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This study proposed a new ratio estimator for estimating population mean in stratified double sampling using the principle of multivariate calibration weightings. The bias and Mean Square Error (MSE) expressions for the proposed estimator are obtained under large sample approximation. Analytical results showed that under certain prescribed conditions, the new estimator is more efficient than all related existing estimators under review. The relative performances of the new estimator with a corresponding Global Estimator were evaluated through a simulation study. Numerical and simulation results proved the dominance of the new estimator.

Keywords: efficiency, global estimator, means square error, multivariate calibration weightings, ratio estimator.

1. Introduction

The incorporation of auxiliary information is very important for the construction of efficient estimators for the estimation of population parameters and increasing the efficiency of the estimators in different sampling designs. Using the knowledge of the auxiliary variables, several authors have developed different estimation techniques for estimating the finite population mean of the study variable; Cochran [16],Singh and Tailor[33], Kadilar and Cingi [23], Gupta and Shabbir [20], Sharma and Tailor [30], Diana *et al*. [18], Solanki *et al*. [36], Swain [37], Haq and Shabbir [21], Singh and Audu [34], Shittu and Adepoju [31], Lone and Tailor [28]; Clement [3], [4], Clement and Enang [9], Clement *et al* [14] and Inyang and Clement [22], among others, have worked on the estimation of population parameters using auxiliary information.

The concept of calibration estimator introduced by Deville and Sarndal [17], uses auxiliary information to adjust the original design weights to improve the precision of survey estimates of population or subpopulation parameters. The calibration weights are chosen to minimize a given distance measure (or loss function) and these weights satisfy the constraints related auxiliary variable information. Many authors have defined some modified calibration estimators in survey sampling using univariate auxiliary information (univariate calibration weightings). A few key references are Wu and Sitter [40], Arnab and Singh [1], Kott [26], Kim *et al.* [24], Kim and Park [25], Rao *et al.* [29], Koyuncu and Kadilar [27], Clement *et al.* [13], Clement and Enang [10], Clement [6], [7], [8], Clement and Inyang [11], [12], Enang and Clement [19] and Clement and Etukudoh [15].

Tracy *et al.* [38] introduced the concept of calibration estimator to stratified double sampling using multi-parametric calibration weightings. Multi-parametric calibration weightings is the formulation of calibration constraints with respect to a given distance measure to obtain expression of calibration weights using information from two or more parameters of the same auxiliary variable. Work in this aspect include: Tracy *et al.* [38], Koyuncu and Kadilar [27], Clement [5].

However, it has been observed that the use of two or more auxiliary variables to formulate calibration constraints gives more precise and efficient calibration estimators than the use of different parameters of the same auxiliary variable (Multi-parametric calibration weightings)

In the progression to improve calibration estimation, the present study proposed a new improved calibration ratio estimator of population mean under the stratified double sampling using multivariate calibration weightings. Multivariate calibration weightings is the use of the same parameter of two or more auxiliary variables in formulating calibration constraints. The choice is obvious, because in the presence of two or more auxiliary variables, the calibration estimation meets the objective of reducing both the non-response bias and the sampling error, thereby increasing the efficiency of the calibration estimator.

2. Materials and Methods

2.1 Multivariate Stratified Double Sampling Design

Consider a heterogeneous population of size N divided into H strata. The hth (h = 1, 2…, H) stratum consists of N_h units, such that $\sum_{h=1}^{H} N_h = N$. y_{hi} is the ith value of the response variable y_h, and (x_{1hi}, x_{2hi}) are the ith values of the auxiliary variables x_{1hi}, x_{2hi} are in the hth stratum. Under simple random sampling without replacement, a sample of size n_h is drawn in each stratum with

$$\sum_{h=1}^{H} n_h = n \,.\, \bar{y}_h = \frac{\sum_{i=1}^{n_h} y_{hi}}{n_h}, \bar{x}_{1h} = \frac{\sum_{i=1}^{n_h} x_{1hi}}{n_h}, \text{and } \bar{x}_{2h} = \frac{\sum_{i=1}^{n_h} x_{2hi}}{n_h} \text{ are the sample}$$

means corresponding to the population means $\bar{Y}_h = \frac{\sum_{i=1}^{N_h} y_{hi}}{N_h}, \bar{X}_{1h} = \frac{\sum_{i=1}^{N_h} x_{1hi}}{N_h}$, and $\bar{X}_{2h} = \frac{\sum_{i=1}^{n_h} x_{2hi}}{N_h}$ in the *h*th stratum; $\bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$ is the combined sample mean corresponding to the population mean $\bar{Y} = \sum_{h=1}^{H} W_h \bar{Y}_h$, and $\bar{x}_{1st} = \sum_{h=1}^{H} W_h \bar{x}_{1h}, \bar{x}_{2st} = \sum_{h=1}^{H} W_h \bar{x}_{2h}$ are the combined sample means corresponding to the population means $\bar{X}_1 = \sum_{h=1}^{H} W_h \bar{X}_{1h}$,

$\bar{X}_2 = \sum_{h=1}^{H} W_h \bar{X}_{2h}$, where $W_h = \frac{N_h}{N}$ is the known stratum weight.

Double sampling allows higher efficiency when at least the population mean of one of the auxiliary variables x_{1h} or x_{2h} is unknown in the *h*th stratum. “Double” or “two-phase” sampling is used when the population characteristics of the auxiliary variables are not known in advance. A large sample of size n' is drawn in advance, called “first-phase sample,” to estimate the population characteristics of the auxiliary variables. Then a subsample of this first-phase sample is drawn of size *n* ($n<n'$), called “second-phase” sample, to estimate a characteristic of the variable of interest. Under double sampling, a large sample of size $n_h' \,(< N_h)$ is drawn at random to estimate $\bar{X}_{1h}$ and $\bar{X}_{2h}$ $(h = 1,2 \ldots, H)$ only in the first phase. Then a sub-sample of size $n_h (< n_h')$ is drawn to obtain $\bar{y}_h$, $\bar{x}_{1h}$ and $\bar{x}_{2h} (h = 1,2 \ldots, H)$.

Consider $\bar{x}_{1st}' = \sum_{h=1}^{H} W_h \bar{x}_{1h}'$ *and* $\bar{x}_{2st}' = \sum_{h=1}^{H} W_h \bar{x}_{2h}'$ the combined sample estimators of the population means $\bar{X}_1$ and $\bar{X}_2$, where $\bar{x}_{1h}' = \frac{\sum_{i=1}^{n_h'} x_{1hi}}{n_h'}$ and $\bar{x}_{2h}' = \frac{\sum_{i=1}^{n_h'} x_{2hi}}{n_h'}$ are based on the first phase sample in the *h*th stratum.

Consider the following notations and definitions:

$$e_{hy} = \left(\frac{\bar{y}_h - \bar{Y}_h}{\bar{Y}_h}\right) \textit{ so that } \bar{y}_h = \bar{Y}_h(1 + e_{hy})$$

$$e_{hx_1} = \left(\frac{\bar{x}_{1h} - \bar{X}_{1h}}{\bar{X}_{1h}}\right) \textit{ so that } \bar{x}_{1h} = \bar{X}_{1h}(1 + e_{hx_1})$$

$$e_{hx_2} = \left(\frac{\bar{x}_{2h} - \bar{X}_{2h}}{\bar{X}_{2h}}\right) \textit{ so that } \bar{x}_{2h} = \bar{X}_{2h}(1 + e_{hx_2})$$

$$e_{hx_1}' = \left(\frac{\bar{x}_{1h}' - \bar{X}_{1h}}{\bar{X}_{1h}}\right) \textit{ so that } \bar{x}_{1h}' = \bar{X}_{1h}(1 + e_{hx_1}')$$

$$e_{hx_2}' = \left(\frac{\bar{x}_{2h}' - \bar{X}_{2h}}{\bar{X}_{2h}}\right) \textit{ so that } \bar{x}_{2h}' = \bar{X}_{2h}(1 + e_{hx_2}')$$

$$\gamma_h' = \left(\frac{1}{n_h'} - \frac{1}{N_h}\right), \quad \gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) and\ \gamma_h^{*'} = \left(\gamma_h - \gamma_h'\right) = \left(\frac{1}{n_h} - \frac{1}{n_h'}\right)$$

$$E\ (e_{hx_1}') = E\left(e_{hy}\right) = E\ (e_{hx_1}) = (e_{hx_2}') = (e_{hx_2}) = 0$$

$$E(e_{hy}^2) = \frac{\gamma_h S_{hy}^2}{\bar{Y}_h^2} \qquad E(e_{hx_2}'^2) = \frac{\gamma_h' S_{hx_2}^2}{\bar{X}_{2h}^2}$$

$$E(e_{hx_1}^2) = \frac{\gamma_h S_{hx_1}^2}{\bar{X}_{1h}^2} \qquad E(e_{hy} e_{hx_1}) = \frac{\gamma_h S_{hyx_1}}{\bar{Y}_h \bar{X}_{1h}}$$

$$E(e_{hx_2}^2) = \frac{\gamma_h S_{hx_2}^2}{\bar{X}_{2h}^2} \qquad E(e_{hy} e_{hx_2}) = \frac{\gamma_h S_{hyx_2}}{\bar{Y}_h \bar{X}_{2h}}$$

$$E(e_{hx_1}'^2) = \frac{\gamma_h' S_{hx_1}^2}{\bar{X}_{1h}^2} \qquad E(e_{hy} e_{hx_1}') = \frac{\gamma_h' S_{hyx_1}}{\bar{Y}_h \bar{X}_{1h}}$$

$$E(e_{hy} e_{hx_2}') = \frac{\gamma_h' S_{hyx_2}}{\bar{Y}_h \bar{X}_{2h}} \qquad E(e_{hx_1} e_{hx_2}') = \frac{\gamma_h' S_{hx_1x_2}}{\bar{X}_{1h} \bar{X}_{2h}}$$

$$E\left(e_{hx_1} e_{hx_2}\right) = \frac{\gamma_h S_{hx_1x_2}}{\bar{X}_{1h} \bar{X}_{2h}} \qquad E(e_{hx_1}' e_{hx_2}') = \frac{\gamma_h' S_{hx_1x_2}}{\bar{X}_{1h} \bar{X}_{2h}}$$

$$E(e_{hx_1} e_{hx_1}') = \frac{\gamma_h' S_{hx_1}^2}{\bar{X}_{1h}^2} \qquad E(e_{hx_2} e_{hx_2}') = \frac{\gamma_h' S_{hx_2}^2}{\bar{X}_{2h}^2}$$

$$C_{hx_1}^2 = \frac{S_{hx_1}^2}{\bar{X}_{1h}^2} \quad C_{hx_2}^2 = \frac{S_{hx_2}^2}{\bar{X}_{2h}^2} W_h = \frac{N_h}{N} R_{1h} = \frac{\bar{Y}_h}{\bar{X}_{1h}} R_{2h} = \frac{\bar{Y}_h}{\bar{X}_{2h}}$$

$$\rho_{hyx_1} = \frac{S_{hyx_1}}{S_{hy} S_{hx_1}} \qquad \rho_{hyx_2} = \frac{S_{hyx_2}}{S_{hy} S_{hx_2}} \qquad \rho_{hx_1x_2} = \frac{S_{hx_1x_2}}{S_{hx_1} S_{hx_2}}$$

$$\eta_h = (S_{hx_1}^2 S_{hx_2}^2 - S_{hx_1x_2}^2) \qquad \sigma_{hx_1} = \sqrt{S_{hx_1}^2} = S_{hx_1}$$

where the parameters are defined wherever they appear as the following:

$\bar{y}_h$ - The second phase sample stratum mean of the study variable.

$\bar{Y}_h$ - The second phase population stratum mean of the study variable.

$\bar{x}_{1h}$ - The second phase sample stratum mean of the first auxiliary variable.

$\bar{X}_{1h}$ - The second phase population stratum mean of the first auxiliary variable.

$\bar{x}_{2h}$ - The second phase sample stratum mean of the second auxiliary variable.

$\bar{X}_{2h}$ - The second phase population stratum mean of the second auxiliary variable.

$\bar{x}_{1h}'$ - The first phase sample stratum mean of the first auxiliary variable.

$\bar{x}_{2h}^{'}$ - The first phase sample stratum mean of the second auxiliary variable.

$s_{hx_1}^2$ - The sample stratum variance of the first auxiliary variable.

$s_{hx_2}^2$ - The sample stratum variance of the second auxiliary variable.

$S_{hx_1}^2$ - The population stratum variance of the first auxiliary variable.

$S_{hx_2}^2$ - The population stratum variance of the second auxiliary variable.

σ_{hx_1} - Stratum standard deviation of the first auxiliary variable

C_{hx_1} - Stratum coefficient of variation of the first auxiliary variable

$\beta_{1h(x_1)}$ - Stratum coefficient of skewness of the first auxiliary variable

$\beta_{2h(x_1)}$ - Stratum coefficient of kurtosis of the first auxiliary variable

ρ_{hyx_1} - Stratum Coefficient of correlation between the study variable and the first auxiliary variable

ρ_{hyx_2} - Stratum coefficient of correlation between the study variable and the second auxiliary variable

$\rho_{hx_1x_2}$ - Stratum coefficient of correlation between the first auxiliary variable and the second auxiliary variable

3. Some Existing Multivariate Estimators in Stratified Double Sampling

This section gives a summary of some existing multivariate estimators of population mean relevant to the study under the stratified double sampling design with their minimum mean square error (MSE) expressions.

3.1 Chand [2] Chain Ratio Type Estimator

Chand [2] proposed a chain ratio type estimator of the population mean ($\bar{Y}$) in two variables as:

$$\hat{\hat{\tau}}_1 = \sum_{h=1}^{H} W_h \bar{y}_h \left(\frac{\bar{x}_{1h}^{'}}{\bar{X}_{1h}}\right)\left(\frac{\bar{X}_{2h}}{\bar{x}_{2h}^{'}}\right) \quad (1)$$

with mean square error expression given as:

$$MSE\ (\hat{\hat{\tau}}_1) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h} S_{hyx_1}\right) + \gamma_h^{'}\left(R_{2h}^2 S_{hx_2}^2 - 2R_{2h} S_{hyx_2}\right)\right] \quad (2)$$

3.2 Singh and Upadhyaya [35] Estimator

Singh and Upadhyaya [35] proposed the traditional multivariate ratio estimator of the population mean ($\bar{Y}$) in two variables as:

$$\hat{\hat{\tau}}_2 = \sum_{h=1}^{H} W_h \bar{y}_h \left(\frac{\bar{x}_{1h}^{'}}{\bar{X}_{1h}}\right)\left(\frac{\bar{X}_{2h}+C_{hx_2}}{\bar{x}_{2h}^{'}C_{hx_2}}\right) \quad (3)$$

with mean square error expression given as:

$$MSE\ (\hat{\hat{\tau}}_2) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h}S_{hyx_1}\right)+\gamma_h^{'}\left(\theta_h^2 R_{2h}^2 S_{hx_2}^2 - 2\theta_h R_{2h}S_{hyx_2}\right)\right] \quad (4)$$

where $\theta_h = \frac{\bar{X}_{2h}}{\bar{X}_{2h}C_{hx_2}}$

3.3 Upadhyaya and Singh [39] Estimator

Upadhyaya and Singh [39] proposed a multivariate ratio-type estimator of the population mean $(\bar{Y})$ in two variables as:

$$\hat{\hat{\tau}}_3 = \sum_{h=1}^{H} W_h \bar{y}_h \left(\frac{\bar{x}_{1h}^{'}}{\bar{X}_{1h}}\right)\left(\frac{\beta_{2h}(x_2)\bar{X}_{2h}+C_{hx_2}}{\beta_{2h}(x_2)\bar{x}_{2h}^{'}C_{hx_2}}\right) \quad (5)$$

with mean square error expression given as:

$$MSE\ (\hat{\hat{\tau}}_3) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h}S_{hyx_1}\right)\right.$$
$$\left.+\gamma_h^{'}\left(\varphi_h^2 R_{2h}^2 S_{hx_2}^2 - 2\varphi_h R_{2h}S_{hyx_2}\right)\right] \quad (6)$$

where $\varphi_h = \frac{\beta_{2h}(x_2)\bar{X}_{2h}}{\beta_{2h}(x_2)\bar{X}_{2h}C_{hx_2}}$

3.4 Singh [32] Estimator I

Singh [32] proposed a multivariate ratio estimator of the population mean $(\bar{Y})$ in two variables as:

$$\hat{\hat{\tau}}_4 = \sum_{h=1}^{H} W_h \bar{y}_h \left(\frac{\bar{x}_{1h}^{'}}{\bar{X}_{1h}}\right)\left(\frac{\bar{X}_{2h}+\sigma_{hx_2}}{\bar{x}_{2h}^{'}\sigma_{hx_2}}\right) \quad (7)$$

with mean square error expression given as:

$$MSE\ (\hat{\hat{\tau}}_4) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h}S_{hyx_1}\right)\right.$$
$$\left.+\gamma_h^{'}\left(\mathrm{K}_h^2 R_{2h}^2 S_{hx_2}^2 - 2\mathrm{K}_h R_{2h}S_{hyx_2}\right)\right] \quad (8)$$

where $\mathrm{K}_h = \frac{\bar{X}_{2h}}{\bar{X}_{2h} + \sigma_{hx_2}}$

3.5 Singh [32] Estimator II

Singh [32] proposed another multivariate ratio estimator of the population mean $(\bar{Y})$ in two variables as:

$$\hat{\hat{\tau}}_5 = \sum_{h=1}^{H} W_h \bar{y}_h \left(\frac{\bar{x}_{1h}^{'}}{\bar{X}_{1h}}\right)\left(\frac{\beta_{1h}(x_2)\bar{X}_{2h}+\sigma_{hx_2}}{\beta_{1h}(x_2)\bar{x}_{2h}^{'}\sigma_{hx_2}}\right) \quad (9)$$

with mean square error expression given as:

$$MSE\ (\hat{\hat{\tau}}_5) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h} S_{hyx_1}\right)\right.$$

$$\left. +\gamma_h^{'}\left(\phi_h^2 R_{2h}^2 S_{hx_2}^2 - 2\phi_h R_{2h} S_{hyx_2}\right)\right] \quad (10)$$

where $\phi_h = \dfrac{\beta_{1h}(x_2)\bar{X}_{2h}}{\beta_{1h}(x_2)\bar{X}_{2h}\sigma_{hx_2}}$

4. Proposed Calibration Ratio Estimator

The proposed calibration ratio estimator for population mean is defined as:

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} \Omega_h^* \bar{y}_h \quad (11)$$

where $\Omega_h^*$is a weight selected so that chi-square-type loss function,

$$\psi = \sum_{h=1}^{H} \sum_{i=1}^{2} \frac{\left(\Omega_h^* - W_h\right)^2}{W_h q_{hi}} \quad (12)$$

is minimized subject to the calibration constraints

$$\sum_{h=1}^{H} W_h \bar{x}_{1h} \quad = \quad \sum_{h=1}^{H} \Omega_h^* \bar{x}_{1h}^{'} \quad (13)$$

$$\sum_{h=1}^{H} W_h \bar{x}_{2h} \quad = \quad \sum_{h=1}^{H} \Omega_h^* \bar{x}_{2h}^{'} \quad (14)$$

The Lagrange's multiplier is given as:

$$\mathrm{L}\ (\Omega_h, \psi) = \sum_{h=1}^{H} \sum_{i=1}^{2} \frac{\left(\Omega_h^* - W_h\right)^2}{W_h q_{hi}} + 2\lambda_1^*\left(\sum_{h=1}^{H} W_h \bar{x}_{1h} - \sum_{h=1}^{H} \Omega_h^* \bar{x}_{1h}^{'}\right)$$

$$+ 2\lambda_2^*\left(\sum_{h=1}^{H} W_h \bar{x}_{2h} - \sum_{h=1}^{H} \Omega_h^* \bar{x}_{2h}^{'}\right) \quad (15)$$

$$\frac{\partial L(\Omega_h^*, \psi)}{\partial \Omega_h^*} = \frac{2(\Omega_h^* - W_h)}{W_h q_{hi}} - 2\lambda_1^* \bar{x}_{1h}^{'} - 2\lambda_2^* \bar{x}_{2h}^{'} = 0$$

$$\Omega_h^* = W_h + \lambda_1^* W_h q_{hi} \bar{x}_{1h}^{'} + \lambda_2^* W_h q_{hi} \bar{x}_{2h}^{'} \qquad \mathrm{i} = 1,2 \quad (16)$$

Putting Equation (16) in Equation (13) and Equation (14) gives the following system of Equations.

$$\lambda_1^* \sum_{h=1}^{H} W_h\, \mathrm{q}_{1\mathrm{h}} \bar{x}_{1h}^2 + \lambda_2^* \sum_{h=1}^{H} W_h\, \mathrm{q}_{1\mathrm{h}} \bar{x}_{1\mathrm{h}} \bar{x}_{2h}^{'} = \sum_{h=1}^{H} W_h \left(\bar{x}_{1h}^{'} - \bar{x}_{1\mathrm{h}}\right) \quad (17)$$

$$\lambda_1^* \sum_{h=1}^{H} W_h\, \mathrm{q}_{2\mathrm{h}} \bar{x}_{1h}^{'} \bar{x}_{2\mathrm{h}} + \lambda_2^* \sum_{h=1}^{H} W_h\, \mathrm{q}_{2\mathrm{h}} \bar{x}_{2h}^2 = \sum_{h=1}^{H} W_h \left(\bar{x}_{2h}^{'} - \bar{x}_{2\mathrm{h}}\right) \quad (18)$$

$$\begin{pmatrix} \sum_{h=1}^{H} W_h\, \mathrm{q}_{1\mathrm{h}} \bar{x}_{1h}^2 & \sum_{h=1}^{H} W_h\, \mathrm{q}_{1\mathrm{h}} \bar{x}_{1h} \bar{x}_{2h}^{'} \\ \sum_{h=1}^{H} W_h\, \mathrm{q}_{2\mathrm{h}} \bar{x}_{1h}^{'} \bar{x}_{2h} & \sum_{h=1}^{H} W_h\, \mathrm{q}_{2\mathrm{h}} \bar{x}_{2h}^2 \end{pmatrix} \begin{pmatrix} \lambda_1^* \\ \lambda_2^* \end{pmatrix} = \begin{pmatrix} \sum_{h=1}^{H} W_h \left(\bar{x}_{1h}^{'} - \bar{x}_{1\mathrm{h}}\right) \\ \sum_{h=1}^{H} W_h \left(\bar{x}_{2h}^{'} - \bar{x}_{2\mathrm{h}}\right) \end{pmatrix}$$

So that

$$\lambda_1^* = \frac{\left(\sum_{h=1}^{H} W_h\, q_{2h}\bar{x}_{2h}^2\right)\left(\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)\right)}{\left(\sum_{h=1}^{H} W_h\, q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h\, q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h\, q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}^{'}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} \quad (19)$$

and

$$\lambda_2^* = \frac{\left(\sum_{h=1}^{H} W_h\, q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)\right)}{\left(\sum_{h=1}^{H} W_h\, q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h\, q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h\, q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}^{'}\bar{x}_{2h}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} \quad (20)$$

Putting Equation (19) and Equation (20) in Equation (16) gives the calibration weight in stratified double sampling as:

$$\Omega_h^* = W_h + W_h q_{1h}\bar{x}_{1h}^{'}\left[\frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}^{'}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right] + W_h q_{2h}\bar{x}_{2h}^{'}\left[\frac{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}\bar{x}_{1h}^{'}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right] \quad (21)$$

Putting Equation (21) in Equation (11) gives the proposed calibration estimator of population mean in stratified double sampling as:

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} W_h\,\bar{y}_h + \sum_{h=1}^{H} W_h\, q_{1h}\bar{x}_{1h}^{'}\bar{y}_h\left[\frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}^{'}\bar{x}_{2h}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right] + \sum_{h=1}^{H} W_h\, q_{2h}\bar{x}_{2h}^{'}\bar{y}_h\left[\frac{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}^{'}\bar{x}_{2h}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right] \quad (22)$$

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} W_h\,\bar{y}_h + \sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'} - \bar{x}_{1h}\right)\left[\frac{\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right)\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^{'}\bar{y}_h}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2} - \frac{\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}^{'}\bar{x}_{2h}\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^{'}\bar{y}_h\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right]$$

$$+\sum_{h=1}^{H} W_h \left(\bar{x}_{2h}^{'} - \bar{x}_{2h}\right)\left[\frac{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^{'}\bar{y}_h}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right)-\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right.$$
$$\left.-\frac{\sum_{h=1}^{H} W_h q_{1h}\,\bar{x}_{2h}^{'}\bar{x}_{1h}\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^{'}\bar{y}_h\right)}{\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}^2\right)\left(\sum_{h=1}^{H} W_h q_{2h}\bar{x}_{2h}^2\right)-\left(\sum_{h=1}^{H} W_h q_{1h}\bar{x}_{1h}\bar{x}_{2h}\right)^2}\right] \quad (23)$$

Setting the tuning parameters in Equation (23) as $q_{1h} = \frac{1}{\bar{x}_{1h}}$ and $q_{2h} = \frac{1}{\bar{x}_{2h}}$ accordingly gives the proposed calibration ratio estimator as:

$$\hat{\bar{\tau}}_p = \sum_{h=1}^{H} W_h\,\bar{y}_h + \left(\frac{\sum_{h=1}^{H} W_h\bar{y}_h}{\sum_{h=1}^{H} W_h\bar{x}_{1h}}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'}-\bar{x}_{1h}\right) + \left(\frac{\sum_{h=1}^{H} W_h\bar{y}_h}{\sum_{h=1}^{H} W_h\bar{x}_{2h}}\right)\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'}-\bar{x}_{2h}\right)$$

$$\hat{\bar{\tau}}_p = \sum_{h=1}^{H} W_h\,\bar{y}_h\left[1+\frac{\sum_{h=1}^{H} W_h\left(\bar{x}_{1h}^{'}-\bar{x}_{1h}\right)}{\sum_{h=1}^{H} W_h\bar{x}_{1h}}+\frac{\sum_{h=1}^{H} W_h\left(\bar{x}_{2h}^{'}-\bar{x}_{2h}\right)}{\sum_{h=1}^{H} W_h\bar{x}_{2h}}\right] \quad (24)$$

4.1 Bias and Mean Square Error (MSE) Expressions for the proposed Estimator

Let;

$$\left.\begin{aligned} \bar{y}_h &= \bar{Y}_h\left(1+e_{hy}\right)\\ \bar{x}_{1h} &= \bar{X}_{1h}\left(1+e_{hx_1}\right)\\ \bar{x}_{1h}^{'} &= \bar{X}_{1h}\left(1+e_{hx_1}^{'}\right)\\ \bar{x}_{2h} &= \bar{X}_{2h}\left(1+e_{hx_2}\right)\\ \bar{x}_{2h}^{'} &= \bar{X}_{2h}\left(1+e_{hx_2}^{'}\right)\end{aligned}\right] \quad (25)$$

Putting Equation (25) in Equation (24) gives:

$$\hat{\bar{\tau}}_p = \sum_{h=1}^{H} W_h\left[\bar{Y}_h\left(1+e_{hy}\right)\right] + \frac{\sum_{h=1}^{H} W_h\left(\bar{Y}_h\left(1+e_{hy}\right)\right)}{\sum_{h=1}^{H} W_h\left(\bar{X}_{1h}\left(1+e_{hx_1}\right)\right)}\sum_{h=1}^{H} W_h\left[\bar{X}_{1h}\left(1+e_{hx_1}^{'}\right) - \bar{X}_{1h}\left(1+e_{hx_1}\right)\right] + \frac{\sum_{h=1}^{H} W_h\left(\bar{Y}_h\left(1+e_{hy}\right)\right)}{\sum_{h=1}^{H} W_h\left(\bar{X}_{2h}\left(1+e_{hx_2}\right)\right)}\sum_{h=1}^{H} W_h\left[\bar{X}_{2h}\left(1+e_{hx_2}^{'}\right)-\bar{X}_{2h}\left(1+e_{hx_2}\right)\right] \quad (26)$$

$$\hat{\bar{\tau}}_p = \sum_{h=1}^{H} W_h\left[\bar{Y}_h\left(1+e_{hy}\right)\right] + \frac{\sum_{h=1}^{H} W_h\left(\bar{Y}_h\left(1+e_{hy}\right)\right)}{\sum_{h=1}^{H} W_h\left(\bar{X}_{1h}\left(1+e_{hx_1}\right)\right)}\sum_{h=1}^{H} W_h\,\bar{X}_{1h}\left(e_{hx_1}^{'}-e_{hx_1}\right) + \frac{\sum_{h=1}^{H} W_h\left(\bar{Y}_h\left(1+e_{hy}\right)\right)}{\sum_{h=1}^{H} W_h\left(\bar{X}_{2h}\left(1+e_{hx_2}\right)\right)}\sum_{h=1}^{H} W_h\,\bar{X}_{2h}\left(e_{hx_2}^{'}-e_{hx_2}\right) \quad (27)$$

$$\hat{\bar{\tau}}_p = \sum_{h=1}^{H} W_h\,\bar{Y}_h\left(1+e_{hy}\right)\left[1+\frac{\sum_{h=1}^{H} W_h\bar{X}_{1h}\left(e_{hx_1}^{'}-e_{hx_1}\right)}{\sum_{h=1}^{H} W_h\bar{X}_{1h}\left(1+e_{hx_1}\right)}+\frac{\sum_{h=1}^{H} W_h\bar{X}_{2h}\left(e_{hx_2}^{'}-e_{hx_2}\right)}{\sum_{h=1}^{H} W_h\bar{X}_{2h}\left(1+e_{hx_2}\right)}\right]$$

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} W_h\, \bar{Y}_h\left(1 + e_{hy}\right)\left[1 + \left(e'_{hx_1} - e_{hx_1}\right)\left(1 + e_{hx_1}\right)^{-1} + \left(e'_{hx_2} - e_{hx_2}\right)\left(1 + e_{hx_2}\right)^{-1}\right] \tag{28}$$

Using Taylor's series expansion, Equation (28) becomes

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} W_h\, \bar{Y}_h\left(1 + e_{hy}\right)\left[1 + \left(e'_{hx_1} - e_{hx_1}\right)\left(1 - e_{hx_1} + e_{hx_1}^2\right) + \left(e'_{hx_2} - e_{hx_2}\right)\left(1 - e_{hx_2} + e_{hx_2}^2\right)\right] \tag{29}$$

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} \mathrm{W_h}\, \bar{Y}_h\left(1 + e_{hy}\right)\left[1 + \left(e'_{hx_1} - e'_{hx_1}e_{hx_1} - e'_{hx_1}e_{hx_1}^2 + e_{hx_1}^2 - e_{hx_1} - e_{hx_1}e_{hx_1}^2\right) + (e'_{hx_2} - e_{hx_2}e'_{hx_2} + e_{hx_2}^2 e'_{hx_2} - e_{hx_2} + e_{hx_2}^2 - e_{hx_2}e_{hx_2}^2\right] \tag{30}$$

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} \mathrm{W_h}\, \bar{Y}_h\left(1 + e_{hy}\right)\left[1 + \left(e'_{hx_1} - e'_{hx_1}e_{hx_1} - e'_{hx_1}e_{hx_1}^2 - e_{hx_1} + e_{hx_1}^2 - e_{hx_2}e_{hx_2}^2\right) + (e'_{hx_2} - e_{hx_2}e'_{hx_2} + e_{hx_2}^2 e'_{hx_2} - e_{hx_2} - e_{hx_2}e_{hx_2}^2 + e_{hx_2}^2)\right] \tag{31}$$

$$\hat{\hat{\tau}}_p = \sum_{h=1}^{H} \mathrm{W_h}\, \bar{Y}_h\left(1 + e_{hy}\right)\left[1 + e'_{hx_1} - e'_{hx_1}e_{hx_1} - e_{hx_1} - e'_{hx_1}e_{hx_1}^2 + e_{hx_1}^2 + e'_{hx_2} - e_{hx_2}e'_{hx_2} - e_{hx_2}e_{hx_2}^2 + e_{hx_2}^2 e'_{hx_2} - e_{hx_2} + e_{hx_2}^2 - e_{hx_2}e_{hx_2}^2)\right] \tag{32}$$

Retaining terms to the first degree of approximation gives:

$$\left(\hat{\hat{\tau}}_p - \bar{Y}\right) = \sum_{h=1}^{H} \mathrm{W_h}\, \bar{Y}_h\left[e_{hy} + e'_{hx_1}e_{hy} - e_{hx_1}e_{hy} + e'_{hx_2}e_{hy} - e_{hx_2}e_{hy} + e'_{hx_1} - e'_{hx_1}e_{hx_1} - e_{hx_1} + e_{hx_1}^2 + e'_{hx_2} - e_{hx_2}e'_{hx_2} - e_{hx_2} + e_{hx_2}^2\right] \tag{33}$$

Since $\bar{Y} = \sum_{h=1}^{H} \mathrm{W_h}\bar{Y}_h$

Taking expectation of both sides of Equation (33) gives:

$$E\left(\hat{\hat{\tau}}_p - \bar{Y}\right) = \sum_{h=1}^{H} \mathrm{W_h}\, \bar{Y}_h\, \mathrm{E}\left[e_{hy} + e'_{hx_1}e_{hy} - e_{hx_1}e_{hy} + e'_{hx_2}e_{hy} - e_{hx_2}e_{hy} + e'_{hx_1} - e'_{hx_1}e_{hx_1} - e_{hx_1} + e_{hx_1}^2 + e'_{hx_2} - e_{hx_2}e'_{hx_2} - e_{hx_2} + e_{hx_2}^2\right] \tag{34}$$

$$\mathrm{Bias}\left(\hat{\hat{\tau}}_p\right) = \sum_{h=1}^{H} \mathrm{W_h}\, \bar{Y}_h\left[\gamma_h \frac{s_{hx_1}^2}{\bar{X}_{1h}^2} - \gamma'_h \frac{s_{hx_1}^2}{\bar{X}_{1h}^2} - \gamma'_h \frac{s_{hx_2}^2}{\bar{X}_{2h}^2} + \gamma_h \frac{s_{hx_2}^2}{\bar{X}_{2h}^2} - \gamma_h \frac{s_{hyx_1}}{\bar{X}_{1h}\bar{Y}_h} + \gamma'_h \frac{s_{hyx_1}}{\bar{X}_{1h}\bar{Y}_h} + \gamma'_h \frac{s_{hyx_2}}{\bar{X}_{2h}\bar{Y}_h} - \gamma_h \frac{s_{hyx_2}}{\bar{X}_{2h}\bar{Y}_h}\right] \tag{35}$$

$$\mathrm{Bias}\ (\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} \mathrm{W_h}\left[\frac{\gamma_h \mathrm{R_{1h}} S_{hx_1}^2}{\bar{X}_{1h}} - \frac{\gamma'_h \mathrm{R_{1h}} S_{hx_1}^2}{\bar{X}_{1h}} - \frac{\gamma'_h \mathrm{R_{2h}} S_{hx_2}^2}{\bar{X}_{2h}} + \frac{\gamma_h \mathrm{R_{2h}} S_{hx_2}^2}{\bar{X}_{2h}} - \frac{\gamma_h S_{hyx_1}}{\bar{X}_{1h}} + \frac{\gamma'_h S_{hyx_1}}{\bar{X}_{1h}} - \frac{\gamma_h S_{hyx_2}}{\bar{X}_{2h}} + \frac{\gamma'_h S_{hyx_2}}{\bar{X}_{2h}}\right] \tag{36}$$

$$\mathrm{Bias}\ (\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} \mathrm{W_h}\left[\left(\gamma_h - \gamma'_h\right)\frac{\mathrm{R_{1h}} S_{hx_1}^2}{\bar{X}_{1h}} + \left(\gamma_h - \gamma'_h\right)\frac{\mathrm{R_{2h}} S_{hx_2}^2}{\bar{X}_{2h}} + \left(\gamma'_h - \gamma_h\right)\frac{S_{hyx_1}}{\bar{X}_{1h}} + \left(\gamma'_h - \gamma_h\right)\frac{S_{hyx_2}}{\bar{X}_{2h}}\right]$$

$$\mathrm{Bias}\ (\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} \mathrm{W_h}\left[\frac{\gamma_h^{*'} \mathrm{R_{1h}} S_{hx_1}^2}{\bar{X}_{1h}} - \frac{\gamma_h^{*'} S_{hyx_1}}{\bar{X}_{1h}} + \frac{\gamma_h^{*'} \mathrm{R_{2h}} S_{hx_2}^2}{\bar{X}_{2h}} - \frac{\gamma_h^{*'} S_{hyx_2}}{\bar{X}_{2h}} - \right] \tag{37}$$

$$\text{Bias}\ (\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} \mathrm{W_h} \left[\gamma_h^{*'} \frac{\left(\mathrm{R_{1h}} S_{hx_1}^2 - S_{hyx_1}\right)}{\bar{X}_{1h}} + \gamma_h^{*'} \frac{\left(\mathrm{R_{2h}} S_{hx_2}^2 - S_{hyx_2}\right)}{\bar{X}_{2h}}\right] \quad (38)$$

Equation (38) is the bias of the proposed calibration ratio estimator of population mean in stratified double sampling.

Squaring both sides of Equation (33) and retaining terms to power of 2 gives:

$$(\hat{\hat{\tau}}_p - \bar{Y})^2 = \left[\sum_{h=1}^{H} \mathrm{W}_h^2 \bar{Y}_h^2 \left[e_{hy} + e'_{hx_1} e_{hy} - e_{hx_1} e_{hy} + e'_{hx_2} e_{hy} - e_{hx_2} e_{hy} + e'_{hx_1} - e'_{hx_1} e_{hx_1} - e_{hx_1} + e_{hx_1}^2 + e'_{hx_2} - e_{hx_2} e'_{hx_2} - e_{hx_2} + e_{hx_2}^2\right]\right]^2 \quad (39)$$

$$(\hat{\hat{\tau}}_p - \bar{Y})^2 = \sum_{h=1}^{H} \mathrm{W_h^2} \bar{\mathrm{Y}}_\mathrm{h}^2 \left[e_{hy}^2 + e_{hx_1}^2 + e_{hx_2}^2 + e_{hx_1}^{'2} + e_{hx_2}^{'2} + 2e_{hx_1} e_{hy} - 2e'_{hx_1} e_{hx_1} + 2e_{hx_1} e_{hx_2} - 2e_{hx_1} e'_{hx_2} + 2e'_{hx_1} e'_{hx_2} - 2e'_{hx_1} e_{hx_2} - 2e'_{hx_1} e_{hy} - 2e_{hx_2} e'_{hx_2} - 2e'_{hx_2} e_{hy} + 2e_{hx_2} e_{hy}\right] \quad (40)$$

Taking expectation of both sides of Equation (40) gives:

$$\mathrm{MSE}(\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} \mathrm{W_h^2} \bar{\mathrm{Y}}_\mathrm{h}^2 \left[\gamma_h \frac{S_{hy}^2}{\bar{Y}_h^2} + \gamma_h \frac{S_{hx_1}^2}{\bar{X}_1^2} + \gamma_h \frac{S_{hx_2}^2}{\bar{X}_2^2} + \gamma'_h \frac{S_{hx_1}^2}{\bar{X}_1^2} + \gamma'_h \frac{S_{hx_2}^2}{\bar{X}_2^2} + 2\gamma_h \frac{S_{hyx_1}}{\bar{X}_1 \bar{Y}_h} + 2\gamma_h \frac{S_{hyx_2}}{\bar{X}_2 \bar{Y}_h} - 2\gamma'_h \frac{S_{hyx_1}}{\bar{X}_1 \bar{Y}_h} - 2\gamma'_h \frac{S_{hyx_2}}{\bar{X}_2 \bar{Y}_h} + 2\gamma_h \frac{S_{hx_1x_2}}{\bar{X}_1 \bar{X}_2} - 2\gamma'_h \frac{S_{hx_1}^2}{\bar{X}_1^2} - 2\gamma'_h \frac{S_{hx_1x_2}}{\bar{X}_1 \bar{X}_2} - 2\gamma'_h \frac{S_{hx_1x_2}}{\bar{X}_1 \bar{X}_2} + 2\gamma'_h \frac{S_{hx_1x_2}}{\bar{X}_1 \bar{X}_2} - 2\gamma'_h \frac{S_{hx_2}^2}{\bar{X}_2^2}\right] \quad (41)$$

$$\mathrm{MSE}(\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h R_{1h}^2 S_{hx_1}^2 + \gamma_h R_{2h}^2 S_{hx_2}^2 - \gamma'_h R_{1h}^2 S_{hx_1}^2 - \gamma'_h R_{2h}^2 S_{hx_2}^2 + 2\gamma_h R_{1h} S_{hyx_1} + 2\gamma_h R_{2h} S_{hyx_2} - 2\gamma'_h R_{1h} S_{hyx_1} - 2\gamma'_h R_{2h} S_{hyx_2} + 2\gamma_h R_{1h} R_{2h} S_{hx_1x_2} - 2\gamma'_h R_{1h} R_{2h} S_{hx_1x_2}\right] \quad (42)$$

$$\mathrm{MSE}(\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h R_{1h}^2 S_{hx_1}^2 - \gamma'_h R_{1h}^2 S_{hx_1}^2 + \gamma_h R_{2h}^2 S_{hx_2}^2 - \gamma'_h R_{2h}^2 S_{hx_2}^2 + 2\gamma_h R_{1h} S_{hyx_1} - 2\gamma'_h R_{1h} S_{hyx_1} + 2\gamma_h R_{2h} S_{hyx_2} - 2\gamma'_h R_{2h} S_{hyx_2} + 2\gamma_h R_{1h} R_{2h} S_{hx_1x_2} - 2\gamma'_h R_{1h} R_{2h} S_{hx_1x_2}\right] \quad (43)$$

$$\mathrm{MSE}(\hat{\hat{\tau}}_p) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + (\gamma_h - \gamma'_h) R_{1h}^2 S_{hx_1}^2 + (\gamma_h - \gamma'_h) R_{2h}^2 S_{hx_2}^2 + 2(\gamma_h - \gamma'_h) R_{1h} S_{hyx_1} + 2(\gamma_h - \gamma'_h) R_{2h} S_{hyx_2} + 2(\gamma_h - \gamma'_h) R_{1h} R_{2h} S_{hx_1x_2}\right] \quad (44)$$

Since $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$ and $\gamma'_h = \left(\frac{1}{n'_h} - \frac{1}{N_h}\right)$

Then

$$(\gamma_h - \gamma'_h) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) - \left(\frac{1}{n'_h} - \frac{1}{N_h}\right) = \left(\frac{1}{n_h} - \frac{1}{n'_h}\right)$$

Let,

$$\gamma_h^{*'} = \left(\frac{1}{n_h} - \frac{1}{n'_h}\right) \quad (45)$$

Putting Equation (45) in Equation (44) gives:

$$\text{MSE}\left(\hat{\hat{\tau}}_p\right) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'} R_{1h}^2 S_{hx_1}^2 + \gamma_h^{*'} R_{2h}^2 S_{hx_2}^2 + 2\gamma_h^{*'} R_{1h} S_{hyx_1} + 2\gamma_h^{*'} R_{2h} S_{hyx_2} + 2\gamma_h^{*'} R_{1h} R_{2h} S_{hx_1x_2}\right] \quad (46)$$

$$MSE\ \left(\hat{\hat{\tau}}_p\right) = \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*'} (R_{1h}^2 S_{hx_1}^2 + R_{2h}^2 S_{hx_2}^2 + 2R_{1h} S_{hyx_1} + 2R_{2h} S_{hyx_2} + 2R_{1h} R_{2h} S_{hx_1x_2})\right] \quad (47)$$

Equation (47) is the mean square error (MSE) of the proposed calibration ratio estimator of population mean in stratified double sampling.

5. Analytical Comparison

This section compares the mean square error of the proposed calibration ratio estimator of mean with the mean square errors of some existing ratio and regression-type estimators of population mean listed in sections 3.

5.1 Comparison with Chand [2] Chain Ratio Estimator

The proposed calibration ratio estimator of population mean would be more efficient than the Chand [2] Chain Ratio Estimator if $MSE\left(\hat{\hat{\tau}}_p\right) \leq MSE(\hat{\hat{\tau}}_1)$.

$$\sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1} (R_{1h}^2 S_{hx_1}^2 + R_{2h}^2 S_{hx_2}^2 + 2R_{1h} S_{hyx_1} + 2R_{2h} S_{hyx_2} + 2R_{1h} R_{2h} S_{hx_1x_2}\right] \leq \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h} S_{hyx_1}\right) + \gamma_h^{'}\left(R_{2h}^2 S_{hx_1}^2 - 2R_{2h} S_{hyx_2}\right)\right]$$

So that

$$\sum_{h=1}^{H} W_h^2 \gamma_h^{*1} \left(R_{2h}^2 S_{hx_2}^2 + 4R_{1h} S_{hyx_1} + 2R_{2h} S_{hyx_2} + 2R_{1h} R_{2h} S_{hx_1x_2}\right) \leq \sum_{h=1}^{H} W_h^2\ \gamma_h^{'}\left(R_{2h}^2 S_{hx_1}^2 - 2R_{2h} S_{hyx_2}\right) \quad (48)$$

If Equation (48) holds, then the proposed calibration ratio estimator is more efficient than the Chand [2] Chain Ratio Estimator.

5.2 Comparison with Singh and Upadhyaya [35] Estimator

The proposed calibration ratio estimator of population mean would be more efficient than the Singh and Upadhyaya [35] Estimator if $MSE\left(\hat{\hat{\tau}}_p\right) \leq MSE(\hat{\hat{\tau}}_2)$.

$$\sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1} (R_{1h}^2 S_{hx_1}^2 + R_{2h}^2 S_{hx_2}^2 + 2R_{1h} S_{hyx_1} + 2R_{2h} S_{hyx_2} + 2R_{1h} R_{2h} S_{hx_1x_2})\right] \leq \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h} S_{hyx_1}\right) + \gamma_h^{'}\left(\theta_h^2 R_{2h}^2 S_{hx_2}^2 - 2\theta_h R_{2h} S_{hyx_2}\right)\right]$$

So that

$$\sum_{h=1}^{H} W_h^2 \gamma_h^{*1} \left(R_{2h}^2 S_{hx_2}^2 + 4R_{1h} S_{hyx_1} + 2R_{2h} S_{hyx_2} + 2R_{1h} R_{2h} S_{hx_1x_2}\right) \leq \sum_{h=1}^{H} W_h^2\ \gamma_h^{'}\left(\theta_h^2 R_{2h}^2 S_{hx_2}^2 - 2\theta_h R_{2h} S_{hyx_2}\right) \quad (49)$$

If Equation (49) holds, then the proposed calibration ratio estimator is more efficient than the Singh and Upadhyaya [35] Estimator.

5.3 Comparison with Upadhyaya and Singh [39] Estimator

The proposed calibration ratio estimator of population mean would be more efficient than the Upadhyaya and Sing [39] if $MSE\left(\hat{\hat{\tau}}_p\right) \leq MSE(\hat{\hat{\tau}}_3)$.

$$\sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}\left(R_{1h}^2 S_{hx_1}^2 + R_{2h}^2 S_{hx_2}^2 + 2R_{1h}S_{hyx_1} + 2R_{2h}S_{hyx_2} + 2R_{1h}R_{2h}S_{hx_1x_2}\right)\right] \leq \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h}S_{hyx_1}\right) + \gamma_h^{'}\left(\varphi_h^2 R_{2h}^2 S_{hx_2}^2 - 2\varphi_h R_{2h} S_{hyx_2}\right)\right]$$

So that

$$\sum_{h=1}^{H} W_h^2 \gamma_h^{*1} \left(R_{2h}^2 S_{hx_2}^2 + 4R_{1h}S_{hyx_1} + 2R_{2h}S_{hyx_2} + 2R_{1h}R_{2h}S_{hx_1x_2}\right) \leq \sum_{h=1}^{H} W_h^2\, \gamma_h^{'}\left(\varphi_h^2 R_{2h}^2 S_{hx_2}^2 - 2\varphi_h R_{2h} S_{hyx_2}\right) \quad (50)$$

If Equation (50) holds, then the proposed calibration ratio-estimator is more efficient than the Upadhyaya and Singh [39] estimator.

5.4 Comparison with Singh [32] Estimator I

The proposed calibration ratio estimator of population mean would be more efficient than the Singh [32] if $MSE\left(\hat{\hat{\tau}}_p\right) \leq MSE(\hat{\hat{\tau}}_4)$.

$$\sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}(R_{1h}^2 S_{hx_1}^2 + R_{2h}^2 S_{hx_2}^2 + 2R_{1h}S_{hyx_1} + 2R_{2h}S_{hyx_2} + 2R_{1h}R_{2h}S_{hx_1x_2})\right] \leq \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h}S_{hyx_1}\right) + \gamma_h^{'}\left(K_h^2 R_{2h}^2 S_{hx_2}^2 - 2K_h R_{2h} S_{hyx_2}\right)\right]$$

So that

$$\sum_{h=1}^{H} W_h^2 \gamma_h^{*1} \left(R_{2h}^2 S_{hx_2}^2 + 4R_{1h}S_{hyx_1} + 2R_{2h}S_{hyx_2} + 2R_{1h}R_{2h}S_{hx_1x_2}\right) \leq \sum_{h=1}^{H} W_h^2\, \gamma_h^{'}\left(K_h^2 R_{2h}^2 S_{hx_2}^2 - 2K_h R_{2h} S_{hyx_2}\right) \quad (51)$$

If Equation (51) holds, then the proposed calibration ratio-estimator is more efficient than Singh [32] Estimator I.

5.5 Comparison with Singh [32] Estimator II

The proposed calibration ratio estimator of population mean would be more efficient than the Singh [32] Estimator II if $MSE\left(\hat{\hat{\tau}}_p\right) \leq MSE(\hat{\hat{\tau}}_5)$.

$$\sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}(R_{1h}^2 S_{hx_1}^2 + R_{2h}^2 S_{hx_2}^2 + 2R_{1h}S_{hyx_1} + 2R_{2h}S_{hyx_2} + 2R_{1h}R_{2h}S_{hx_1x_2})\right] \leq \sum_{h=1}^{H} W_h^2 \left[\gamma_h S_{hy}^2 + \gamma_h^{*1}\left(R_{1h}^2 S_{hx_1}^2 - 2R_{1h}S_{hyx_1}\right) + \gamma_h^{'}\left(\phi_h^2 R_{2h}^2 S_{hx_2} - 2\phi_h R_{2h} S_{hyx_2}\right)\right]$$

So that

$$\sum_{h=1}^{H} W_h^2 \gamma_h^{*1} \left(R_{2h}^2 S_{hx_2}^2 + 4R_{1h} S_{hyx_1} + 2R_{2h} S_{hyx_2} + 2R_{1h} R_{2h} S_{hx_1x_2}\right) \leq \sum_{h=1}^{H} W_h^2 \gamma_h^{'} \left(\phi_h^2 R_{2h}^2 S_{hx_2}^2 - 2\phi_h R_{2h} S_{hyx_2}\right) \quad (52)$$

If Equation (52) holds, then the proposed calibration ratio-estimator is more efficient than the Singh [32] estimator II.

6. Data analysis and discussion

6.1 Empirical study

To test the optimal performance as well as illustrate the general results from the analytical study, a data set in Table 1.

Table 1: Data Statistic

Parameter	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5	Stratum 6
N_h	80	76	52	82	70	64
$n_h^{'}$	24	20	15	25	18	16
n_h	7	6	3	7	5	4
$\bar{Y}_h$	210.34	182.62	164.32	184.44	174.68	170.26
$\bar{X}_{1h}$	16.34	15.62	12.84	15.64	15.86	16.12
$\bar{X}_{2h}$	15.92	14.32	13.42	14.26	16.38	15.82
$S_{hx_1}^2$	214.86	176.80	125.32	182.42	212.42	214.48
$S_{hx_2}^2$	202.42	165.46	141.22	158.06	240.42	196.82
S_{hy}^2	204.38	206.26	198.82	216.32	158.92	186.34
S_{hyx_1}	154.38	170.22	134.82	163.36	172.48	145.62
S_{hyx_2}	164.28	160.32	154.46	148.28	168.44	132.36
$S_{hx_1x_2}$	204.62	168.92	130.46	160.26	210.92	198.92

From Table 1, the sample information in Table 2 were obtained.

Table 2: Sample Information

Parameter	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5	Stratum 6
$\beta_{1h}(x_1)$	4.26	2.64	5.38	6.43	3.83	4.32
$\beta_{2h}(x_1)$	6.42	3.46	2.44	5.05	4.92	5.03
$\beta_{1h}(x_2)$	2.67	3.45	2.82	4.02	3.86	2.44
$\beta_{2h}(x_2)$	3.24	2.86	2.32	3.82	2.96	2.32
$C_{hx_1}^2$	0.8047	0.7246	0.7601	0.7458	0.8445	0.8254
$C_{hx_2}^2$	0.7987	0.8069	0.7841	0.7729	0.8961	0.7864
γ_h	0.1304	0.1535	0.3141	0.1307	0.1857	0.2344
$\gamma_h^{'}$	0.0292	0.0368	0.0474	0.0278	0.0143	0.0469
$\gamma_h^{*'}$	0.1011	0.1167	0.2667	0.1029	0.1444	0.1875
ρ_{hyx_1}	0.7367	0.8914	0.8541	0.8224	0.9388	0.7284
ρ_{hyx_2}	0.8077	0.8678	0.9218	0.8019	0.8617	0.6911
$\rho_{hyx_1x_2}$	0.9812	0.9876	0.9807	0.9438	0.9333	0.9682
θ_h	0.9468	0.9409	0.9381	0.9419	0.9454	0.9469
φ_h	0.9829	0.9785	0.9723	0.9841	0.9809	0.9764

λ_h	0.5281	0.5268	0.5304	0.5315	0.5137	0.5299
ϕ_h	0.7492	0.7934	0.7610	0.8201	0.8031	0.7256
W_h	0.1887	0.1792	0.1226	0.1934	0.1651	0.1509
R_{1h}	12.8727	11.6914	12.7975	11.7928	11.0139	10.5620
R_{2h}	13.2123	12.7528	12.2441	12.9341	10.6642	10.7623
π_h	1622.62	719.36	677.88	3150.04	6582.77	2644.79

6.2 Percent Relative Efficiency (PRE)

The percent relative efficiency (PRE) of an estimator $(\hat{\hat{Y}}_i)$ with respect to the traditional multivariate ratio estimator of population mean in stratified double sampling by Chand (1975) $(\hat{\hat{Y}}_1)$ is defined by

$$PRE\ (\hat{\hat{\tau}}_1, \hat{\hat{\tau}}_i) = \frac{MSE\ (\hat{\hat{\tau}}_1)}{MSE\ (\hat{\hat{\tau}}_i)} \text{x } 100, \quad i = 1, 2, 3, 4, 5$$

Table 3: Performance of estimators from analytical study

S/N	Estimator	MSE	PRE
1	$\hat{\hat{\tau}}_1$	647.57	100
2	$\hat{\hat{\tau}}_2$	632.84	102
3	$\hat{\hat{\tau}}_3$	642.07	101
4	$\hat{\hat{\tau}}_4$	551.81	117
5	$\hat{\hat{\tau}}_5$	596.77	109
6	$\hat{\hat{\tau}}_p$	496.86	130

6.3 Simulation Study

6.3.1 Comparisons with a global estimator

For a given estimator (say) $\hat{\hat{\tau}}_i^*$, let $\hat{\hat{\tau}}_i^{*(m)}$ be the estimate of $\hat{\hat{\tau}}_i^*$ in the m-th simulation run;

m =1, 2… M (=2,500). Five performance criteria namely; Bias (B), Relative Bias (RB), Mean Square Error (MSE), Average Length of Confidence Interval (AL) and Coverage Probability (CP) of $\hat{\hat{\tau}}_i^*$ were used to compare the performance of the proposed calibration ratio estimator with the GREG-estimator. Each measuring criterion is calculated as follows:

(i) $B(\hat{\hat{\tau}}_i^*) = \bar{\hat{\hat{\tau}}}_i^* - \hat{\tau}_i^{*(m)}$

where $\bar{\hat{\hat{\tau}}}_i^* = \frac{1}{M}\sum_{m=1}^{M} \hat{\hat{\tau}}_i^{*(m)}$

(ii) $RB(\hat{\hat{\tau}}_i^*) = \frac{1}{M}\sum_{m=1}^{M}\left(\frac{\hat{\hat{\tau}}_i^{*(m)} - \bar{\hat{\hat{\tau}}}_i^*}{\bar{\hat{\hat{\tau}}}_i^*}\right)$

(iii) $MSE(\hat{\hat{\tau}}_i^*) = \sum_{m=1}^{M}\left(\hat{\hat{\tau}}_i^{*(m)} - \bar{\hat{\hat{\tau}}}_i^*\right)^2/M$

where $\hat{\tau}_i^{*(m)}$ is the estimated total based on sample m and $M$is the total number of samples drawn for the simulation.

(iv) $CP(\hat{\hat{\tau}}_i^*) = \frac{1}{M}\sum_{m=1}^{M}\left(\hat{\hat{\tau}}_L^{*(m)} < \hat{\hat{\tau}}_i^{*(m)} < \hat{\hat{\tau}}_U^{*(m)}\right)$

where $\hat{\hat{\tau}}_L^{*(m)}$is the lower confidence limit and $\hat{\hat{\tau}}_U^{*(m)}$ is the upper confidence limit. For each estimator of $\hat{\hat{\tau}}_i^*$, a 95% Confidence Interval $\left(\hat{\hat{\tau}}_L^{*(m)}, \hat{\hat{\tau}}_U^{*(m)}\right)$ is constructed, where

$$\hat{\hat{\tau}}_L^{*(m)} = \hat{\hat{\tau}}_i^{*(m)} - 1.96\sqrt{Var\left(\hat{\hat{\tau}}_i^{*(m)}\right)}, \hat{\hat{\tau}}_U^{*(m)} = \hat{\hat{\tau}}_i^{*(m)} + 1.96\sqrt{Var\left(\hat{\hat{\tau}}_i^{*(m)}\right)}$$

$$\text{and } Var\left(\hat{\hat{\tau}}_i^{*(m)}\right) = \frac{1}{M-1}\sum_{m=1}^{M}\left(\hat{\hat{\tau}}_i^{*(m)} - \bar{\hat{\hat{\tau}}}_i^*\right)^2.$$

(v) $ALCI(\hat{\hat{\tau}}_i^*) = \frac{1}{M}\sum_{m=1}^{M}\left(\hat{\hat{\tau}}_U^{*(m)} - \hat{\hat{\tau}}_L^{*(m)}\right)$.

The corresponding GREG-estimator and calibration ratio estimator of the population mean $\bar{Y}$ are computed: $\bar{\tau}_{GREG}^{*(m)}$ and $\bar{\tau}_p^{*(m)}$.The results of the analysis are given in Table 4.

Table 4: Performance of estimators from simulation study

Estimators	B	RB	MSE	ALCI	CP
$\bar{\tau}_{GREG}^*$	0.0036	0.0312	3234	1442.60	0.7426
$\bar{\tau}_p^*$	0.0064	0.0120	1864	1032.62	0.5992

7. Results and Discussion

Analytical study showed that under certain prescribed conditions, the new estimator is more efficient than all related existing estimators under review. These results also showed that the proposed calibration ratio estimator is substantially superior in terms of efficiency.

Numerical results for the percent relative efficiency (*PREs*) in Table 3 showed that the proposed calibration ratio estimator ($\hat{\hat{\tau}}_p$) has 30 percent efficiency gain while the Singh and Upadhyaya [35] estimator has 2 percent efficiency gain with respect to the traditional multivariate ratio estimator of population mean in stratified double sampling by Chand [2]. This means that in using the proposed calibration ratio estimator of population mean, one would have 28 percent efficiency gains over the Singh and Upadhyaya [35] estimator. Similarly, the proposed calibration ratio estimator ($\hat{\hat{\tau}}_p$) has 29 percent efficiency gain over the Upadhyaya and Singh [39] estimator which has 1 percent efficiency gain with respect to the traditional multivariate ratio estimator of population mean in stratified double sampling by Chand [2].

Again, the proposed calibration ratio estimator ($\hat{\hat{\tau}}_p$) has 13 percent efficiency gain over Singh [32] Estimator I which has 17 percent efficiency gain with respect to the traditional multivariate ratio estimator of population mean in stratified double sampling.

Also, in using our proposed calibration ratio estimator ($\hat{\hat{\tau}}_p$), one would have 21 percent efficiency gain over Singh [32] Estimator II which has 9 percent efficiency gain with respect to the Chand [2] multivariate ratio estimator.

Analysis for the comparison of performance of estimators showed that the biases of 0.36 percent and 0.64 percent respectively for the proposed estimator and the GREG-estimator are negligible. But the bias of the GREG-estimator though negligible is the most biased among the two estimators considered. The relative bias for the proposed estimator is relatively smaller than that of the GREG-estimator. The variance for the GREG-estimator is significantly larger than that of the proposed calibration ratio estimator, as is indicated by their respective mean square errors in Table 4. The average length of the confidence interval for the calibration ratio estimator is significantly smaller than that of the GREG-estimator. The coverage probability of the calibration ratio estimator is also smaller than that of the GREG-estimator. These results showed that there is greater variation in the estimates made by the GREG-estimator than the calibration ratio estimator.

8. Conclusion

This study proposed a new ratio estimator for estimating population mean in stratified double sampling using the principle of multivariate calibration weightings. The bias and Mean Square Error (MSE) expressions for the proposed estimator are obtained under large sample approximation. Analytical results showed that under certain prescribed conditions, the new estimator is more efficient than all related existing estimators under review.

Sequel to the discussion of results above, we conclude that the proposed calibration ratio estimator ($\hat{\hat{\tau}}_p$), fares better than the Generalized Regression (GREG) estimator both in efficiency and biasedness. This is against an established fact in survey sampling literature that generally the regression estimator is more efficient than both the ratio and product estimators. These results proved the dominance of the new proposal over existing estimators.

Conflicts of Interest:

We do not have any conflicts of interest associated with the publication.

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