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In this paper, we have proposed an estimator for domain mean using auxiliary character when domain mean of the auxiliary character is unknown. The some members of the proposed estimator for domain mean using auxiliary character with unknown domain mean are studied. The expressions for bias and mean square error of the proposed estimator have been studied. The value of the minimum mean square error of the proposed estimator has been explained with the relevant estimators. The proposed estimator is found more efficient than the relevant estimators. The simulation study is also obtained in the terms of an absolute relative bias and simulated relative standard error using the data (Sarndal et al. (1992, Appendix B)). The simulation study shows that the proposed estimator is more efficient than the relevant estimators for those domains for which the synthetic assumption of the proposed estimator meet closely.

Keywords: Synthetic estimator, Domain, Simulation, Synthetic estimator.

1. Introduction

If we are interested in the estimation of mean of sub-populations instead of the whole population then in these cases sub-populations are treated as domains such as census area, block, district or states. The synthetic estimator plays an important role in the estimation of small domains for small area estimation. The synthetic estimator's demand is increasing due to its wide importance specially, when less information is received for small domains. Sometimes the available information may be sufficient in the domain in such cases direct estimator for domain mean is more efficient than synthetic estimator (Purcell and Linacre (1976), and Singh and Tessier (1976)). But, sometimes the information are not sufficiently available in the domain, then we use indirect estimator instead of the direct estimator. In indirect estimator, we select a sample from population instead of the domain because we use the surrounding information about the domain. The literatures about indirect estimators have been discussed by Rao (2003) and Longford (2005). The indirect estimators are of three types: synthetic estimator, composite estimator and James-Stein estimator. First, an estimator is termed as synthetic estimator if a reliable estimator for large area, covering several small areas is used to derive an indirect estimator for large area under the assumption that the small areas characters have the characteristic as large area (Gonzalez (1973)). The synthetic estimator using auxiliary character has been proposed by Ghangrude and Singh (1978) and Rai and Pandey (2013). But due to an increase of the complexity in the equation of mean square error and bias, we are using synthetic assumption and obtain the performance of estimator via simulation study (Tikkiwal and Ghiya (2000) and Khare and Ashutosh (2017, 2018)). Now, another estimator has been developed with the help of the synthetic estimator is termed as composite estimator. The composite estimator is a convex combination of direct estimator and synthetic estimator. The composite estimator has been discussed by Schaible (1978). Due to increased problem in small area estimation, we need a modified synthetic estimator which performs better under the synthetic assumption than the synthetic estimator. Now motivated by Searl (1964) and Kumar (1993), we proposed a synthetic estimator for domain mean using information on the domain means of auxiliary character based on a large sample from the domain when domain mean of auxiliary character is not known.

In this paper, we proposed an improved generalized synthetic estimator for domain mean using auxiliary character when the domain mean of the auxiliary character is unknown and also, discussed several members of the estimator as special cases for domain mean. The better performance of the proposed estimator has been justified in relation to the relevant estimators based on a simulation study for the data (Sarndal et al. (1992), Appendix B).

2. Methodology

We have a finite population U:($1, 2, 3, ...N$), is classified into K non-overlapping domains U_a of size N_a (a=1, 2, 3,……K), where we denote the study character by y and the auxiliary character by x. We selected a sample s of size n from S of size N using simple random sampling without replacement (SRSWOR) method of sampling from y and x, and information of y and x are collected. Again a sample $s_a^{'}$ of size $n_a^{'}$ is selected from S_a of the domain U_a have size N_a and the information $n_a^{'}$ units for x is obtained.

Let

$$\sum_{a=1}^{K} N_a = N \quad \text{and} \quad \sum_{a=1}^{K} n_a = n.$$

The following notations are used in the present context:

$\overline{Y}$: Population mean of the study character y.

$\overline{Y}_a$: a^{th} domain mean of the study character.

$\overline{y}$: Sample mean of the study character based on sample of size n from the population of size N.

$\overline{X}$: Population mean of the auxiliary character x.

$\overline{X}_a$: a^{th} domain mean of auxiliary character x.

$\overline{x}$: Sample mean of the auxiliary character x of a^{th} domain based on sample of size $n_a^{'}$.

The population mean square for y and x, covariance between y and x for a^{th} domain and the coefficient of variation for y and x for population U are given as follows:

$$S_Y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \overline{Y})^2 , S_X^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(X_i - \overline{X})^2 ,$$

$$S_{YX} = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \overline{Y})(X_i - \overline{X}) , C_Y = \frac{S_Y}{\overline{Y}} , C_X = \frac{S_X}{\overline{X}} \text{ and } C_{XY} = \frac{S_{YX}}{\overline{X}\overline{Y}} .$$

The domain mean square for y and x, covariance between y and x for a^{th} domain and the coefficient of variation for y and x of a^{th} domain U_a are given as follows:

$$S_{Y_a}^2 = \frac{1}{(N_a-1)}\sum_{i=1}^{N_a}(Y_{a_i} - \overline{Y}_a)^2 ,\ S_{X_a}^2 = \frac{1}{(N_a-1)}\sum_{i=1}^{N_a}(X_{a_i} - \overline{X}_a)^2 ,$$

$$S_{Y_aX_a} = \frac{1}{(N_a-1)}\sum_{i=1}^{N_a}(Y_{a_i} - \overline{Y}_a)(X_{a_i} - \overline{X}_a) \ , C_{Y_a} = \frac{S_{Y_a}}{\overline{Y}_a} , C_{X_a} = \frac{S_{X_a}}{\overline{X}_a} \text{ and}$$

$$C_{X_aY_a} = \frac{S_{Y_aX_a}}{\overline{X}_a\overline{Y}_a} .$$

where, X_{a_i} and Y_{a_i} are the i^{th} value of a^{th} domain for the character y and x.

2.1 Synthetic estimators for domain mean using auxiliary character

Ratio synthetic estimator for domain mean ($T_{RS,a}$)

$$T_{RS,a} = \frac{\overline{y}}{\overline{x}}\overline{X}_a \qquad \text{Rao (2003)}$$

$$Bias(T_{RS,a}) = \left(\frac{\overline{Y}}{\overline{X}}\overline{X}_a - \overline{Y}_a\right) - \frac{N-n}{Nn}\frac{\overline{Y}}{\overline{X}}\overline{X}_a\left(C_X^2 - C_{YX}\right)$$

$$MSE(T_{RS,a}) = \left(\frac{\overline{Y}}{\overline{X}}\overline{X}_a - \overline{Y}_a\right)^2 + \frac{N-n}{Nn}\frac{\overline{Y}}{\overline{X}}\overline{X}_a\left\{\frac{\overline{Y}}{\overline{X}}\overline{X}_a\left(3C_X^2 + C_Y^2 - 4C_{YX}\right) - 2\overline{Y}_a\left(C_X^2 - C_{YX}\right)\right\}$$

Generalized synthetic estimator for domain mean ($T_{GS,\beta,a}$)

$$T_{GS,\beta,a} = \bar{y}\left(\frac{\bar{x}}{\bar{\bar{X}}_a}\right)^{\beta}$$ Tikkiwal and Ghiya (2000)

$$Bias(T_{GS,\beta,a}) = \bar{Y}\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\beta}\left[1+\frac{N-n}{Nn}\left\{\frac{\beta(\beta-1)}{2}C_X^2 + \beta C_{YX}\right\}\right] - \bar{Y}_a$$

$$MSE(T_{GS,\beta,a}) = \bar{Y}^2\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{2\beta}\left[1+\frac{N-n}{Nn}\left\{\left(2\beta^2-\beta\right)C_X^2 + C_Y^2 + 4\beta C_{YX}\right\}\right] + \bar{Y}_a^2$$

$$-2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\beta}\left[1+\frac{N-n}{Nn}\left\{\frac{\beta(\beta-1)}{2}C_X^2 + \beta C_{YX}\right\}\right]$$

2.2 Proposed estimator for domain mean ($T_{PS,\alpha,B,a}$)

We proposed a class of the improved generalized synthetic estimator for domain mean using auxiliary character when domain mean of the auxiliary character is unknown given as follows:

$$T_{PS,\alpha,B,a} = B\bar{y}\left(\frac{\bar{x}}{\bar{x}_a'}\right)^{\alpha}$$, where B is a suitable chosen value and α are constant. (1)

The cases of the proposed estimator are given as follows:

$$T_{2,a} = \bar{y}\left(\frac{\bar{x}_a'}{\bar{x}}\right)$$, if $B=1$ and $\alpha=-1$ (2)

$$T_{3,a} = \bar{y}\left(\frac{\bar{x}}{\bar{x}_a'}\right)$$, if $B=1$ and $\alpha=1$ (3)

$$T_{4,a} = \bar{y}\left(\frac{\bar{x}}{\bar{x}_a'}\right)^{\delta}$$, if $B=1$ and $\alpha=\delta$. (4)

We assume large sample approximations which are given as follows:

$\bar{y}=\bar{Y}(1+\varepsilon_0)$, $\bar{x}=\bar{X}(1+\varepsilon_1)$ and $\bar{x}_a' = \bar{X}_a(1+\varepsilon_2)$, such that $E(\varepsilon_0)=0$, $E(\varepsilon_1)=0$,

$E(\varepsilon_2)=0$, $E(\varepsilon_0\varepsilon_2)=0$, $E(\varepsilon_1\varepsilon_2)=0$ and $E(\varepsilon_0^2)=\frac{(N-n)}{Nn}C_Y^2$, $E(\varepsilon_1^2)=\frac{(N-n)}{Nn}C_X^2$,

$E(\varepsilon_2^2)=\frac{(N_a-n_a')}{N_a n_a'}C_{X_a}^2$ and $E(\varepsilon_0\varepsilon_1)=\frac{(N-n)}{Nn}C_{YX}$. (5)

Bias and Mean Square Error of the ratio synthetic estimator $(T_{2,a})$ and generalized synthetic estimator $(T_{4,a})$ are given as:

$$Bias(T_{2,a}) = \left(\frac{\overline{Y}}{\overline{\overline{X}}}\overline{X}_a - \overline{Y}_a\right) - \left(\frac{1}{n} - \frac{1}{N}\right)\frac{\overline{Y}}{\overline{\overline{X}}}\overline{X}_a\left(C_X^2 - C_{YX}\right) \tag{6}$$

$$\begin{aligned} MSE(T_{2,a}) &= \left(\frac{\overline{Y}}{\overline{\overline{X}}}\overline{X}_a - \overline{Y}_a\right)^2 \\ &+ \left(\frac{1}{n} - \frac{1}{N}\right)\frac{\overline{Y}}{\overline{\overline{X}}}\overline{X}_a\left\{\frac{\overline{Y}}{\overline{\overline{X}}}\overline{X}_a\left(C_X^2 + C_Y^2 - 2C_{YX}\right) - 2\overline{Y}_a\left(C_X^2 - C_{YX}\right)\right\} \\ &+ \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\left(\frac{\overline{Y}}{\overline{\overline{X}}}\overline{X}_a\right)^2 C_{X_a}^2 \end{aligned} \tag{7}$$

Under synthetic assumption of $\frac{\overline{Y}_a}{\overline{\overline{Y}}} = \left(\frac{\overline{X}_a}{\overline{\overline{X}}}\right)$, the equation (7) is reduced to

$$MSE(T_{2,a}) = \overline{Y}_a^2\left\{\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 - C_X^2\right) + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)C_{X_a}^2\right\} \tag{8}$$

Bias and MSE of the conventional generalized synthetic estimator for domain mean are given as follows:

$$Bias(T_{4,a}) = \overline{Y}\left(\frac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\delta}\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{\delta(\delta - 1)}{2}C_X^2 + \delta C_{YX}\right)\right] - \overline{Y}_a \tag{9}$$

Under the synthetic assumption

$$\frac{\overline{Y}_a}{\overline{\overline{Y}}} = \left(\frac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\delta} \tag{10}$$

Equation (9) is reduced to

$$Bias(T_{4,a}) = \overline{Y}_a\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{\delta(\delta - 1)}{2}C_X^2 + \delta C_{YX}\right)\right] \tag{11}$$

$$\begin{aligned} MSE(T_{4,a}) = \left(\overline{Y}\left(\frac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\delta}\right)^2 \Bigg[1 &+ \left(\frac{1}{n} - \frac{1}{N}\right)\left\{C_Y^2 + \delta(2\delta - 1)C_X^2 + 4\delta C_{YX}\right\} \\ &+ \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\delta(2\delta + 1)C_{X_a}^2\Bigg] + \overline{Y}_a^2 \end{aligned}$$

$$-2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\delta}\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)\left(\delta C_{YX}+\frac{\delta(\delta-1)}{2}C_X^2\right)+\left(\frac{1}{n_a^{'}}-\frac{1}{N_a}\right)\frac{\delta(\delta+1)}{2}C_{X_a}^2\right] \quad (12)$$

Now differentiate $MSE(T_{4,a})$ with respect to δ and equating to zero. Then the value of δ does not comes in the closed form. Therefore we proceed to an another process, we take the synthetic assumption from equation (10) and we have

$$MSE(T_{4,a})=\bar{Y}_a^2\left[\left(\frac{1}{n}-\frac{1}{N}\right)\left\{C_Y^2+\delta^2C_X^2+2\delta C_{YX}\right\}+\left(\frac{1}{n_a^{'}}-\frac{1}{N_a}\right)\delta^2C_{X_a}^2\right] \quad (13)$$

Now we are solving $\frac{\partial MSE(T_{4,a})}{\partial\delta}=0$ the optimum value of δ which minimizes the equation (13) are given as follows:

$$\delta=\frac{-\left(\frac{1}{n}-\frac{1}{N}\right)C_{YX}}{\left\{\left(\frac{1}{n}-\frac{1}{N}\right)C_X^2+\left(\frac{1}{n_a^{'}}-\frac{1}{N_a}\right)C_{X_a}^2\right\}}\;. \quad (14)$$

Bias and MSE of the proposed estimator for domain mean ($T_{PS,B,\alpha,a}$):

$$Bias(T_{PS,B,\alpha,a})=B\bar{Y}\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\alpha}\left[1+\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\alpha(\alpha-1)}{2}C_X^2+\alpha C_{YX}\right)\right]-\bar{Y}_a \quad (15)$$

We take the synthetic assumption

$$\frac{\bar{Y}_a}{\bar{\bar{Y}}}\propto\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\alpha}, \quad (16)$$

and one of the assumption is

$$\frac{\bar{Y}_a}{\bar{\bar{Y}}}=A\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\alpha} \quad (17)$$

However,

$\frac{\bar{Y}_a}{\bar{\bar{Y}}}\cong\left(\frac{\bar{X}}{\bar{\bar{X}}_a}\right)^{\alpha}$, may also be a case of (16), which may takes in the present study (18)

We put the value from equation (18) in the equation (15), we have

$$Bias(T_{PS,B,\alpha,a})\cong B\bar{Y}_a\left[\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\alpha(\alpha-1)}{2}C_X^2+\alpha C_{YX}\right)\right]-\bar{Y}_a \quad (19)$$

$\frac{\partial MSE(T_{PS,B,\alpha,a})}{\partial \alpha} = 0$, Bias will minimum if

$$\alpha = \frac{1}{2} - \frac{C_{YX}}{C_X^2} \text{ and } B \neq 0 \tag{20}$$

$$MSE(T_{PS,B,\alpha,a}) = \left(B\overline{Y}\left(\frac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\alpha}\right)^2 \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left\{\alpha(2\alpha - 1)C_X^2 + C_Y^2 + 4\alpha C_{YX}\right\} + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\alpha(2\alpha+1)C_{X_a}^2\right] + \overline{Y}_a^2$$

$$-2B\overline{Y}_a\overline{Y}\left(\frac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\alpha}\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\alpha C_{YX} + \frac{\alpha(\alpha-1)}{2}C_X^2\right) + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\frac{\alpha(\alpha+1)}{2}C_{X_a}^2\right] \tag{21}$$

Now using the synthetic assumption from equation (18) in equation (21), we have

$$MSE(T_{PS,B,\alpha,a}) \cong \overline{Y}_a^2\left[B^2\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left\{\alpha(2\alpha-1)C_X^2 + C_Y^2 + 4\alpha C_{YX}\right\} + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\alpha(2\alpha+1)C_{X_a}^2\right]\right.$$
$$\left. -2B\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left\{\alpha C_{YX} + \frac{\alpha(\alpha-1)}{2}C_X^2\right\} + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\frac{\alpha(\alpha+1)}{2}C_{X_a}^2\right]\right] \tag{22}$$

Now solving $\frac{\partial MSE(T_{PS,B,\alpha,a})}{\partial \alpha} = 0$ and $\frac{\partial MSE(T_{PS,B,\alpha,a})}{\partial B} = 0$, we obtain the optimum value of α and B which are given as follows:

$$\alpha_{opt} = \frac{\frac{(1-B)}{2}\left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)C_{X_a}^2 - \left(\frac{1}{n} - \frac{1}{N}\right)\left\{\frac{(1-B)}{2}C_X^2 + (2B-1)C_{YX}\right\}}{(2B-1)\left(\frac{1}{n} - \frac{1}{N}\right)C_X^2 + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)C_{X_a}^2} \tag{23}$$

$$B_{opt} = \frac{\left(\frac{1}{n} - \frac{1}{N}\right)\left(\alpha C_{YX} + \frac{\alpha(\alpha-1)}{2}C_X^2\right) + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\frac{\alpha(\alpha+1)}{2}C_{X_a}^2}{\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \alpha(\alpha-1)C_X^2 + 4\alpha C_{YX}\right) + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\alpha(2\alpha+1)C_{X_a}^2} \tag{24}$$

The equation (23) and equation (24) are very tedious to solve because it do not come in the closed form. Therefore, we consider the value of α from equation (14) and a suitable value of B which minimizes the MSE of the proposed estimator has been obtained for these values. The performance of the proposed estimator and relevant estimators has been obtained by using simulation study.

3. Simulation study

For the purpose of simulation study, we consider the data (Sarndal et al. (1992), in appendix B). We take five geographical areas (2, 3, 4, 5 and 6) out of eight geographical areas (1, 2, 3, 4, 5, 6, 7 and 8) having sizes 48, 32, 38, 56 and 41 respectively. Following the formulations of problem in section 2, we use same procedure in the simulation study. We select a sample of 20% units (43 out of 215) from study and auxiliary character y and x from the population, and from each a^{th} domain, we select a sample of size $n_a^{'}$ (approximately 50%) foe the auxiliary character. This process is repeated by 10000 times.

The absolute relative bias (ARB) and simulation relative standard error (SRSE) of the proposed estimator for domain mean, ratio synthetic estimator for domain mean and generalized synthetic estimator for domain mean have been obtained which are given as follows:

$$ARB(T_{PS,B,\alpha,a}) = \frac{abs\left(\sum_{s=1}^{10000} T_{PS,B,\alpha,a}^{s} - \overline{Y}_a\right)}{\overline{Y}_a} \times 100 \quad (25)$$

$$SRSE(T_{PS,B,\alpha,a}) = \frac{\sqrt{SMSE(T_{PS,B,\alpha,a})}}{\overline{Y}_a} \times 100 \quad (26)$$

where, Simulated Mean Square Error (*SMSE*) is given as:

$$SMSE(T_{PS,B,\alpha,a}) = \frac{1}{10000} \sum_{s=1}^{10000} \left(T_{PS,B,\alpha,a}^{s} - \overline{Y}_a\right)^2 \quad (27)$$

Population 1

The informations about the population are given as follows:

y: Real estate values according to 1984 assessment (in millions of Kronor).

x: Number of municipal employees in 1984.

The parameters about the population are given as follows:

N =215, $\overline{Y}$ = 2758.94, $\overline{X}$ = 1626.81, S_Y^2 = 12509261, S_X^2 = 14335002, S_{YX} =12514709 and ρ_{YX} = 0.935, where ρ_{YX} is the correlation between y and x in the population.

Table 1: The values of the domains parameters of all domains (1, 2, 3, 4 and 5):

Domain Parameters	Domain Values				
	1	2	3	4	5
N_a	48	32	38	56	41
$\overline{Y}_a$	2970.958	2498.750	2915.526	3046.946	2175.317
$\overline{X}_a$	1658.708	1316.938	1937.711	1950.393	1101.220
$S_{Y_a}^2$	11118969	4164522	9575690	27861139	2869024
$S_{X_a}^2$	4601899	1989177	15986129	38786393	1025355
$S_{Y_a X_a}$	6920194	2681882	11697914	31770431	1675340
$\rho_{Y_a X_a}$	0.967	0.932	0.945	0.966	0.977

Population 2

We consider another population for the purpose of the empirical study. The information about the population 2 and domains are given as follows:

y: Real estate values according to 1984 assessment (in millions of Kronor).

x: 1975 population (in thousands).

N =215, $\overline{Y}$ =2758.94, $\overline{X}$ =27.302, S_Y^2 =288027.4, S_X^2 =1520.93, S_{YX} =134815.2 and ρ_{YX} =0.977.

Table 2: The domain parameters values for different domains (1, 2, 3, 4 and 5) (for population 2)

Domain Parameters	Domain Values				
	1	2	3	4	5
N_a	48	32	38	56	41
$\overline{Y}_a$	2970.958	2498.750	2915.526	3046.946	2175.317
$\overline{X}_a$	29.17	23.94	30.63	28.71	20.98
$S_{Y_a}^2$	11118969	4164522	9575690	27861139	2869024
$S_{X_a}^2$	1228.23	437.16	1721.16	3565.12	300.92
$S_{Y_a X_a}$	112977.5	40499.05	126072.1	308468.1	28661.96
$\rho_{Y_a X_a}$	0.967	0.949	0.982	0.979	0.975

Table 3: The synthetic assumption of the ratio synthetic estimator for domain mean of different domains (1, 2, 3, 4 and 5) (for Population 1):

Domain	$\frac{\overline{Y}_a}{\overline{\overline{Y}}}$	$\frac{\overline{X}_a}{\overline{\overline{X}}}$	$abs\left(\frac{\overline{Y}_a}{\overline{\overline{Y}}} - \frac{\overline{X}_a}{\overline{\overline{X}}}\right)$
1	1.07685	1.01961	0.05724
2	0.90569	0.80951	0.09618
3	1.05675	1.19111	0.13436
4	1.10439	1.19890	0.09451
5	0.78846	0.67692	0.11154

Table 4: The synthetic assumption of the generalized synthetic estimator for domain mean of all domains (1, 2, 3, 4 and 5) (for population 1):

Domain	$\frac{\overline{Y}_a}{\overline{\overline{Y}}}$	$\frac{\overline{X}_a}{\overline{\overline{X}}}$	$abs\left(\frac{\overline{Y}_a}{\overline{\overline{Y}}} - \left(\frac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\delta}\right)$
1	1.07685	1.00867	0.06818
2	0.90569	0.91065	0.00496
3	1.05675	1.06156	0.00481
4	1.10439	1.05254	0.05185
5	0.78846	0.83154	0.04308

It is observed that the amount of an absolute difference of the synthetic assumption of the generalized synthetic for domains (2, 3, 4 and 5) is lower than the amount of an absolute difference of the ratio synthetic estimator and slightly deviate for domain 1. The performance of the proposed estimator is considered under the synthetic assumption of the generalized synthetic estimator.

Table 5: The absolute difference under the synthetic assumption of the ratio synthetic assumption for domains (1, 2, 3, 4 and 5) (for Population 2):

Domain	$\frac{\overline{Y}_a}{\overline{\overline{Y}}}$	$\frac{\overline{X}_a}{\overline{\overline{X}}}$	$abs\left(\frac{\overline{Y}_a}{\overline{\overline{Y}}} - \frac{\overline{X}_a}{\overline{\overline{X}}}\right)$
1	1.07685	1.06828	0.00857
2	0.90569	0.87676	0.02893
3	1.05675	1.12194	0.06518
4	1.10439	1.05172	0.05267
5	0.78846	0.76827	0.02019

Table 6: The absolute difference under the synthetic assumption of the generalized synthetic estimator for different domains (1, 2, 3, 4 and 5) (Population 2):

Domain	$\dfrac{\overline{Y}_a}{\overline{Y}}$	$\left(\dfrac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\delta}$	$abs\left(\dfrac{\overline{Y}_a}{\overline{Y}} - \left(\dfrac{\overline{X}}{\overline{\overline{X}}_a}\right)^{\delta}\right)$
1	1.076846	1.043482	0.03336372
2	0.9056907	0.914217	0.00852628
3	1.056754	1.06597	0.00921587
4	1.104389	1.023213	0.08117583
5	0.788460	0.8235112	0.03505121

From the table 5.5 and table 5.6 are shows that the absolute difference under the synthetic assumption of the generalized synthetic estimator is lower than the absolute difference under the synthetic assumption of the ratio synthetic estimator for domains (2 and 3) and approximately near for domains (4 and 5). Hence, the proposed estimator under the generalized synthetic estimator is better than the relevant estimators.

Table 7: The absolute relative bias (ARB) and simulated related standard error (SRSE) of the proposed estimator, generalized synthetic estimator and ratio synthetic estimator for the domains (1, 2, 3, 4 and 5): (for population 1 and population 2)

Population 1							
Estimator	$T_{2,a}$		$T_{4,a}$		$T_{PS,B,\alpha,a}$		
Domains	ARB	SRSE	ARB	SRSE	ARB	SRSE	B
1	10.156	53.999	1.862	31.413	9.713	30.541	(0.84, 1) 0.92*
2	4.084	50.132	5.499	34.109	7.688	30.442	(0.72, 1) 0.88
3	31.758	82.059	3.145	33.156	6.344	30.133	(0.78, 1) 0.91
4	26.050	85.644	3.894	30.527	8.119	29.880	(0.89, 1) 0.96
5	0.162	46.570	11.248	37.695	7.664	30.929	(0.63, 1) 0.83
Population 2							
Estimator	$T_{2,a}$		$T_{4,a}$		$T_{PS,B,\alpha,a}$		B
1	6.191	44.557	0.048	33.407	9.957	31.672	(0.84,1) .92*
2	4.731	42.934	5.557	36.035	7.136	32.138	(0.75, 1) 0.88
3	14.243	52.483	3.352	34.058	9.088	30.681	(0.74, 1) 0.87
4	3.273	49.216	5.206	30.686	10.419	30.718	(0.92, 1) 0.95
5	4.537	40.936	9.035	37.694	3.504	32.576	(0.65, 1) 0.89

The following results are obtained for population 1 and population 2:

1. The amount of SRSE of the proposed estimator is lower than the amount of SRSE of $T_{2,a}$ and $T_{4,a}$ for all domains (1, 2, 3, 4 and 5) and the amount of ARB of the

proposed estimator is lower than the amount of the ARB of $T_{2,a}$ for domains (1, 3 and 4). (for population 1)

2. The amount of the SRSE of the proposed estimator is varies (29.880, 30.440), $T_{2,a}$ varies (50.132, 85.644) and $T_{4,a}$ varies (30.527, 34.109) i.e. the SRSE of the proposed estimator is least variations for domains (2, 3 and 4) and amount of ARB of the proposed estimator is varies (6.344, 8.119), $T_{2,a}$ varies (4.084, 26.050) and $T_{4,a}$ varies (3.145, 5.499) i.e. amount of the ARB and SRSE of the proposed estimator is lower variations between domains (2, 3 and 4) where synthetic assumption of the generalized synthetic estimator meets closely. (for population 1)

3. The amount of SRSE of the proposed estimator is lower than the amount of SRSE of $T_{2,a}$ and $T_{4,a}$ for all domains and the amount of ARB of the proposed estimator is lower than the amount of the ARB of $T_{2,a}$ and $T_{4,a}$ for domains (3 and 5) and domain 5 respectively. (for population 2)

4. The amount of the SRSE of the proposed estimator is varies (30.681, 32.128), $T_{2,a}$ varies (42.934, 52.483) and $T_{4,a}$ varies (34.527, 36.035) and amount of the ARB of the proposed estimator is varies (7.136, 9.088), $T_{2,a}$ varies (4.731, 14.243) and $T_{4,a}$ varies (3.357, 5.552) i.e. amount of the ARB and SRSE of the proposed estimator is lower variations for domains (2 and 3) where synthetic assumption of the generalized synthetic estimator meets closely. (for population 2)

4. Conclusion

It is conclude that the proposed estimator is more efficient than the ratio synthetic estimator for domain mean and generalized synthetic estimator for domain mean. From the analysis of the results, it is also shows that the proposed estimator is a kind of two-phase technique has superior than the relevant two-phase estimators for which domains where the synthetic assumption meet closely. The main advantage of the two-phase is shows in this paper, if domain mean of the auxiliary character is unknown and population mean of the study character and auxiliary character is known. The proposed estimator is potentially may use in the small area estimation. Also, proposed estimator will help full for multi-auxiliary characters for domain mean.

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