j}}{e^{-\theta y_j}}\right)^2\right)} \qquad (12)$$

Using (11) and (12) in (10), we get, we get estimate of θ as $\hat{\theta}$

Substituting $\hat{\theta}$, we obtain the MLEs of γ_1 and γ_2 as:

$$\hat{\gamma}_1 = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1-e^{-\hat{\theta} x_i}}{e^{-\hat{\theta} x_i}}\right)^2\right)} \qquad (13)$$

$$\hat{\gamma}_2 = \sqrt{\left(\frac{1}{m}\sum_{j=1}^{m}\left(\frac{1-e^{-\hat{\theta} y_j}}{e^{-\hat{\theta} y_j}}\right)^2\right)} \qquad (14)$$

Hence, we get the MLE of $R_{s,k}$ as:

$$\hat{R}_{s,k} = \hat{\gamma}^{\,2} \sum_{i=s}^{k} \frac{k!\left(i+\hat{\gamma}^{\,2}-1\right)}{i!\,(k+\hat{\gamma}^{\,2})} \tag{15}$$

Further, the asymptotic variance (AV) of the MLEs is given as:

$$V(\hat{\gamma}_{\mathbf{1}}) = \left[E\left(-\frac{\partial^2 l}{\partial \gamma_1}\right)\right]^{-1} = \frac{4n}{\gamma_1^2}$$

$$V(\hat{\gamma}_{\mathbf{2}}) = \left[E\left(-\frac{\partial^2 l}{\partial \gamma_2}\right)\right]^{-1} = \frac{4m}{\gamma_2^2}$$

The AV of an estimate of $R_{s,k}$ which is the function of two independent statistic (γ_1, γ_2) as obtained by Rao (1973) is given as:

$$AV\left(\hat{R}_{s,k}\right) = V(\hat{\gamma}_{\mathbf{1}})\left(\frac{\partial R_{s,k}}{\partial \gamma_1}\right)^2 + V(\hat{\gamma}_{\mathbf{2}})\left(\frac{\partial R_{s,k}}{\partial \gamma_2}\right)^2$$

For simplicity of the derivation of $R_{s,k}$, we obtain derivatives of $R_{s,k}$ at $(s,k) = (1{,}3)\ and\ (2{,}4)$ and are given as:

$$\frac{\partial \hat{R}_{1,3}}{\partial \gamma_1} = \frac{12\gamma^2\left(3\gamma^4+12\gamma^2+11\right)}{\gamma_2(\gamma^2+1)^2(\gamma^2+2)^2(\gamma^2+3)^2}$$

$$\frac{\partial \hat{R}_{1,3}}{\partial \gamma_2} = \frac{-12\gamma\left(3\gamma^4+12\gamma^2+11\right)}{\gamma_2(\gamma^2+1)^2(\gamma^2+2)^2(\gamma^2+3)^2}$$

$$\frac{\partial \hat{R}_{2,4}}{\partial \gamma_1} = \frac{-48\gamma\left(3\gamma^4+18\gamma^2+26\right)}{\gamma_2(\gamma^2+2)^2(\gamma^2+3)^2(\gamma^2+4)^2}$$

$$\frac{\partial \hat{R}_{2,4}}{\partial \gamma_2} = \frac{-48\gamma^2\left(3\gamma^4+18\gamma^2+26\right)}{\gamma_2(\gamma^2+2)^2(\gamma^2+3)^2(\gamma^2+4)^2}$$

$$AV\left(\hat{R}_{1,3}\right) = \frac{36\hat{\gamma}^2\left(3\hat{\gamma}^4+12\hat{\gamma}^2+11\right)^2}{\left((\hat{\gamma}^2+1)(\hat{\gamma}^2+2)(\hat{\gamma}^2+3)\right)^4}\left(\frac{\hat{\gamma}^2}{n}+\frac{1}{m}\right)$$

$$AV\left(\hat{R}_{2,4}\right) = \frac{576\hat{\gamma}^2\left(3\hat{\gamma}^4+18\hat{\gamma}^2+26\right)^2}{\left((\hat{\gamma}^2+1)(\hat{\gamma}^2+2)(\hat{\gamma}^2+3)\right)^4}\left(\frac{\hat{\gamma}^2}{n}+\frac{1}{m}\right)$$

$$As\ m_1 \to \infty, m_2 \to \infty, \qquad \frac{\hat{R}_{s,k}-R_{s,k}}{AV\left(\hat{R}_{s,k}\right)} \to N(0,1)$$

The asymptotic 95% (CI) is given as $\hat{R}_{s,k} \pm 1.96\sqrt{AV\left(\hat{R}_{s,k}\right)}$

The asymptotic 95% CI for $\hat{R}_{1,3}\ and\ \hat{R}_{2,4}$ respectively is given by

$$\hat{R}_{1,3} \pm 1.96\,\frac{6\hat{\gamma}\left(3\hat{\gamma}^4+12\hat{\gamma}^2+11\right)}{\left((\hat{\gamma}^2+1)(\hat{\gamma}^2+2)(\hat{\gamma}^2+3)\right)^4}\sqrt{\left(\frac{\hat{\gamma}^2}{n}+\frac{1}{m}\right)}$$

$$\hat{R}_{2,4} \pm 1.96\,\frac{24\hat{\gamma}\left(3\hat{\gamma}^4+18\hat{\gamma}^2+20\right)}{\left((\hat{\gamma}^2+2)(\hat{\gamma}^2+3)(\hat{\gamma}^2+4)\right)^4}\sqrt{\left(\frac{\hat{\gamma}^2}{n}+\frac{1}{m}\right)}$$

7. Simulation Study

Results are obtained from Monte-Carlo simulation on R Software, R Core Team (2017), to access the performance of $R_{s,k}$ by changing the sample sizes using the following algorithm:

7.1 Algorithm:

To study the behaviour of $R_{s,k}$, the following algorithm has been used

1) Compute $R_{1,3}$ and $R_{2,4}$ from (7) for given values of (γ_1, γ_2)
2) Generate 3000 r.s of sizes $10(5)35$ using Monte Carlo Simulation.
3) Obtain $\hat{\gamma}_1$ and $\hat{\gamma}_2$ using (13) and (14).
4) Compute $\hat{R}_{1,3}$ and $\hat{R}_{2,4}$ from (15).
5) Calculate Average Bias, Average Mean Square Error (MSE), Asymptotic 95% CI for $R_{1,3}$ and $R_{2,4}$.

3,000 random samples of varying sizes $10(5)35$ were generated for $(\gamma_1, \gamma_2) = (2.5, 0.5), (2.0, 0.5), (1.5, 0.5), (1.0,0.5), (0.5,0.5), (0.5, 1.0), (0.5, 1.5), (0.5,2.0)$, for both stress and strength population. In order to get the multicomponent reliability for the pairs $(s, k) = (1,3), (2,4)$, the ML estimators of γ_1, γ_2 are substituted in γ. We considered, θ to be known $(\theta = 1)$ in the simulation study. The average bias, average MSE are present in Tables 2 and 3. Average confidence length (ACL) of $R_{s,k}$ is given in table 4. The true values of multicomponent SSR for given combination of (γ_1, γ_2) for

$(s, k) = (1,3)$ are $0.9996, 0.9989, 0.9954, 0.9714, 0.75, 0.3435,$ 0.1778 and 0.1059 and for

$(s, k) = (2,4)$ are $0.9989, 0.9964, 0.9860, 0.9285, 0.6, 0.2277, 0.111$

and 0.0647. From Table 1, it is observed that as θ_2 (stress parameter) increases, keeping θ_1 (strength parameter) constant, the reliability of multicomponent stress strength model decreases whereas, as θ_1 increases, keeping θ_2 constant, the multicomponent SSR increases. Further as sample size increases, the bias, MSE and length of CI decreases.

Table 1. Estimated Reliability $R_{s,k}$									
		(γ_1,γ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.9992	0.9978	0.9917	0.9587	0.7333	0.3525	0.1884	0.1141
	(15.15)	0.9994	0.9983	0.9932	0.9637	0.7400	0.3515	0.1859	0.1120
	(20,20)	0.9995	0.9985	0.9940	0.9664	0.7457	0.3522	0.1856	0.1115
	(25,25)	0..9995	0.9986	0.9942	0.9670	0.7470	0.3508	0.1842	0.1105
	(30,30)	0.9996	0.9987	0.9945	0.9680	0.7469	0.3493	0.1829	0.1096
	(35,35)	0.9996	0.9987	0.9945	0.9682	0.7460	0.3473	0.1815	0.1086
(2,4)	(10,10)	0.9976	0.9933	0.9775	0.9090	0.5938	0.2390	0.1196	0.0704
	(15.15)	0.9982	0.9946	0.9809	0.9167	0.5984	0.2366	0.1174	0.0688
	(20,20)	0.9984	0.9951	0.9826	0.9209	0.6018	0.2365	0.1170	0.0685
	(25,25)	0.9985	0.9956	0.9834	0.9227	0.6019	0.2349	0.1159	0.0677
	(30,30)	0.9986	0.9957	0.9837	0.9233	0.6011	0.2335	0.1150	0.0672
	(35,35)	0.9985	0.9957	0.9839	0.9234	0.5999	0.2318	0.1139	0.0665

Table 2. Average bias of $R_{s,k}$									
		(γ_1,γ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	-0.0004	-0.0011	-0.0037	-0.0126	-0.0166	0.0089	0.0106	0.0081
	(15.15)	-0.0003	-0.0006	-0.0021	-0.0076	-0.0090	0.0079	0.0081	0.0060
	(20,20)	-0.0002	-0.0004	-0.0014	-0.0050	-0.0042	0.0086	0.0078	0.0056
	(25,25)	-0.0002	-0.0003	-0.0010	-0.0038	-0.0039	0.0072	0.0064	0.0045
	(30,30)	-0.0001	-0.0002	-0.0009	-0.0033	-0.0030	0.0057	0.0051	0.0037
	(35,35)	-0.0001	-0.0002	-0.0008	-0.0032	-0.0029	0.0037	0.0037	0.0027
(2,4)	(10,10)	-0.0012	-0.0031	-0.0084	-0.0194	-0.0061	0.0113	0.0085	0.0057
	(15.15)	-0.0006	-0.0018	-0.0050	-0.0118	-0.0015	0.0089	0.0062	0.0041
	(20,20)	-0.0004	-0.0011	-0.0033	-0.0076	-0.0018	0.0087	0.0058	0.0038
	(25,25)	-0.0003	-0.0008	-0.0025	-0.0057	-0.0019	0.0072	0.0047	0.0030
	(30,30)	-0.0002	-0.0007	-0.0022	-0.0052	0.0011	0.0058	0.0038	0.0025
	(35,35)	-0.0002	-0.0007	-0.0020	-0.0051	-0.0005	0.0041	0.0028	0.0018

Table 3. Average MSE of estimates of $R_{s,k}$

		(γ_1,γ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.0039	0.0084	0.0399	0.0016	0.0155	0.0141	0.0060	0.0026
	(15.15)	0.0051	0.0120	0.0380	0.0008	0.0102	0.0093	0.0038	0.0016
	(20,20)	0.0023	0.0058	0.0204	0.0005	0.0076	0.0072	0.0029	0.0012
	(25,25)	0.0014	0.0035	0.0132	0.0003	0.0060	0.0057	0.0022	0.0009
	(30,30)	0.0010	0.0027	0.0101	0.0003	0.0052	0.0048	0.0019	0.0007
	(35,35)	0.0010	0.0026	0.0084	0.0002	0.0045	0.0041	0.0016	0.0006
(2,4)	(10,10)	0.0097	0.0616	0.0006	0.0051	0.0194	0.0087	0.0029	0.0011
	(15.15)	0.0137	0.0260	0.0003	0.0029	0.0131	0.0056	0.0017	0.0006
	(20,20)	0.0066	0.0136	0.0002	0.0020	0.0100	0.0043	0.0013	0.0005
	(25,25)	0.0040	0.0087	0.0002	0.0014	0.0080	0.0033	0.0010	0.0003
	(30,30)	0.0031	0.0068	0.0001	0.0012	0.0069	0.0028	0.0008	0.0003
	(35,35)	0.0024	0.0053	0.0001	0.0010	0.0060	0.0024	0.0007	0.0002

Table 4. ACL of estimates of $R_{s,k}$

		(γ_1,γ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.0022	0.0066	0.0225	0.0895	0.3162	0.3067	0.1984	0.1306
	(15.15)	0.0014	0.0043	0.0156	0.0677	0.2620	0.2554	0.1631	0.1068
	(20,20)	0.0010	0.0032	0.0122	0.0559	0.2292	0.2239	0.1427	0.0936
	(25,25)	0.0008	0.0027	0.0104	0.0490	0.2067	0.2015	0.1281	0.8416
	(30,30)	0.0007	0.0024	0.0093	0.0445	0.1898	0.1846	0.1173	0.0772
	(35,35)	0.0006	0.0022	0.0085	0.0412	0.1769	0.1713	0.1087	0.0718
(2,4)	(10,10)	0.0035	0.0093	0.0273	0.0834	0.1799	0.1190	0.0675	0.0420
	(15.15)	0.0023	0.0063	0.0198	0.0659	0.1508	0.0980	0.0550	0.0341
	(20,20)	0.0017	0.0049	0.0160	0.0559	0.1322	0.0856	0.0479	0.0298
	(25,25)	0.0014	0.0041	0.0139	0.0496	0.1194	0.0767	0.0429	0.0267
	(30,30)	0.0012	0.0037	0.0125	0.0453	0.1096	0.0701	0.0392	0.0245
	(35,35)	0.0011	0.0034	0.0115	0.0421	0.1019	0.0648	0.0363	0.0228

8. Comparative Analysis

An effort has been made to assess our results with existing distributions Rao et al. (2017), Rao (2017) at $(\gamma_1, \gamma_2) = (2.5, 0.5)$ which is displayed in Table 5. The results indicate average bias, MSE, and average lengths of CI are lesser as compared to previous results. Thus, our results for R-E(LL) distribution acts well with respect to the study of multicomponent SSR as compared to the existing models.

Table 5. Comparison of bias, MSE, ACL at (γ_1, γ_2) = (2.5, 0.5) for three models.

(s,k)	(1,3)				(2,4)		
	(n,m)	$Bias$	MSE	ACL	$Bias$	MSE	ACL
Raleigh Exponential Log-logistic Distribution	(10,10)	-0.0004	0.0039	0.0022	-0.0012	0.0097	0.0035
	(15,15)	-0.0003	0.0051	0.0014	-0.0006	0.0137	0.0023
	(20,20)	-0.0002	0.0023	0.0010	-0.0004	0.0066	0.0017
	(25,25)	-0.0002	0.0014	0.0008	-0.0003	0.0040	0.0014
	(30,30)	-0.0001	0.0010	0.0007	-0.0002	0.0031	0.0012
	(35,35)	-0.0001	0.0010	0.0006	-0.0002	0.0024	0.0011
Erlang-Truncated Exponential Distribution	(10,10)	0.0097	0.0127	0.4339	0.0113	0.0078	0.3361
	(15,15)	0.0082	0.0089	0.3585	0.0090	0.0053	0.2752
	(20,20)	0.0064	0.0067	0.3118	0.0069	0.0040	0.2380
	(25,25)	0.0029	0.0053	0.2786	0.0038	0.0031	0.2114
	(30,30)	0.0045	0.0044	0.2558	0.0047	0.0026	0.1940
	(35,35)	0.0040	0.0037	0.2373	0.0042	0.0021	0.1796
Two Parameter Exponentiated Weibull Distribution	(10,10)	-0.0061	0.0100	0.1095	-0.0096	0.0026	0.1816
	(15,15)	-0.0031	0.0050	0.0873	-0.0049	0.0015	0.1454
	(20,20)	-0.0030	0.0040	0.0753	-0.0048	0.0010	0.1255
	(25,25)	-0.0020	0.0030	0.0667	-0.0032	0.0008	0.1115
	(30,30)	-0.0019	0.0030	0.0608	-0.0025	0.0007	0.1016
	(35,35)	-0.0019	0.0020	0.0556	-0.0031	0.0006	0.0930

9. Conclusion:

In this article, a new lifetime model known as Rayleigh-Exponential (Log Logistic) Distribution has been introduced. The proposed model acts well in terms of lifetime distribution as compared to existing models. Different reliability measures of the said distribution have been obtained. The SSR for single and multicomponent model has been derived. The Reliability model parameters are estimated using MLE method. MonteCarlo simulation technique has been used to compute large-sample CI. We observe that greater the sample size, lesser is the bias, MSE. Also with increase in sample size, CI decreases. Further, to examine the efficacy of the model, our work has been compared with previous work and concluded that R-E(LL) performs well in survival analysis in consideration to multicomponent SSR.

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