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This paper studies ratio and product type estimators for two population means. The BIAS and MSE of the developed estimator have been derived. The developed estimator has been compared with usual estimator, ratio and product type estimators for ratio of two population means.

Keywords: Ratio and Product type estimators, two population means, BIAS and MSE.

1. Introduction

Sometimes, we would like to stratify on a key variable but cannot place the units into their correct strata until the units are sampled. For instance, in a telephone interview, the respondents can not be placed into a male or female stratum until after the respondent is contacted. Poststratification (stratification after the sample has been selected by simple random sampling) is often appropriate when a simple random sample is not properly balanced by the representation.

Unlike stratification, post-stratification relies on the data obtained in the survey itself that were not available before sampling, and adjusts the weights so that the totals in each group are equal to the known population totals. It still needs the post-stratification cells to be mutually exclusive and cover the whole population.

Singh (1967) utilized information on population mean of two auxiliary variables and defined ratio-cum-product type estimator for population mean in SRSWOR. In stratified random sampling, Tailor et al. (2016) studied ratio-cum-product type estimator of finite population mean in case of post-stratification. Gupta & Tailor (2021) examined ratio in ratio type exponential strategy for the estimation of population mean. The following authors worked in area of post-stratification with the help of ratio & product type exponential estimators and ratio-cum-product type estimator: Lone & Tailor (2015), Tailor, Tailor & Chouhan (2017), Gupta & Tailor

(2021), Vishwakarma (2017), Frey & Feeman (2012), Ovidio et al. (2021), Agrawal & Panda (1993), Singh & Ruiz (2003), Shukla & Trivedi (2006), Singh & Vishwakarma (2010), Holt & Smitch (1979), Agers, Oden & Trulsson (1985) and Vishwakarma & Singh (2019) also studied in the areas of post-stratification.

The usual estimator fro ratio of two population means in post-stratification is expressed as

$$\hat{R}_{ps} = \frac{\bar{y}_{0ps}}{\bar{y}_{1ps}}. \tag{1}$$

A ratio and product type estimators for ratio of two population means in case of post-stratification are given as

$$\hat{R}_{Rps} = \hat{R}_{ps}\left(\frac{\bar{X}_1}{\bar{x}_{1ps}}\right) \tag{2}$$

and

$$\hat{R}_{Pps} = \hat{R}_{ps}\left(\frac{\bar{x}_{2ps}}{\bar{X}_2}\right) \tag{3}$$

where $\bar{y}_{0ps} = \sum_{h=1}^{L} W_h \bar{y}_{0h}$ and $\bar{y}_{1ps} = \sum_{h=1}^{L} W_h \bar{y}_{1h}$ are the unbiased estimates of population means $\bar{Y}_0$ and $\bar{Y}_1$ respectively.

The BIAS and MSE of $\hat{R}_{ps}$, $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$ are given as

$$B(\hat{R}_{ps}) = R\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(\frac{S_{y_1h}^2}{\bar{Y}_1^2} - \frac{S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}\right), \tag{4}$$

$$B(\hat{R}_{Rps}) = R\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(\frac{S_{y_1h}^2}{\bar{Y}_1^2} + \frac{S_{x_1h}^2}{\bar{X}_1^2} - \frac{S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1} - \frac{S_{y_0x_1h}}{\bar{Y}_0\bar{X}_1} + \frac{S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}\right), \tag{5}$$

$$B(\hat{R}_{Pps}) = R\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2} - \frac{S_{y_0x_1h}}{\bar{Y}_0\bar{Y}_1} - \frac{S_{y_1x_2h}}{\bar{Y}_1\bar{X}_2} + \frac{S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}\right), \tag{6}$$

$$MSE(\hat{R}_{ps}) = R^2\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2} + \frac{S_{y_1h}^2}{\bar{Y}_1^2} - \frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}\right), \tag{7}$$

$$MSE(\hat{R}_{Rps}) = R^2\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2} + \frac{S_{y_1h}^2}{\bar{Y}_1^2} + \frac{S_{x_1h}^2}{\bar{X}_1^2} - \frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1} + \frac{2S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1} - \frac{2S_{y_0x_1h}}{\bar{Y}_0\bar{X}_1}\right), \tag{8}$$

$$MSE(\hat{R}_{Pps}) = R^2\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2} + \frac{S_{y_1h}^2}{\bar{Y}_1^2} + \frac{S_{x_2h}^2}{\bar{X}_2^2} - \frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1} - \frac{2S_{y_1x_2h}}{\bar{Y}_1\bar{X}_2} + \frac{2S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}\right), \tag{9}$$

2. Developed Estimator

Assuming that population means of two auxiliary variables i.e., $\bar{X}_1$ and $\bar{X}_2$ are known, a ratio-cum-product type estimator for ratio of two population means is developed as

$$\hat{R}_{RP}^{ps} = \hat{R}_{ps}\left(\frac{\bar{X}_1}{\bar{x}_{1ps}}\right)\left(\frac{\bar{x}_{2ps}}{\bar{X}_2}\right) \tag{10}$$

To obtain the BIAS and MSE of the developed estimator $\hat{R}_{RP}^{ps}$, it is assumed that $\bar{y}_{0h} = \bar{Y}_{0h}(1+e_{0h})$, $\bar{y}_{1h} = \bar{Y}_{1h}(1+e_{1h})$, $\bar{x}_{1h} = \bar{X}_{1h}(1+e_{2h})$ and $\bar{x}_{2h} = \bar{X}_{2h}(1+e_{3h})$ such that $E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = E(e_{3h}) = 0$ and

$$E(e_{0h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{y_0h}^2,\ E(e_{1h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{y_1h}^2,\ E(e_{2h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{x_1h}^2,$$

$$E(e_{3h}^2) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)C_{x_2h}^2,\ E(e_{0h}e_{1h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{y_0y_1h}C_{y_0h}C_{y_1h},$$

$$E(e_{0h}e_{2h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{y_0x_1h}C_{y_0h}C_{x_1h},\ E(e_{0h}e_{3h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{y_0x_2h}C_{y_0h}C_{x_2h},$$

$$E(e_{1h}e_{2h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{y_1x_1h}C_{y_1h}C_{x_1h},\ E(e_{1h}e_{3h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{y_1x_2h}C_{y_1h}C_{x_2h},$$

and

$$E(e_{2h}e_{3h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{x_1x_2h}C_{x_1h}C_{x_2h}.$$

Expressing $\hat{R}_{RP}^{ps}$ in terms of e_{ih} , provides

$$\hat{R}_{RP}^{ps} = R(1+e_0)(1+e_1)^{-1}(1+e_2)^{-1}(1+e_3) \tag{11}$$

Where $e_0 = \dfrac{\sum_{h=1}^{L} W_h\bar{Y}_{0h}e_{0h}}{\bar{Y}_0}$, $e_1 = \dfrac{\sum_{h=1}^{L} W_h\bar{Y}_{1h}e_{1h}}{\bar{Y}_1}$, $e_2 = \dfrac{\sum_{h=1}^{L} W_h\bar{X}_{1h}e_{2h}}{\bar{X}_1}$ and

$$e_3 = \frac{\sum_{h=1}^{L} W_h\bar{X}_{2h}e_{3h}}{\bar{X}_2}$$

such that $E(e_0)=E(e_1)=E(e_2)=E(e_3)=0$ and $E(e_0^2)=\frac{1}{\bar{Y}_0^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_0h}^2$,

$E(e_1^2)=\frac{1}{\bar{Y}_1^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_1h}^2$, $E(e_2^2)=\frac{1}{\bar{X}_1^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{x_1h}^2$,

$E(e_3^2)=\frac{1}{\bar{X}_2^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{x_2h}^2$, $E(e_0e_1)=\frac{1}{\bar{Y}_0\bar{Y}_1}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_0y_1h}^2$,

$E(e_0e_2)=\frac{1}{\bar{Y}_0\bar{X}_1}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_0x_1h}^2$, $E(e_0e_3)=\frac{1}{\bar{Y}_0\bar{X}_2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_0x_2h}^2$,

$E(e_1e_2)=\frac{1}{\bar{Y}_1\bar{X}_1}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_1x_1h}^2$, $E(e_1e_3)=\frac{1}{\bar{Y}_1\bar{X}_2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{y_1x_2h}^2$,

$E(e_2e_3)=\frac{1}{\bar{X}_1\bar{X}_2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h S_{x_1x_2h}^2$.

Finally, the BIAS and MSE of $\hat{R}_{RP}^{ps}$ are obtained as

$$B\left(\hat{R}_{RP}^{ps}\right)=R\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left[\frac{S_{y_{1h}}^2}{\bar{Y}_1^2}+\frac{S_{x_{1h}}^2}{\bar{X}_1^2}-\frac{S_{x_1x_2h}}{\bar{X}_1\bar{X}_2}+\frac{S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}-\frac{S_{y_1x_2h}}{\bar{Y}_1\bar{X}_2}-\frac{S_{y_0x_1h}}{\bar{Y}_0\bar{X}_1}+\frac{S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}-\frac{S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}\right] \tag{12}$$

and

$$MSE\left(\hat{R}_{RP}^{ps}\right)=R^2\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left[\frac{S_{y_0h}^2}{\bar{Y}_0^2}+\frac{S_{y_1h}^2}{\bar{Y}_1^2}+\frac{S_{x_1h}^2}{\bar{X}_1^2}+\frac{S_{x_2h}^2}{\bar{X}_2^2}-\frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}+\frac{2S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}-\frac{2S_{x_1x_2h}}{\bar{X}_1\bar{X}_2}+\frac{2S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}\right]$$

. (13)

3. Theoretical Comparison for $\hat{R}_{RP}^{ps}$

Comparison of (1.7), (2.4) indicates that the developed ratio-cum-product type estimator $\hat{R}_{RP}^{ps}$ would be higher efficient than usual estimator $\hat{R}_{ps}$ if

(i) $MSE\left(\hat{R}_{RP}^{ps}\right)-MSE\left(\hat{R}_{ps}\right)<0$

$$\sum_{h=1}^{L}W_h\left(\frac{S_{x_1h}^2}{\bar{X}_1^2}+\frac{S_{x_2h}^2}{\bar{X}_2^2}+\frac{2S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}-\frac{2S_{x_1x_2h}}{\bar{X}_1\bar{X}_2}+\frac{2S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}\right)<0. \tag{14}$$

Comparison of (1.8), (2.4) indicates that the developed estimator $\hat{R}_{RP}^{ps}$ would be higher efficient than $\hat{R}_{Rps}$ if

(ii) $MSE\left(\hat{R}_{RP}^{ps}\right)-MSE\left(\hat{R}_{Rps}\right)<0$

$$\sum_{h=1}^{L} W_h\left(\frac{S_{x_2h}^2}{\overline{X}_2^2}-\frac{4S_{y_0y_1h}}{\overline{Y}_0\overline{Y}_1}-\frac{2S_{x_1x_2h}}{\overline{X}_1\overline{X}_2}+\frac{2S_{y_0x_2h}}{\overline{Y}_0\overline{X}_2}\right)<0. \tag{15}$$

Comparison of (1.9), (2.4) indicates that the developed estimator $\hat{R}_{RP}^{ps}$ would be higher efficient than $\hat{R}_{Pps}$ if

(iii) $MSE\left(\hat{R}_{RP}^{ps}\right)-MSE\left(\hat{R}_{Pps}\right)<0$

$$\sum_{h=1}^{L} W_h\left(\frac{S_{x_1h}^2}{\overline{X}_1^2}+\frac{2S_{y_1x_1h}}{\overline{Y}_1\overline{X}_1}-\frac{2S_{x_1x_2h}}{\overline{X}_1\overline{X}_2}+\frac{2S_{y_1x_2h}}{\overline{Y}_1\overline{X}_2}\right)<0. \tag{16}$$

4. Numerical Illustration

To examine the developed ratio-cum-product type estimator in comparison to alternative estimators, we examine a natural population data set.

Population Source: [K.C. Bhuyan, P. No. 4]

y_0 : Number of everborn children

y_1 : Number of dead children under age 5

x_1 : Level of education of father (in complete years)

x_2 : Level of education of mother (in complete years)

Constant	**Stratum I**	**Stratum II**
N_h	15	13
n_h	10	10
$\overline{Y}_{0h}$	6.33	3.07
$\overline{Y}_{1h}$	0.8	0.15
$\overline{X}_{1h}$	8.66	11.92
$\overline{X}_{2h}$	5.66	9.92
S_{y0h}	1.44	1.03
S_{y1h}	0.94	0.37
S_{x1h}	4.70	4.78
S_{x2h}	4.77	4.53
S_{y0y1h}	0.85	0.15
S_{y0x1h}	-3.66	-2.99
S_{y0x2h}	-2.66	-2.82
S_{y1x1h}	-2.85	-0.30

S_{y1x2h}	-1.14	-0.40
S_{x1x2h}	15.80	18.91

Table 1: The MSE and Percent Relative Efficiencies of $\hat{R}_{ps}$, $\hat{R}_{Rps}$, $\hat{R}_{Pps}$ and $\hat{R}_{RP}^{ps}$ with respect to $\hat{R}_{ps}$.

Estimator	MSE	PRE
$\hat{R}_{ps}$	0.202	100.00
$\hat{R}_{Rps}$	0.251	80.24
$\hat{R}_{Pps}$	0.274	73.57
$\hat{R}_{RP}^{ps}$	0.127	158.38

5. Conclusion

Expressions (3.1), (3.2) and (3.3) are the conditions under which the developed estimator $\hat{R}_{Rp}^{ps}$ has less mean squared error in comparison to usual unbiased estimator $\hat{R}_{ps}$, ratio estimator $\hat{R}_{Rps}$ and product estimator $\hat{R}_{Pps}$.

Table 1 shows that the developed ratio and product type estimator for ratio of two population means has better percent relative efficiency as compared to other usual estimator, ratio estimator and product estimator. Thus there is a substantial gain in efficiency by using developed estimator $\hat{R}_{RP}^{ps}$ over usual estimator $\hat{R}_{ps}$, ratio estimator $\hat{R}_{Rps}$ and product estimator $\hat{R}_{Pps}$.

Therefore, the developed estimator $\hat{R}_{RP}^{ps}$ is recommended for the use in practice for the estimation of ratio of two population means when conditions obtained in section 3 are satisfied.

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