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In the present study, we propose a new estimator for population mean using Kadilar and Cingi (2005) estimators using two auxiliary variables. The results for the mean square error of the introduced estimator in this article are find for the first order of approximation. The mean square error's results have also been checked through numerical illustration. It is observed that our proposed estimator is having less mean square error than the other estimators.

Keywords: Mean square error (MSE), Auxiliary variables, Ratio estimator and Regression estimator.

1. Introduction

In statistics, we are interested to know the behavior of the population based on a sample. Sample results cannot be accurate as the population results. Every time there is a difference between the results of sample and the results of population. Throughout this paper, we are trying to minimize this mean square error. For this we have proposed an estimator using auxiliary information for two variables. Our target is to find Mean square errors of some estimators which is already given in literature, is always more than our proposed estimator.

The auxiliary variable's information are effectively used to increase the efficiency of the estimator of the population mean. Ratio, product, and regression estimators are utilized in many conditions, see for example Cochran (1977) and Murthy (1967) Modified ratio estimators are developed to achieve further improvements on the ratio estimator with known parameters of the auxiliary variable, which include Sisodia & Dwivedi (1981) with known Co-efficient of Variation, Singh et. (2004) with the known Skewness, Subramani and Kumarapandiyan (2012) with the known median and its linear combinations with the other known parameters. The given article is dealing with an improved regression estimator with known correlation coefficient and skewness for two auxiliary variables.

We have Ratio estimator and regression estimators for two auxiliary variables respectively given in (1) and (2). Kadilar and Cingi (2005) introduced an estimator given in (3).

$$t_1 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right) \tag{1}$$

$$t_2 = \bar{y} + b_1(\bar{X} - \bar{x}) + b_2(\bar{Z} - \bar{z}) \tag{2}$$

$$t_3 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha_1}\left(\frac{\bar{Z}}{\bar{z}}\right)^{\alpha_2} + b_1(\bar{X} - \bar{x}) + b_2(\bar{Z} - \bar{z}) \tag{3}$$

The mean squared error of t_1, t_2, and t_3 after taking first order of approximation is hereby

$$MSE(t_1) = \bar{Y}^2\left(\frac{1-f}{n}\right)[C_x^2 + C_y^2 + C_z^2 - 2\rho_{xy}C_xC_y - 2\rho_{yz}C_yC_z + 2\rho_{xz}C_xC_z]$$

$$MSE(t_2) = \left(\frac{1-f}{n}\right)[\bar{Y}^2C_y^2 + b_1^2\bar{X}^2C_x^2 + b_2^2\bar{Z}^2C_z^2 + 2b_1b_2\bar{X}\bar{Z}\rho_{xz}C_xC_z - 2\bar{X}\bar{Y}b_1\rho_{xy}C_xC_y - 2\bar{Y}\bar{Z}b_2\rho_{yz}C_yC_z]$$

$$MSE(t_3) = \left(\frac{1-f}{n}\right)[\alpha_1^2(\bar{Y}^2C_x^2) + \alpha_2^2(\bar{Y}^2C_z^2) + 2\alpha_1\alpha_2(\bar{Y}^2\rho_{xz}C_xC_z) - 2\alpha_1\left(\bar{Y}^2\rho_{xy}C_xC_y - \bar{Y}B_1\bar{X}C_x^2 - \bar{Y}B_2\bar{Z}\rho_{xz}C_xC_z\right) - 2\alpha_2\left(\bar{Y}^2\rho_{yz}C_yC_z - \bar{Y}B_2\bar{Z}C_z^2 - \bar{Y}B_1\bar{X}\rho_{xz}C_xC_z\right) + (B_1^2\bar{X}^2C_x^2 + B_2^2\bar{Z}^2C_z^2 + 2B_1B_2\bar{X}\bar{Z}\rho_{xz}C_xC_z) + \left(\bar{Y}^2C_y^2 - 2\bar{Y}B_1\bar{X}\rho_{xy}C_xC_y - 2\bar{Y}B_2\bar{Z}\rho_{yz}C_yC_z\right)]$$

2. The proposed estimator

Using (1), (2) and adapting Kadilar and Cingi's (3) estimator for two auxiliary variables

$$t_{akm} = \bar{y}\left(\frac{\bar{X}+\theta_1}{\bar{x}+\theta_1}\right)^{\alpha_1}\left(\frac{\bar{Z}+\theta_2}{\bar{z}+\theta_2}\right)^{\alpha_2} + b_1(\bar{X} - \bar{x}) + b_2(\bar{Z} - \bar{z}) \tag{4}$$

To find the Mean square error (MSE) of t_{akm} up to the first order of approximation, we have

$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $\bar{z} = \bar{Z}(1+e_2)$

$e_0 = \frac{\bar{y}}{\bar{Y}} - 1$, $e_1 = \frac{\bar{x}}{\bar{X}} - 1$, $e_2 = \frac{\bar{z}}{\bar{Z}} - 1$

$E(e_0) = 0$, $E(e_1) = 0$, $E(e_2) = 0$

$E(e_0^2) = \left(\frac{1-f}{n}\right)C_y^2$, $E(e_1^2) = \left(\frac{1-f}{n}\right)C_x^2$, $E(e_2^2) = \left(\frac{1-f}{n}\right)C_z^2$,

$E(e_0e_1) = \left(\frac{1-f}{n}\right)\rho_{xy}C_xC_y$, $E(e_0e_2) = \left(\frac{1-f}{n}\right)\rho_{yz}C_yC_z$, $E(e_1e_2) = \left(\frac{1-f}{n}\right)\rho_{xz}C_xC_z$

$b_1 = \frac{s_{yx}}{s_x^2}$ and $b_2 = \frac{s_{yz}}{s_z^2}$, where s_{yx} and s_{yz} are the sample covariances between y and x and between z respectively.

Using above notations, we get

$$t_{akm} = \bar{Y}(1+e_0) \left(\frac{\bar{X}+\theta_1}{\bar{X}(1+e_1)+\theta_1}\right)^{\alpha_1} \left(\frac{\bar{Z}+\theta_2}{\bar{Z}(1+e_2)+\theta_2}\right)^{\alpha_2} + b_1(\bar{X} - \bar{X} - \bar{X}e_1) + b_2(\bar{Z} - \bar{Z} - \bar{Z}e_2)$$

$$t_{akm} = \bar{Y}(1+e_0) \left(\frac{\bar{X}+\theta_1}{\bar{X}+\bar{X}e_1+\theta_1}\right)^{\alpha_1} \left(\frac{\bar{Z}+\theta_2}{\bar{Z}+\bar{Z}e_2+\theta_2}\right)^{\alpha_2} - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

$$t_{akm} = \bar{Y}(1+e_0) \left(\frac{\bar{X}+\theta_1}{\bar{X}+\theta_1\left[1+\frac{\bar{X}e_1}{\bar{X}+\theta_1}\right]}\right)^{\alpha_1} \left(\frac{\bar{Z}+\theta_2}{\bar{Z}+\theta_2\left[1+\frac{\bar{Z}e_2}{\bar{Z}+\theta_2}\right]}\right)^{\alpha_2} - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

Let $k_1 = \frac{\bar{X}}{\bar{X}+\theta_1}$ and $k_2 = \frac{\bar{Z}}{\bar{Z}+\theta_2}$, θ_1 and θ_2 are known parameters.

$$t_{akm} = \bar{Y}(1+e_0) \left(\frac{1}{1+k_1e_1}\right)^{\alpha_1} \left(\frac{1}{1+k_2e_2}\right)^{\alpha_2} - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

$$t_{akm} = \bar{Y}(1+e_0)\,(1 + k_1e_1)^{-\alpha_1}(1 + k_2e_2)^{-\alpha_2} - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

$$t_{akm} = \bar{Y}(1+e_0) \left[1 - \alpha_1k_1e_1 + \frac{\alpha_1(\alpha_1+1)}{2!}k_1^2e_1^2\right]\left[1 - \alpha_2k_2e_2 + \frac{\alpha_2(\alpha_2+1)}{2!}k_2^2e_2^2\right] - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

$$t_{akm} = \bar{Y}(1+e_0) \left[1 - \alpha_1k_1e_1 + \frac{\alpha_1(\alpha_1+1)}{2!}k_1^2e_1^2 - \alpha_2k_2e_2 + \alpha_1\alpha_2k_1k_2e_1e_2 + \frac{\alpha_2(\alpha_2+1)}{2!}k_2^2e_2^2\right] - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

$$t_{akm} = \bar{Y}\left[1 - \alpha_1k_1e_1 + \frac{\alpha_1(\alpha_1+1)}{2!}k_1^2e_1^2 - \alpha_2k_2e_2 + \alpha_1\alpha_2k_1k_2e_1e_2 + \frac{\alpha_2(\alpha_2+1)}{2!}k_2^2e_2^2 + e_0 - \alpha_1k_1e_0e_1 - \alpha_2k_2e_0e_2\right] - b_1\bar{X}e_1 - b_2\bar{Z}e_2$$

Subtracting $\bar{Y}$ from both sides,

$$(t_{akm} - \bar{Y}) = \bar{Y}[e_0 - \alpha_1k_1e_1 - \alpha_2k_2e_2] - [b_1\bar{X}e_1 - b_2\bar{Z}e_2]$$

Squaring both sides and taking expectation, we get

$$E(t_{akm} - \bar{Y})^2 = E\{\bar{Y}[e_0 - \alpha_1k_1e_1 - \alpha_2k_2e_2] - [b_1\bar{X}e_1 - b_2\bar{Z}e_2]\}^2$$

$$MSE(t_{akm}) = E\{\bar{Y}^2[e_0 - \alpha_1k_1e_1 - \alpha_2k_2e_2]^2 + [b_1\bar{X}e_1 - b_2\bar{Z}e_2]^2 - 2\bar{Y}[e_0 - \alpha_1k_1e_1 - \alpha_2k_2e_2][b_1\bar{X}e_1 - b_2\bar{Z}e_2]\}$$

$$MSE(t_{akm}) = E[\bar{Y}^2(e_0^2 + \alpha_1^2k_1^2e_1^2 + \alpha_2^2k_2^2e_2^2 - 2\alpha_1k_1e_0e_1 + 2\alpha_1\alpha_2k_1k_2e_1e_2 - 2e_0\alpha_2k_2e_2) + (b_1^2\bar{X}^2e_1^2 + b_2^2\bar{Z}^2e_2^2 + 2b_1b_2\bar{X}\bar{Z}e_1e_2) - 2\bar{Y}(b_1\bar{X}e_0e_1 + b_2\bar{Z}e_0e_2 - b_1\alpha_1k_1\bar{X}e_1^2 - b_2\alpha_1k_1\bar{Z}e_1e_2 - b_1\alpha_2k_2\bar{X}e_1e_2 - b_2\alpha_2k_2\bar{Z}e_2^2)]$$

After solving up to the first order of approximation the MSE of t_{ak} is given below

$$(t_{akm}) == \bar{Y}^2fC_y^2 + \alpha_1^2A_1 + \alpha_2^2A_2 + 2\alpha_1\alpha_2A_3 - 2\alpha_1A_4 - 2\alpha_2A_5 + A_6 \tag{5}$$

where, $A_1 = \bar{Y}^2k_1^2fC_x^2$

$A_2 = \bar{Y}^2 k_2^2 f C_z^2$

$A_3 = \bar{Y}^2 k_1 k_2 f \rho_{xz} C_x C_z$

$A_4 = \bar{Y}^2 k_1 f \rho_{xy} C_x C_y - \bar{Y} B_1 k_1 \bar{X} f C_x^2 - \bar{Y} B_2 k_1 f \bar{Z} \rho_{xz} C_x C_z$

$A_5 = \bar{Y}^2 k_2 f \rho_{yz} C_y C_z \bar{Y} B_2 k_2 \bar{Z} f C_z^2 - \bar{Y} B_1 k_2 \bar{X} f \rho_{xz} C_x C_z$

$A_6 = B_1^2 \bar{X}^2 f C_x^2 + B_2^2 \bar{Z}^2 f C_z^2 + 2B_1 B_2 \bar{X}\bar{Z} f \rho_{xz} C_x C_z + \bar{Y}^2 f C_y^2 - 2\bar{Y} B_1 \bar{X} f \rho_{xy} C_x C_y - 2\bar{Y} B_2 \bar{Z} f \rho_{yz} C_y C_z$

Differentiating with respect to α_1 and α_2

$$2\alpha_1 A_1 + 2\alpha_2 A_3 = 2A_4 \tag{6}$$

$$2\alpha_2 A_2 + 2\alpha_1 A_3 = 2A_5 \tag{7}$$

On solving equation (6) and (7), we get

$$\alpha_1 = \frac{A_2 A_4 - A_3 A_5}{A_1 A_2 - A_3^2} \text{ and } \alpha_2 = \frac{A_3 A_4 - A_1 A_5}{A_3^2 - A_1 A_2}$$

3. Application

The performance of the proposed improve regression estimator through ratio estimator are assessed with that of the sample mean and the existing ratio estimators for two different populations. The population 1 is taken from Singh & Chaudhary (1986) given in page 177 and population 2 is taken from taken from the Cingi & Kadilar (2009) given in page 117 is shown in Table 1. In first population area under wheat (1974) is our study variable, area under wheat (1971) is first variable and area under wheat (1973) is the second auxiliary variable. For the second population the population mean of the height of the fish is our study variable, the population mean of the length of the head is first variable and the population mean of the length of the fin is the second auxiliary variable. The results MSE values of different estimators is shown in Table 2.

Table 1. Data Statistics

Population I	**Population II**
N = 34	N = 25
n = 20	n = 10
$\bar{X}$ = 208.88	$\bar{X}$ = 14.3
$\bar{Y}$ = 856.41	$\bar{Y}$ = 75.28
$\bar{Z}$ = 199.44	$\bar{Z}$ = 6.82
S_x = 150.22	S_x = 3.17
S_y = 733.14	S_y = 17.27
S_z = 150.22	S_z = 1.53
ρ_{yx}= 0.45	ρ_{yx}= 0.99
ρ_{yz}= 0.45	ρ_{yz}= 0.89

ρ_{xz} = 0.98	ρ_{xz} = 0.92
B_1 = 2.19	B_1 = 2.60
B_2 = 2.19	B_2 = 10.04
β_{21} = 2.91	β_{21} = 4.26
β_{22} = 3.73	β_{22} = 4.35
β_{12} = 1.28	β_{12} = 0.86
β_{11} = 0.87	β_{11} = 1.24

Table 2. Results MSE values of different estimators

Estimators	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	**MSE (Population I)**	**MSE (Population II)**
t_1			25154.60	8.12
t_2			10976.42	4.87
t_3			8802.54	4.38
t_{akm}	ρ_{xz}	ρ_{xz}	**8792.02**	**4.10**
	β_{22}	β_{22}	**8769.16**	**2.04**

4. Conclusion

In the present paper, we have introduced an estimator for finding the population mean of study variable using available information of two auxiliary variables. From the results given in Table 2 we can have an idea that the introduced estimator t_{akm} is performing better than other estimators in literature under correlation coefficient and coefficient of skewness of auxiliary variables.

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