Journal of Modern Applied Statistical Methods

Volume 23 | Issue 1 Article 14

Normality of the T-tests for Buy and Sell Days from Moving Average Trading Rules on the NASDAQ

Luh Yu Ren
College of Business Administration, University of Houston-Victoria, USA, ren800us@yahoo.com

Peter Ren
College of Business, University of Houston-Downtown, USA, renp@uhd.edu

Recommended Citation

Luh Yu Ren, Peter Ren (2024). Normality of the T-tests for Buy and Sell Days from Moving Average Trading Rules on the NASDAQ. Journal of Modern Applied Statistical Methods, 23(1), https://doi.org/10.56801/Jmasm.V23.i1.14

Journal of Modern Applied Statistical Methods
2024, Vol. 23, No. 1,
Doi: 10.56801/Jmasm.V23.i1.14

ISSN 1538 – 9472

Normality of the T-tests for Buy and Sell Days from Moving Average Trading Rules on the NASDAQ

Luh Yu Ren
College of Business Administration,
University of Houston-Victoria, USA

Peter Ren
College of Business, University of
Houston-Downtown, USA

In this article, we examine the sampling distributions of the T-ratios commonly used to support the exam of the market efficiency. From the p-values and the fixed block bootstrap methods, it shows that those T-ratios are far from the standard normal distribution claimed by numerous previous studies. Therefore, the T-ratios should not be used to test whether or not the market is efficient.

Keywords: Efficient market hypothesis; Fixed block bootstrap method; Moving average trading rule; T-ratio.

1. Introduction

Under the efficient market hypothesis (EMH), stock prices are expected to fully reflect all available information about the market. Fama (1970) classifies the market into “strong”, “semi-strong”, and “weak” forms. Under the strong form of the EMH, all information that is public or non-public is incorporated into the calculation of a stock’s current price; therefore, investors cannot gain abnormal profits from any public or non-public information. Under the semi-strong form of the EMH, only information that is public is incorporated into the calculation of a stock’s current price; therefore, investors cannot gain abnormal profits from any public information. Finally, under the weak form of the EMH, only historical information is incorporated into the calculation of a stock’s current price; therefore, investors cannot use technical analysis to obtain abnormal profits.

To assess whether the markets are at least weak form efficient, numerous studies have investigated whether investors can apply the simple moving average trading rules to gain abnormal profits. See for reference: Bessembinder, and Chan (1998), Brock, Lakonishok, and LeBaron (1992), Hudson, Dempsey, and Keasey (1996), Kwon, and Kish (2002), Metghalchi, Chang, and Du (2011), Metghalchi, Chen, and Hajilee (2016), and Yu, Nartea, Gan, and Yao (2013).

Following the aforementioned studies, a day is labeled as a Buy day if the short-term moving average price of length S exceeds the long-term moving average price of length L. Various moving average trading rules also specify the differences by a band of α% for eliminating "whiplash" signals (Brock, et al., 1992) when the short and long period moving averages are close. That is,

Buy if $(1+\alpha\%)\frac{1}{S}\sum_{i=0}^{S-1} P_{t-i} \geq (1-\alpha\%)\frac{1}{L}\sum_{i=0}^{L-1} P_{t-i}$; otherwise Sell. (1)

We will denote this trading rule as MA(S, L, α). Popular moving average trading rules (Brock, et al., 1992) are MA(1, 5, 0), MA(1, 15, 0), MA(1, 50, 0), MA(1, 100, 0), MA(1, 200, 0), MA(1, 5, 1), MA(1, 15, 1), MA(1, 50, 1), MA(1, 100, 1), and MA(1, 200, 1).

Following Fama (1965), the daily rate of return at time t, R_t, is defined as in Equation (2):

$$R_t = \ln(P_t) - \ln(P_{t-1}) = \ln\left(\frac{P_t}{P_{t-1}}\right). \quad (2)$$

To test whether abnormal profits exist, three T-tests for the following hypotheses were tested (see Brock, et al., 1992, and numerous articles by Metghalchi, et al., 2011, 2016, 2018, and 2019).

H_0: $\mu_b = \mu_h$, $\mu_s = \mu_h$, $\mu_b = \mu_s$

v.s. H_a: $\mu_b \neq \mu_h$, $\mu_s \neq \mu_h$, $\mu_b \neq \mu_s$,

where μ_b, μ_s, and μ_h are the mean returns for the Buy, Sell days, and buy-and-hold strategy, respectively.

For testing H_0: $\mu_b = \mu_h$ v.s. H_a: $\mu_b \neq \mu_h$, use the first t-test:

$$T_1 = \frac{\overline{X_b} - \mu_h}{\frac{s_b}{\sqrt{n_b}}} \quad (3)$$

For testing H_0: $\mu_s = \mu_h$ v.s. H_a: $\mu_s \neq \mu_h$, use the second t-test:

$$T_2 = \frac{\overline{X_s} - \mu_h}{\frac{s_s}{\sqrt{n_s}}} \quad (4)$$

For testing H_0: $\mu_b = \mu_s$ v.s. H_a: $\mu_b \neq \mu_h$, use the third t-test:

$$T_3 = T_3 = \frac{\overline{X_b} - \overline{X_s}}{\sqrt{\frac{s_b^2}{n_b} + \frac{s_s^2}{n_s}}} \quad (5)$$

where $\overline{X_b}$ and $\overline{X_s}$ are sample means, s_b, s_s are sample standard deviations, and n_b, n_s are sample sizes for daily returns of Buy and Sell Days, respectively.

In this study, we will investigate the sampling distribution of the above T-tests via block bootstrapping method with fixed block length of 20, 50, 100, 200, 500, 1000, and 2,000 on ten popular moving average trading rules: MA(1, 5, 0), MA(1, 15, 0), MA(1, 50, 0), MA(1, 100, 0), MA(1, 200, 0), MA(1, 5, 1), MA(1, 15, 1), MA(1, 50, 1), MA(1, 100, 1), and MA(1, 200, 1).

2. Data and Methodology

Our analysis utilizes NASDAQ daily prices from Yahoo! Finance. Specifically, we investigate prices from three different periods. The first study period contains the most recent 1,000 NASDAQ daily prices from 1/11/2016 to 12/30/2019, a period of about 4 years. The second study period contains the most recent 5,000 NASDAQ daily prices from 2/15/2000 to 12/30/2019, a period of about 20 years. The third study period contains the most recent 8,000 NASDAQ daily prices from 4/4/1988 to 12/30/2019, a period of about 32 years.

Table 1 summarizes the sample sizes n_b and n_s, the sample mean $\overline{X_b}$ and $\overline{X_s}$, the sample standard deviations s_b and s_s for daily returns of Buy and Sell Days, respectively, and the T-ratios from Equations (3)-(5). In all previous studies (Brock, et al., 1992, and Metghalchi, et al., 2011, 2016), T-ratios from Equations (3)-(5) are assumed to follow an approximately normal distribution. From Table 1, almost all the T-ratios are significant at the significant level of α=0.01. except for those with a longer moving period for a short period of observations, e.g., MA(1. 200, 0) and MA(1, 200, 1). From the results shown in Table 1, the NASDAQ market was concluded to be inefficient by all previous studies (Metghalchi, et al., 2011, 2016).

Table 1. p-values of the T-ratios for various MA trading rules on NASDAQ price index

	For 1,000 NASDAQ daily prices from 1/11/2016 to 12/30/2019 for about 4 years									
Trading Rule	n_b	n_s	$\overline{X_b}$	$\overline{X_s}$	s_b	s_s	$\square_h$	T_1	T_2	T_3
MA(1,5,0)	624	372	0.005	-0.006	0.007	0.011	0.0007	13.604**	-12.030**	16.993**
MA(1,15,0)	678	308	0.002	-0.003	0.007	0.014	0.0007	6.635**	-5.080**	6.994**
MA(1,50,0)	725	226	0.002	-0.002	0.007	0.016	0.0007	3.614**	-2.877**	3.658**
MA(1,100,0)	764	137	0.001	-0.003	0.008	0.018	0.0007	2.081*	-2.098*	2.434**
MA(1,200,0)	713	88	0.001	-0.002	0.009	0.018	0.0007	0.879	-1.183	1.314
MA(1,5,1)	158	118	0.010	-0.014	0.009	0.013	0.0007	13.534**	-11.724**	16.860**
MA(1,15,1)	442	173	0.004	-0.006	0.007	0.016	0.0007	9.438**	-5.913**	8.125**
MA(1,50,1)	650	152	0.002	-0.003	0.006	0.017	0.0007	4.282**	-2.864**	3.562**
MA(1,100,1)	727	107	0.001	-0.004	0.007	0.019	0.0007	2.027*	-2.329**	2.598**
MA(1,200,1)	699	75	0.001	-0.002	0.009	0.019	0.0007	0.772	-1.055	1.158
	For 5,000 NASDAQ daily prices from 2/15/2000 to 12/30/2019 for about 20 years									
Trading Rule	n_b	n_s	$\overline{X_b}$	$\overline{X_s}$	s_b	s_s	$\square_h$	T_1	T_2	T_3
MA(1,5,0)	2843	2153	0.007	-0.009	0.012	0.015	0.0001	30.023**	-27.843**	40.041**
MA(1,15,0)	2956	2030	0.004	-0.005	0.012	0.018	0.0001	17.361**	-13.617**	20.187**
MA(1,50,0)	3100	1851	0.002	-0.003	0.010	0.020	0.0001	9.470**	-6.046**	9.097**
MA(1,100,0)	3235	1666	0.001	-0.002	0.010	0.021	0.0001	7.318**	-4.726**	6.842**
MA(1,200,0)	3289	1512	0.001	-0.002	0.010	0.022	0.0001	5.698**	-3.140**	4.631**
MA(1,5,1)	1094	1008	0.012	-0.016	0.015	0.017	0.0001	27.054**	-29.064**	39.658**
MA(1,15,1)	2041	1401	0.005	-0.008	0.012	0.020	0.0001	18.735**	-14.158**	20.922**
MA(1,50,1)	2722	1504	0.002	-0.003	0.010	0.022	0.0001	10.845**	-6.257**	9.476**
MA(1,100,1)	2996	1433	0.002	-0.003	0.010	0.022	0.0001	8.109**	-4.587**	6.742**
MA(1,200,1)	3132	1360	0.001	-0.002	0.009	0.023	0.0001	5.436**	-3.048**	4.388**
	For 8,000 NASDAQ Daily Prices from 4/4/1988 to 12/30/2019 for about 32 years									
Trading Rule	n_b	n_s	$\overline{X_b}$	$\overline{X_s}$	s_b	s_s	$\square_h$	T_1	T_2	T_3

MA(1,5,0)	4631	3365	0.006	-0.008	0.011	0.014	0.0004	37.551**	-34.610**	49.648**
MA(1,15,0)	4872	3114	0.004	-0.005	0.011	0.017	0.0004	21.293**	-17.083**	24.918**
MA(1,50,0)	5191	2760	0.002	-0.003	0.010	0.019	0.0004	11.106**	-8.306**	11.816**
MA(1,100,0)	5373	2528	0.002	-0.002	0.010	0.019	0.0004	7.910**	-6.118**	8.464**
MA(1,200,0)	5681	2120	0.001	-0.002	0.011	0.021	0.0004	5.313**	-4.421**	5.807**
MA(1,5,1)+	1724	1435	0.012	-0.015	0.013	0.016	0.0004	35.283**	-35.178**	35.283**
MA(1,15,1)	3359	2039	0.005	-0.007	0.011	0.019	0.0004	23.714**	-17.614**	25.799**
MA(1,50,1)	4510	2224	0.002	-0.003	0.010	0.020	0.0004	12.795**	-8.231**	12.018**
MA(1,100,1)	4975	2143	0.002	-0.002	0.010	0.021	0.0004	8.694**	-5.855**	8.249**
MA(1,200,1)	5474	1905	0.001	-0.002	0.011	0.021	0.0004	5.253**	-4.320**	5.617**

** indicates that the T-ratio is significant at $\alpha = 0.01$ by comparing with $z = \pm 2.576$ wrongfully in previous studies

n_b, n_s are sample sizes, $\overline{X_b}$ and $\overline{X_s}$ are sample means, s_b, s_s are sample standard deviations for daily returns of Buy and Sell Days, respectively. μ_h is the mean return from buy-and-hold strategy. T-ratios are from Equations:(3)-(5).

+ results were shown in Table 2

The numeric example in Table 2 shows how to apply trading rule MA(1, 5, 1) on 8,000 NASDAQ prices from 4/4/1988 to 12/30/2019 and how to obtain the t-ratios of $T_1 = 35.2835$, $T_2 = -35.1781$, and $T_3 = 49.2913$.

Table 2. A Numeric Example of Applying MA(1, 5, 1) to Find the T-ratios

Date	t	Adjusted NASDAQ Closing	Rate of Return = $\ln(P_t/P_{t-1})$	MA(5)	(1+1%)* MA(5)	(1-1%)* MA(5)	Buy/Sell	Remarks
4/4/1988	-4	371.9						
4/5/1988	-3	373.4						
4/6/1988	-2	377.7						
4/7/1988	-1	378.9						
4/8/1988	0	381.8	0.0076	376.74	380.51	372.97	Buy	381.8 > 380.51
4/11/1988	1	382.5	0.0018	378.86	382.65	375.07		382.5 < 382.65
4/12/1988	2	383.4	0.0024	380.86	384.67	377.05		383.4 < 384.67
4/13/1988	3	383.4	0.0000	382.00	385.82	378.18		383.4 < 385.82
4/14/1988	4	374.5	-0.0235	381.12	384.93	377.31	Sell	374.5 < 377.31
4/15/1988	5	373.9	-0.0016	379.54	383.34	375.74	Sell	373.9 < 375.74
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
12/27/2019	7964	9006.62	-0.0017	8970.50	9060.21	8880.80		9006.62<9060.21
12/30/2019	7965	8945.99	-0.0068	8974.71	9064.45	8884.96		8945.99<9064.45
μ_h	0.000398							
n_b	1,724							
n_s	1,435							
$\overline{X_b}$	0.0117							
$\overline{X_s}$	-0.0147							
s_b	0.0133							
s_s	0.0162							

T_1	$=:0.0117-0.000398)/(\frac{0.0133}{\sqrt{1724}}) = 35.2835$
T_2	$=:-0.0147-0.000398)/\left(\frac{0.0162}{\sqrt{1435}}\right) = -35.171$
T_3	$=:0.0117+0.0147)/\sqrt{\frac{0.0133^2}{1724}+\frac{0.0162^2}{1435}} = 49.2913$

Brock, et al (1992) acknowledges the adoption of bootstrap methods on the simulated null models (i.e., on the models of random walk, AR(1), GARCH-M, and the Exponential GARCH) for stock prices. In contrast to Brock, et al (1992), we directly apply the block bootstrap to the resampling of the NASDAQ price index and observe the distribution of the T-ratios. The distribution of the T-ratios on the NASDAQ has never been shown in Brock et al (1992), Metghalchi et al (2011 and 2016), or in any other academic research paper.

In this study, we use block bootstrap methods to show that the large T-ratios are in fact, not significant under certain appropriately chosen fixed block lengths.

In contrast to the original resampling from independent random samples (Efron, 1979), the block bootstrap method is used when samples are correlated. Because the sample is a time series taken from the NASDAQ price index, the original resampling method for independent observations will not be able to replicate the correlations among the data (Hall and Horowitz, 1996, and Inoue and Shintani, 2001, Kunsch, 1989, and Politis, 2003). The rule of thumb for a block bootstrap is to group data into subgroups of length $l = n^{1/3}$ (Hall and Horowitz, 1996, and Inoue and Shintani, 2001) first, then an overlapping or non-overlapping method can be applied in resampling.

The overlapping or so-called moving block bootstrap method is first introduced by Kunsch (1989). It divides the time series into n-l-1 blocks. Block 1 contains $\{y_1, y_2, \ldots, y_l\}$, Block 2 contains $\{y_2, y_3, \ldots, y_{l+1}\}$, ….., and the last Block, the (n-l+1)th Block contains $\{y_{n-l+1}, y_{n-l+2}, \ldots., y_n\}$. For an easy illustration, let n be divisible by l and let k = n/l. The block bootstrap sample of length n will be generated as $\{y_{i1+1}, y_{i1+2}, \ldots, y_{i1+l}; y_{i2+1}, y_{i2+2}, \ldots, y_{i2+l}; \ldots.., y_{ik+1}, y_{ik+2}, \ldots, y_{ik+l}\}$, where $i_1, i_2, \ldots, i_k$ are k independently and identically distributed (i.i.d.) random samples with or without replacement from {1, 2, ….., n-l+1}. For instance, n = 12, and l = 3. For the overlapping method, Block 1 contains $\{y_1, y_2, y_3\}$, Block 2 contains $\{y_2, y_3, y_4\}$, ….., and the last block, the (12-3+1)=10th Block contains $\{y_{10}, y_{11}, y_{12}\}$. Let k=12/3=4 i.i.d. random samples with or without replacement from {1, 2, …., 10} be chosen. For instance, let the four random samples be {2, 7, 5, 2}. The new time series of length n=12 will be $\{y_2, y_3, y_4; y_7, y_8, y_9; y_5, y_6, y_7;$ y2, y3, y4}.

For the non-overlapping or so-called simple block bootstrap method, we divide the data into k = n/l blocks, where for an easy illustration we assume n is divisible by l. Block 1 contains $\{y_1, y_2, \ldots, y_l\}$, Block 2 contains $\{y_{l+1}, y_{l+2}, \ldots, y_{2l}\}$, ….., and the last Block n/l = k^{th} Block contains $\{y_{n-l+1}, y_{n-l+2}, \ldots., y_n\}$. The block bootstrap sample of length n will be $\{y_{(i1-1)*l+1}, \ldots., y_{i1*l}; Y_{(i2-1)l+2}, \ldots.., y_{i2*l}; \ldots.., y_{(ik-1)*l+1}, \ldots.., y_{ik*l}\}$ where $i_1, i_2, \ldots, i_k$ are k i.i.d. random samples with or without replacement from {1, 2, ….., k}. For instance, n = 12, and l = 3. For the overlapping method, Block 1 contains {y_1, y_2, y_3}, Block 2 contains {y_4, y_5, y_6}, ….., and the last block, the 12/3=4th Block contains {y_{10}, y_{11}, y_{12}}. Let k=12/3=4 i.i.d. random samples with or without replacement from {1, 2, 3, 4} be chosen. For instance, let the four random samples be {1, 4, 3, 1}. The new time series of length n=12 will be {y_1, y_2, y_3; y_{10}, y_{11}, y_{12}; y_7, y_8, y_9; y_1, y_2, y_3}.

From Lahiri (1999) and Andrews (2002), there is little difference in performance on sampling distributions of statistics from overlapping and non-overlapping methods. Under the weak form of the EMH, the stationarity condition is not required. As such, we do not need to vary the block length randomly to keep the stationary structure of the NASDAQ (Politis & Romano (1994), and Politis, 2003). In addition, Brock, et al (1992), Kiefer and Vogelsang (2005), and Shao and Politis (2013) suggest the use of fixed block bootstrap method for heteroskedastic and autocorrelated time series. Therefore, we use a fixed, not random, block length overlapping method when constructing the sampling distributions for ratios T_1, T_2, and T_3 using the statistical software “r.”

In our study, the smaller block length of $n^{1/3}$ = 20 will not be able to keep the correlation relationship of our original time series. A block length of ¼ to ½ of the length of a time series will be able to keep the correlation of the time series. For instance, n = 8,000, the block length of 20 will not show the non-significant property in statistics of ratios T_1, T_2, and T_3. However, with a block length of 2,000, or 4,000, we will be able to show the non-significant property in statistics of ratios T_1, T_2, and T_3. For instance, if we are using a block of length 2,000, then the time series of length 8,000 is classified into 8,000-2,000+1 = 6,001 subgroups. Four i.i.d. random samples from {1, 2, …., 6,001} will be drawn. The block strapping time series of size 8,000 will be composed accordingly for our analysis in “r.”

The p-values for ratios T_1, T_2, and T_3 will depend on the length of the time series, moving periods, and the fixed block length. Tables 3-5 show the p-values for ratios T_1, T_2, and T_3 for various length of time series being n = 1,000, 5,000, and 8,000; moving period of 5, 15, 50, 100, and 200 with and without a "whiplash" signal of □ = 1%; and different fix block length. From our study, the "whiplash" will not make much difference on p-values for ratios T_1, T_2, and T_3.

Despite the discussion of non-independency of observations within and between samples via runs test by Ren, Ren, and Glasure (2018), the p-values for a fixed block length of about ¼ to ½ of the original length of a time series also show the non-significant effect of the T-ratios. In other words, the large T-ratios in Table 1 lead to the conclusion that hypotheses H_0: $\mu_b = \mu_h$, $\mu_s = \mu_h$, and $\mu_b = \mu_s$ cannot be rejected at a significant level of $\alpha = 0.05$ or 0.01 and refutes the previous studies that the market is not efficient. In fact, without needing to show the histograms and the Q-Q probability distribution, from the p-values (tail areas) in Tables 3-5, we can see that the T-ratios T_1, T_2, and T_3 are far from the standard normal distribution claimed by previous studies (Brook, et al., 1992, and Metghalchi, et al., 2011, and 2016).

Table 3. p-values of the T_1-ratios for Various MA Trading Rules from Block Bootstrapping

	For 1,000 NASDAQ Daily Prices from 1/11/2016 to 12/30/2019					
	Fixed Block Length					
Trading Rule	50	100	200	250	400	T_1-ratio
MA(1,5,0)	0.002**	0.002**	0.418	0.659	0.973	13.604
MA(1,15,0)	0.002**	0.108	0.729	0.921	0.755	6.635
MA(1,50,0)	0.212	0.503	0.795	0.645	0.436	3.614
MA(1,100,0)	0.150	0.182	0.150	0.106	0.048*	2.081
MA(1,200,0)	0.000**	0.008**	0.002**	0.010*	0.004**	0.879
MA(1,5,1)	0.002**	0.008**	0.230	0.384	0.637	13.534
MA(1,15,1)	0.002**	0.018*	0.382	0.583	0.905	9.438
MA(1,50,1)	0.006	0.168	0.913	0.891	0.617	4.282
MA(1,100,1)	0.108	0.148	0.126	0.082	0.038*	2.027
MA(1,200,1)	0.000**	0.002**	0.002**	0.002**	0.004**	0.772
	For 5,000 NASDAQ Daily Prices from 2/15/2000 to 12/30/2019					
	Fixed block length					
Trading Rule	250	500	1,000	1,250	2,000	T_1-ratio
MA(1,5,0)	0.002**	0.026*	0.354	0.511	0.821	30.023
MA(1,15,0)	0.002**	0.046*	0.422	0.569	0.767	17.361
MA(1,50,0)	0.002**	0.058	0.464	0.611	0.677	9.470
MA(1,100,0)	0.002**	0.062	0.428	0.565	0.835	7.318
MA(1,200,0)	0.002**	0.070	0.458	0.589	0.827	5.698
MA(1,5,1)	0.002**	0.018*	0.280	0.416	0.997	27.054
MA(1,15,1)	0.002**	0.028*	0.360	0.501	0.849	18.735
MA(1,50,1)	0.002**	0.040*	0.398	0.543	0.831	10.845
MA(1,100,1)	0.002**	0.042*	0.422	0.553	0.753	8.109
MA(1,200,1)	0.002**	0.076	0.470	0.589	0.837	5.436
	For 8,000 NASDAQ Daily Prices from 4/4/1988 to 12/30/2019					
	Fixed Block Length					
Trading Rule	250	500	1,000	2,000	4,000	T_1-ratio
MA(1,5,0)	0.002**	0.002**	0.074	0.591	0.805	37.551
MA(1,15,0)	0.002**	0.008**	0.106	0.661	0.727	21.293
MA(1,50,0)	0.002**	0.008**	0.168	0.807	0.629	11.106
MA(1,100,0)	0.012	0.012*	0.248	0.941	0.521	7.910
MA(1,200,0)	0.012	0.060	0.498	0.841	0.352	5.313
MA(1,5,1)	0.002**	0.002**	0.064	0.523	0.899	35.283
MA(1,15,1)	0.002**	0.002**	0.076	0.573	0.823	23.714
MA(1,50,1)	0.002**	0.004**	0.118	0.711	0.703	12.795
MA(1,100,1)	0.002**	0.006**	0.196	0.861	0.591	8.694
MA(1,200,1)	0.002**	0.060**	0.480	0.863	0.362	5.253

* indicates that the T-ratio is significant at $\alpha = 0.05$; ** indicates that the T-ratio is significant at $\alpha = 0.01$

Table 4. p-values of the T_2-ratios for Various MA Trading Rules from Block Bootstrapping

	For 1,000 NASDAQ Daily Prices from 1/11/2016 to 12/30/2019					
	Fixed Block Length					
Trading Rule	50	100	200	250	400	T_2-ratio
MA(1,5,0)	0.000**	0.000**	0.008**	0.016*	0.000**	-12.030
MA(1,15,0)	0.000**	0.004**	0.010*	0.026*	0.002**	-5.080
MA(1,50,0)	0.038	0.006**	0.012*	0.010*	0.012*	-2.877
MA(1,100,0)	0.757	0.066	0.034*	0.036*	0.016*	-2.098
MA(1,200,0)	0.090	0.593	0.755	0.619	0.398	-1.183
MA(1,5,1)	0.000**	0.000**	0.012*	0.028*	0.014*	-11.724
MA(1,15,1)	0.000**	0.000**	0.010*	0.018*	0.004**	-5.913
MA(1,50,1)	0.056	0.020	0.034*	0.038*	0.028*	-2.864
MA(1,100,1)	0.268	0.026	0.040*	0.034*	0.070	-2.329
MA(1,200,1)	0.046	0.274	0.759	0.931	0.883	-1.055
	For 5,000 NASDAQ Daily Prices from 2/15/2000 to 12/30/2019					
	Fixed Block Length					
Trading Rule	250	500	1,000	1,250	2,000	T_2-ratio
MA(1,5,0)	0.000**	0.002**	0.028*	0.040*	0.076	-27.843
MA(1,15,0)	0.000**	0.004**	0.018*	0.046*	0.064	-13.617
MA(1,50,0)	0.000**	0.002**	0.012*	0.118	0.166	-6.046
MA(1,100,0)	0.004	0.038	0.082	0.056	0.060	-4.726
MA(1,200,0)	0.617	0.398	0.577	0.577	0.378	-3.140
MA(1,5,1)	0.002	0.022	0.022*	0.038*	0.064	-29.064
MA(1,15,1)	0.000**	0.004**	0.014*	0.038*	0.034*	-14.158
MA(1,50,1)	0.002**	0.016	0.070	0.084	0.136	-6.257
MA(1,100,1)	0.008	0.052	0.096	0.094	0.028*	-4.587
MA(1,200,1)	0.807	0.507	0.643	0.649	0.490	-3.048
	For 8,000 NASDAQ Daily Prices from 4/4/1988 to 12/30/2019					
	Fixed block length					
Trading Rule	250	500	1,000	2,000	4,000	T_2-ratio
MA(1,5,0)	0.000**	0.000**	0.000**	0.018*	0.080	-34.610
MA(1,15,0)	0.000**	0.000**	0.000**	0.022*	0.090	-17.083
MA(1,50,0)	0.000**	0.000**	0.000**	0.010*	0.024	-8.306
MA(1,100,0)	0.000**	0.000**	0.000**	0.018*	0.060	-6.118
MA(1,200,0)	0.000**	0.000**	0.000**	0.034*	0.124	-4.421
MA(1,5,1)	0.000**	0.000**	0.000**	0.018*	0.084	-35.178
MA(1,15,1)	0.000**	0.000**	0.000**	0.020*	0.086	-17.614
MA(1,50,1)	0.000**	0.000**	0.000**	0.024*	0.070	-8.231
MA(1,100,1)	0.000**	0.000**	0.000**	0.022*	0.064	-5.855
MA(1,200,1)	0.000**	0.000**	0.002**	0.028*	0.120	-4.320

* indicates that the T-ratio is significant at $\alpha = 0.05$; ** indicates that the T-ratio is significant at $\alpha = 0.01$

Table 5. p-values of the T_3-ratios for Various MA Trading Rules from Block Bootstrapping

	For 1,000 NASDAQ Daily Prices from 1/11/2016 to 12/30/2019					
	Fixed Block Length					
Trading Rule	50	100	200	250	400	T_3-ratio
MA(1,5,0)	0.002**	0.002**	0.010*	0.014*	0.012*	16.993
MA(1,15,0)	0.002**	0.006**	0.028*	0.044*	0.048*	6.994
MA(1,50,0)	0.002**	0.084	0.064	0.068	0.076	3.658
MA(1,100,0)	0.002**	0.627	0.380	0.260	0.188	2.434
MA(1,200,0)	0.002**	0.050	0.256	0.388	0.617	1.314
MA(1,5,1)	0.002**	0.002**	0.012*	0.022*	0.022*	16.860
MA(1,15,1)	0.002**	0.002**	0.018*	0.032*	0.024*	8.125
MA(1,50,1)	0.002**	0.142	0.090	0.120	0.132	3.562
MA(1,100,1)	0.002**	0.945	0.214	0.168	0.164	2.598
MA(1,200,1)	0.002**	0.006**	0.158	0.208	0.328	1.158
	For 5,000 NASDAQ Daily Prices from 2/15/2000 to 12/30/2019					
	Fixed Block Length					
Trading Rule	250	500	1,000	1,250	2,000	T_3-ratio
MA(1,5,0)	0.002**	0.016*	0.022*	0.032*	0.094	40.041
MA(1,15,0)	0.002**	0.020*	0.038*	0.042*	0.094	20.187
MA(1,50,0)	0.002**	0.066	0.102	0.108	0.186	9.097
MA(1,100,0)	0.002**	0.060	0.068	0.078	0.138	6.842
MA(1,200,0)	0.002**	0.364	0.440	0.515	0.591	4.631
MA(1,5,1)	0.002**	0.002**	0.012*	0.012*	0.080	39.658
MA(1,15,1)	0.002**	0.002**	0.016*	0.030*	0.078	20.922
MA(1,50,1)	0.002**	0.006**	0.050	0.076	0.158	9.476
MA(1,100,1)	0.002**	0.010*	0.072	0.074	0.138	6.742
MA(1,200,1)	0.002**	0.388	0.503	0.581	0.713	4.388
	For 8,000 NASDAQ Daily Prices from 4/4/1988 to 12/30/2019					
	Fixed Block Length					
Trading Rule	250	500	1,000	2,000	4,000	T_3-ratio
MA(1,5,0)	0.002**	0.002**	0.002**	0.018*	0.090	49.648
MA(1,15,0)	0.002**	0.002**	0.002**	0.034*	0.110	24.918
MA(1,50,0)	0.002**	0.002**	0.002**	0.028*	0.096	11.816
MA(1,100,0)	0.002**	0.002**	0.002**	0.052	0.148	8.464
MA(1,200,0)	0.002**	0.004**	0.028*	0.098	0.206	5.807
MA(1,5,1)	0.002**	0.002**	0.064	0.523	0.899	35.283
MA(1,15,1)	0.002**	0.002**	0.002**	0.030*	0.108	25.799
MA(1,50,1)	0.002**	0.002**	0.002**	0.042*	0.114	12.018
MA(1,100,1)	0.002**	0.002**	0.002**	0.054	0.144	8.249
MA(1,200,1)	0.006**	0.006**	0.024*	0.094	0.198	5.617

* indicates that the T-ratio is significant at $\alpha = 0.05$; ** indicates that the T-ratio is significant at $\alpha = 0.01$

3. Conclusion

In this article, we examine the sampling distributions of the T-ratios in Equations (3)-(5) used to support market efficiency. From the p-values (tail areas) listed in Tables

(3)-(5), the fixed block bootstrap methods show that those T-ratios are far from the standard normal distribution claimed by numerous previous studies, especially when the length of time series get larger, e.g., 5,000 or 8,000 observations (about 20 years' worth and 30 years' worth of observations, respectively). Therefore, the T-ratios presented in Equations (3)-(5) cannot be used to test whether or not the market is efficient. Numerous studies have applied T-ratios to the NASDAQ and other price indices following a Normal (or a T) distribution to assess whether the markets are at least weak form efficient. Recent studies include Metghalchi, Metghalchi, Hajilee, and Hayes (2018) on the Iceland All-share Index and Metghalchi, Hayes, and Niroomand (2019) on the Morgan Stanley Capital International (MSCI) Emerging Market Index. Brock, et al (1992) acknowledges the adoption of bootstrap methods on the simulated null models (i.e., on the models of random walk, AR(1), GARCH-M, and the Exponential GARCH) for the Dow Jones price index from 1897 to 1986. In contrast to Brock, et al (1992), we directly apply the fixed block length bootstrap method to the resampling of the more recent NASDAQ price index and observe the distribution of the T-ratios. In contrast, we demonstrate that such markets cannot be deemed inefficient due to the inappropriate interpretation of T-ratios as what has claimed in Metghalchi, et al (2011, 2016, 2018, and 2019).

References

Andrews, D. (2002). Higher-order improvements of a computationally attractive k-step bootstrap for extremum estimators. Econometrica, 70(1):119-162.

Bessembinder, H., & Chan, K. (1998) Market efficiency and the returns to technical analysis. Financial Management, 27(2):5-17.

Brock, W., Lakonishok, J., & LeBaron, B. (1992) Simple technical trading rules and the stochastic properties of stock returns. Journal of Finance, 47(5):1731-1764.

Efron, B. (1979) Bootstrap methods: Another look at the jackknife. The Annuals of Statistics, 7(1):1-26.

Fama, E. (1965). The behavior of stock market prices. Journal of Business, 38, 34-105.

Fama, E. (1970) Efficient capital markets: a review of theory and empirical work. Journal of Finance, 25(2):383-417.

Hall, P., & Horowitz, J. (1996) Bootstrap Critical Values for Tests Based on Generalized-Method-of-Moments Estimators. Econometrica, 64(4):891-916.

Hudson, R., Dempsey, M., & Keasey, K. (1996) A note on the weak form efficiency of capital markets: The application of simple technical trading rules to UK stock prices 1935-1994. Journal of Banking and Finance, 20:1121-1132.

Inoue, A., & Shintani, M. (2006) Bootstrapping GMM estimators for time series. Journal of Econometrics, 133(2):531-555.

Kiefer, N., & Vogelsang, T. (2005) A new asymptotic theory for heteroskedasticity-autocorrelation robust tests. Econometric Theory, 21:1130-1164.

Kunsch, H. (1989) The Jackknife and the bootstrap for general stationary observations. The Annual of Statistics, 17(3):1217-1241.

Kwon, K., & Kish, R. (2002) Technical trading strategies and return predictability: NYSE. Applied Financial Economics, 12:639-653.

Lahiri, S. (1999) Theoretical comparisons of block bootstrap methods. The Annals of Statistics, 27(1):386-404.

Metghalchi, M., Chang, Y. & Du, J. (2011) Technical trading rules for NASDAQ composite index. International Research Journal of Finance and Economics, 73:109-121.

Metghalchi, M., Chen, C., & Hajilee, M. (2016) Moving average trading rules for NASDAQ composite index. Applied Finance Letters, 5(2):45-57.

Metghalchi, M., Hajilee, M., & Hayes, L. (2018) Return predictability and market efficiency: evidence from the Bulgarian stock market. Eastern European Economics, 00: 1-18.

Metghalchi, M., Hayes, L., & Niroomand, F. (2019) A technical approach to equity investing in emerging markets. The Review of Financial Economics, 37(3): 389-403.

Politis, D. & Romano, J. (1994) The stationary bootstrap. Journal of the American Statistical Association, 89 (428): 1303–1313.

Politis, D. (2003) The impact of bootstrap methods on time series analysis. Statistical Science, 18(2):219-230.

Ren, L., Ren, P., & Glasure, Y. (2018) Is the simple trading range break-out rule profitable from the NASDAQ index? Benchmarking: An International Journal, 25(2):763-771.

Shao, X., & Politis, D. (2013) Fixed b subsampling and the block bootstrap: Improved confidence sets based on p-value calibration. Journal of the Royal Statistical Society, Series B, 75(1): 161-184.

Yu, H., Nartea, G., Gan, C., & Yao, L. (2013) Predictive ability and profitability of simple technical trading rules: Recent evidence from Southeast Asian stock markets. International Review of Economics and Finance, 25:356-371.