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Generalized inverted Kumaraswamy-Rayleigh Distribution: Properties and Application

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The paper proposes a new four parameter distribution called Generalized inverted Kumaraswamy- Rayleigh Distribution. Various properties of the proposed distribution including moments, mean deviation about mean and median, entropy, L-moments, stress-strength reliability etc. are obtained. The parameter estimation is carried out using Maximum likelihood estimation procedure. Finally, two real life data sets are incorporated to illustrate the usefulness and flexibility of the proposed model.

Keywords: Inverted Kumaraswamy distribution, Rayleigh Distribution, moments, entropy and MLE.

1. Introduction

Statistical distributions are prevalent in many areas such as physics, computer science, insurance, communication etc. Due to the random nature of the data arising in different fields, the well- established distributions fail to provide an acceptable fit. Hence, several new generalizations of existing models have been proposed. For example "Marshal-Olkhin- G family" by Marshal and Olkhin (1997), "Kumaraswamy family-G" by Cordiero et al. (2013),"The gamma generated family" by Zografos and Balakrisnan (2009), "A new method of Generating Families of distributions" by Alzaghal et al. (2013), "Kumaraswamy Transmuted-G family of distribution" by Afify et al. (2016a), "Kumaraswamy odd log logistic Distribution" by Alizadeh et al.(2015a), "Weibull-G" by Bourguignon et al. (2014), "Kumaraswamy Marshal-Olkhin" by Alizadeh et al. (2015b), Generalized Transmute-G family" by Nofal et al. (2015),"Burr X-G" by Yousuf et al. (2016), "The Transmuted Geometric-G family" by Afify et al. (2016b), "Type I Half logistic family" by Cordeiro et al. (2016) etc.

The inverted Kumaraswamy Distribution (IKwD) was introduced by Al-Fattah (2017). He expounded some of the properties of IKwD. Recently, Jamal et al. (2018) proposed "Generalized Inverted Kumaraswamy Generated Family of Distribution" (GIKw-G).

The cdf and pdf of a GIKw-G family are given by (1) and (2) respectively.

$$M(x) = \left\{1 - \left(1 - F^{\gamma}(x)\right)^{\alpha}\right\}^{\beta};\ \alpha, \beta, \gamma > 0 \quad (1)$$

$$m(x) = \alpha\beta\gamma f(x) F^{\gamma-1}(x)(1 - F^{\gamma}(x))^{\alpha-1}\{1 - (1 - F^{\gamma}(x))^{\alpha}\}^{\beta-1} \quad (2)$$

where $F(x)$ and $f(x)$are the cdf and pdf of baseline distribution respectively.

The Rayleigh Distribution was introduced by Rayleigh (1980). Siddiqui (1962) studied properties of Rayleigh distribution. Some other authors who also studied this model are Merovci et al. (2013), Ahmad et al. (2014), Howlader and Hossian (1995), Malik et al. (2019) etc.

The main aim of this paper is to generalize RD using GIKw-G family so that the flexibility of RD can be enhanced in terms of density function and hazard rate. The new distribution is named GIKw-Rayleigh Distribution (GIKw-RD). The new distribution exhibits more complex shapes of hazard rate function (hrf) and also outperforms some well-known distributions in terms of two real life data sets. The rest of the paper is organized as follows: In Section 2, the pdf, cdf and the associated reliability measures of the new model are obtained. Section 3 and 4 deal with the structural properties and stress strength reliability of the proposed model are derived. The L-moments and estimation of parameters are discussed in section 5 and 6 respectively. Finally in section 7, the applicability of the model is established by using two real life data sets.

2. GIKw-RD

The cdf and pdf of GIKw-RD can be obtained by putting $F(x) = 1 - e^{-\frac{x^2}{\theta^2}}$ and f(x) $= \frac{2x}{\theta^2} e^{-\frac{x^2}{\theta^2}}$ in (1) and (2) as follows:

$$M(x) = \left\{1 - \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha}\right\}^{\beta} \quad (3)$$

$$m(x) = \alpha\beta\gamma\ \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}} \left(1 - e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma-1} \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha-1} \times \left\{1 - \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha}\right\}^{\beta-1} \quad (4)$$

where $x, \theta > 0$.

The plots of pdf of GIKw-RD are displayed in Figure 1.

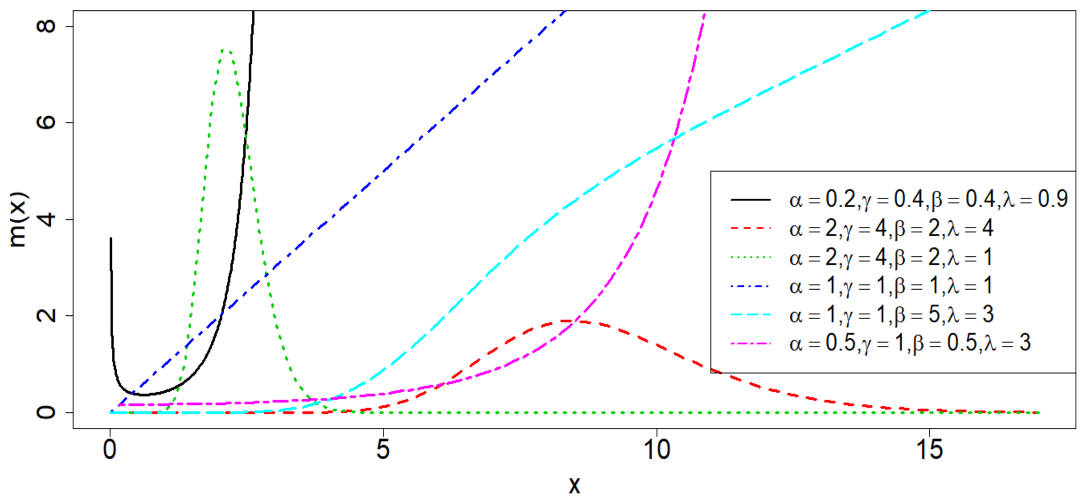

Figure 1. Plots of pdf of GIKw-RD

Figure 1 suggests that GIKw-RD is unimodal and exhibits variety of shapes such as bathtub, constant, increasing decreasing, decreasing-increasing etc. Thus, GIKw-RD can be used to analyse data sets of diverse nature.

The survival function, hrf and reverse hrf of GIKw-RD is given by (5), (6) and (7) respectively. The plots of hrf of GIKw-RD for different parameter combinations are displayed in Figure 2.

$$S(x) = \left\{1 - \left(1 - \left(1 - e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha}\right\}^{\beta}. \tag{5}$$

$$h(x) = \frac{\alpha\beta\gamma\frac{x}{\theta^2}e^{-\frac{x^2}{2\theta^2}}\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma-1}\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha-1}\left\{1-\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha}\right\}^{\beta-1}}{1-\left\{1-\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha}\right\}^{\beta}} \tag{6}$$

$$\varphi(x) = \frac{\alpha\beta\gamma\frac{x}{\theta^2}e^{-\frac{x^2}{2\theta^2}}\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma-1}\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha-1}}{\left\{1-\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^{\gamma}\right)^{\alpha}\right\}} \tag{7}$$

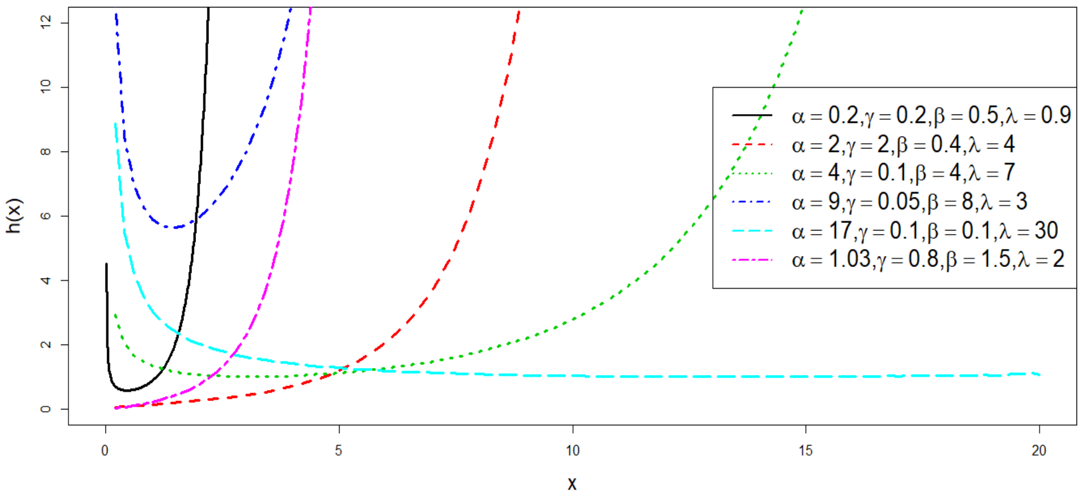

Figure 2. Plots of hrf of GIKw-RD.

Figure 2 suggests that the hrf of GIKw-RD exhibits bathtub, reverse J and constant shapes.

2.1 Mixture Representation

The pdf and cdf of GIKw-RD can be alternatively represented as

$$m(x) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{x}{\theta^2} e^{-\frac{(q+1)x^2}{2\theta^2}} \tag{8}$$

where $\psi(p) = \sum_{m,l=0}^{\infty} \binom{\beta}{m} \binom{\alpha m}{l} (-1)^{l+m} \sum_{p=k}^{\infty} \binom{\gamma l}{k} \binom{k}{p} (-1)^{p+k}$.

$$M(x) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \psi(p)(-1)^q \binom{p-1}{q} e^{-\frac{qx^2}{2\theta^2}} \tag{9}$$

(8) and (9) are very useful in deriving various properties of GIKw-RD.

2.2 Structural Properties

2.2.1 Moments

The r^{th} moment about origin of GIKw-RD is given as

$$\mu_r' = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{\theta^r 2^{\frac{r}{2}} \Gamma\left(\frac{r}{2}+1\right)}{(q+1)^{\left(\frac{r}{2}+1\right)}}.$$

where $\Gamma(a)$ is the gamma function.

2.2.2 Mean

The mean of GIKw-RD given as

$$\mu_1' = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{\theta 2^{\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)}{(q+1)^{\left(\frac{3}{2}\right)}}.$$

2.2.3 Variance

The variance instead of GIKw-RD given as

$$\mu_r = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{2\theta^2}{(q+1)^2} - \left\{\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{\theta 2^{\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)}{(q+1)^{\left(\frac{3}{2}\right)}}\right\}^2.$$

2.2.4 Incomplete Moment

The n^{th}incomplete moment about origin of GIKw-RD is given as

$$\mu_{(n)}' = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{\theta^n 2^{\frac{n}{2}}\,\gamma\left(\frac{n}{2}+1, \frac{(q+1)s^2}{2\theta^2}\right)}{(q+1)^{\left(\frac{n}{2}+1\right)}}.$$

where $\gamma(a,b)$ is the lower incomplete gamma function.

2.2.5 Mean Deviation (MD) about Mean and Median

The MD about mean of GIKw-RD is given as

$$D(\mu) = 2\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{p}{q+1}\, \psi(p)(-1)^q \binom{p-1}{q} \left\{\mu\,\gamma\left(1, \frac{(q+1)\mu^2}{2\theta^2}\right) - \left(\frac{\theta\, 2^{\frac{1}{2}}\,\gamma\left(\frac{3}{2}, \frac{(q+1)\mu^2}{2\theta^2}\right)}{(q+1)^{\frac{1}{2}}}\right)\right\}.$$

Also, the MD about Median of GIKw-RD is of the following form

$$D(M) = \mu - 2\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{\theta\, 2^{\frac{1}{2}}\,\gamma\left(\frac{3}{2}, \frac{(q+1)\mu^2}{2\theta^2}\right)}{(q+1)^{\frac{3}{2}}}.$$

2.2.6 Mean Residual Life (MRL) and Mean Waiting Time (MWT).

The MRL and MWT for GIKw-RD are given by (16) and (17) respectively.

$$MRL = \frac{1 - \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{\theta 2^{\frac{1}{2}}\,\gamma\left(\frac{3}{2}, \frac{(q+1)s^2}{2\theta^2}\right)}{(q+1)^{\left(\frac{3}{2}\right)}}}{1 - \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\, \psi(p)(-1)^q \binom{p-1}{q} \frac{t}{\theta^2} e^{-\frac{(q+1)t^2}{2\theta^2}}} - t\,,$$

$$MWT = t - \frac{\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\,\psi(p)(-1)^q \binom{p-1}{q} \frac{\theta 2^{\frac{1}{2}}\gamma\left(\frac{3}{2},\frac{(q+1)s^2}{2\theta^2}\right)}{(q+1)^{\left(\frac{3}{2}\right)}}}{\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} p\,\psi(p)(-1)^q \binom{p-1}{q} \frac{t}{\theta^2} e^{-\frac{(q+1)t^2}{2\theta^2}}}.$$

2.3 Stress Strength Reliability

If $X_1 \sim GIKw-RD(\alpha,\beta,\gamma,\theta_1)$ and $X_2 \sim GIKw-RD(\alpha,\beta,\gamma,\theta_2)$, then the stress strength reliability denoted by R for GIKw-RD can be obtained as

$$R = \sum_{p,q,r=0}^{\infty}\sum_{s,u,v=0}^{\infty} \alpha\beta\gamma(-1)^{p+q+r}(-1)^{s+u+v}\binom{p-1}{p}\binom{\beta}{q}\binom{\alpha p+\alpha-1}{r} \times \binom{\alpha q}{s}\binom{\gamma+\gamma r}{u}\binom{\gamma s}{v}\int_0^{\infty} \frac{x}{\theta_1^2} e^{-\frac{1}{2}\left(\frac{u}{\theta_1^2}+\frac{v}{\theta_2^2}+1\right)} dx. \tag{10}$$

Upon solving (10) we get,

$$R = \sum_{p,q,r=0}^{\infty}\sum_{s,u,v=0}^{\infty} \frac{\alpha\beta\gamma}{\theta_1^2}(-1)^{p+q+r}(-1)^{s+u+v}\binom{p-1}{p}\binom{\beta}{q}\binom{\alpha p+\alpha-1}{r}\binom{\alpha q}{s}\binom{\gamma+\gamma r}{u}\binom{\gamma s}{v} \times \left(\frac{u}{\theta_1^2}+\frac{v}{\theta_2^2}+1\right)$$

2.4 L-Moments

The L-Moments can be computed using the formula given below:

$$E(X_{i:n}^r) = \int_0^{\infty} x^r\, m_{i:n}(x)dx. \tag{11}$$

We have

$$m_{i:n}(x) = \frac{n!}{(i-1)!\,(n-i)!} M^{i-1}(x)[1-M(x)]^{n-i} m(x).$$

$$m_{i:n}(x) = \frac{n!}{(i-1)!\,(n-i)!}\sum_{u=0}^{n-i}\binom{n-i}{u}(-1)^u M^{u+i-1}(x)m(x).$$

$$m_{i:n}(x) = \sum_{u,v=0}^{\infty}\sum_{w,z=0}^{\infty} \alpha\beta\gamma(-1)^{u+v}(-1)^{w+z}\binom{n-i}{v}\binom{2\beta-1}{u}\binom{\alpha u+\alpha-1}{w}\binom{\gamma w}{z} e^{-\frac{zx^2}{2\theta^2}}. \tag{12}$$

Substituting (12) in (11) and solving we get

$$E(X_{i:n}^r) = \sum_{u,v=0}^{\infty}\sum_{w,z=0}^{\infty} \alpha\beta\gamma(-1)^{u+v}(-1)^{w+z}\binom{n-i}{v}\binom{2\beta-1}{u}\binom{\alpha u+\alpha-1}{w}\binom{\gamma w}{z}\frac{\theta^r 2^{\frac{r}{2}-1}\Gamma\left(\frac{r}{2}\right)}{(z)^{\left(\frac{r}{2}\right)}}$$

Theorem: If $X \sim GIKw - RD(\alpha, \beta, \gamma, \theta)$then the Renyi and Mathai- Haubold entropy for GIKw-RD is respectively given by

$$I_\delta = \frac{1}{1-\delta} log \left\{ \begin{array}{c} \sum_{p=0}^{\infty} (\alpha\beta\gamma)^\delta \sum_{q=0}^{u} \sum_{u=0}^{\infty} \binom{\delta(\beta-1)}{p} \binom{\delta\alpha + \alpha p - \delta}{q} \\ \binom{\delta(\gamma-1)+\gamma q}{u} \frac{2^{\frac{\delta}{2}-1}}{\theta^{\frac{3\delta}{2}}} \frac{\Gamma\left(\frac{\delta}{2}\right)}{(\delta+u)^{\frac{\delta}{2}}} \end{array} \right\} ; \delta > 0, \delta \neq 1.$$

$$g_{MH}(x) = \frac{1}{1-\delta} log \left[\left\{ \begin{array}{c} \sum_{p=0}^{\infty} (\alpha\beta\gamma)^{2-\delta} \sum_{q=0}^{u} \sum_{u=0}^{\infty} \binom{\delta(\beta-1)}{p} \binom{\delta\alpha + \alpha p - \delta}{q} \\ \binom{\delta(\gamma-1)+\gamma q}{u} \frac{2^{\frac{\delta}{2}-1}}{\theta^{\frac{3\delta}{2}}} \frac{\Gamma\left(\frac{\delta}{2}\right)}{(\delta+u)^{\frac{\delta}{2}}} \end{array} \right\} - 1 \right].$$

Proof: The Renyi entropy is defined as

$$I_\delta = \frac{1}{1-\delta} log \int_0^\infty m^\delta(x) dx \tag{13}$$

where $\delta > 0, \delta \neq 1$.

Substituting (2) in (13) we get

$$I_\delta = \frac{1}{1-\delta} log \left[\int_0^\infty \left\{ \alpha\beta\gamma f(x) F^{\gamma-1}(x) (1 - F^\gamma(x))^{\alpha-1} \{1 - (1 - F^\gamma(x))^\alpha\}^{\beta-1} \right\}^\delta dx \right].$$

$$I_\delta = \frac{1}{1-\delta} log \left[\int_0^\infty \left\{ \alpha\beta\gamma f(x) F^{\gamma-1}(x) (1 - F^\gamma(x))^{\alpha-1} \{1 - (1 - F^\gamma(x))^\alpha\}^{\beta-1} \right\}^\delta dx \right].$$

$$I_\delta = \frac{1}{1-\delta} log \left[(\alpha\beta\gamma)^\delta \int_0^\infty f^\delta(x) F^{\delta(\gamma-1)}(x) (1 - F^\gamma(x))^{\delta(\alpha-1)} \{1 - (1 - F^\gamma(x))^\alpha\}^{\delta(\beta-1)} dx \right].$$

$$I_\delta = \frac{1}{1-\delta} log \left\{ \begin{array}{c} \sum_{p=0}^{\infty} (\alpha\beta\gamma)^\delta \sum_{q=0}^{u} \sum_{u=0}^{\infty} \binom{\delta(\beta-1)}{p} \binom{\delta\alpha + \alpha p - \delta}{q} \binom{\delta(\gamma-1)+\gamma q}{u} \\ \int_0^\infty \frac{x^\delta}{\theta^{2\delta}} e^{-\frac{(\delta+u)x^2}{2\theta^2}} \end{array} \right\}$$

$$I_\delta = \frac{1}{1-\delta} log \left\{ \begin{array}{c} \sum_{p=0}^{\infty} (\alpha\beta\gamma)^\delta \sum_{q=0}^{u} \sum_{u=0}^{\infty} \binom{\delta(\beta-1)}{p} \binom{\delta\alpha + \alpha p - \delta}{q} \\ \binom{\delta(\gamma-1)+\gamma q}{u} \frac{2^{\frac{\delta}{2}-1}}{\theta^{\frac{3\delta}{2}}} \frac{\Gamma\left(\frac{\delta}{2}\right)}{(\delta+u)^{\frac{\delta}{2}}} \end{array} \right\}$$

Also, the Mathai- Haubold entropy is defined as

$$g_{MH}(x) = \frac{1}{1-\delta} \left\{ \int_0^\infty m^{2-\delta}(x) dx - 1 \right\}.$$

$$g_{MH}(x) = \frac{1}{1-\delta}\left\{\int_0^\infty \left\{ \alpha\beta\gamma f(x) F^{\gamma-1}(x)(1-F^\gamma(x))^{\alpha-1}\{1 - (1-F^\gamma(x))^\alpha\}^{\beta-1}\right\}^{2-\delta} dx - 1\right\}$$

$$g_{MH}(x) = \frac{1}{1-\delta}\left\{(\alpha\beta\gamma)^\delta \int_0^\infty f^{2-\delta}(x) F^{(2-\delta)(\gamma-1)}(x)(1-F^\gamma(x))^{(2-\delta)(\alpha-1)}\{1 - (1-F^\gamma(x))^\alpha\}^{(2-\delta)(\beta-1)} dx \quad - 1\right\}.$$

$$g_{MH}(x) = \frac{1}{1-\delta} log \left\{ \begin{array}{c} \sum_{p=0}^{\infty} (\alpha\beta\gamma)^{2-\delta} \sum_{q=0}^{u}\sum_{u=0}^{\infty} \binom{(2-\delta)(\beta-1)}{p}\binom{(2-\delta)(\alpha-1)+\alpha p}{q} \\ \binom{(2-\delta)(\gamma-1)+\gamma q}{u} \int_0^\infty \frac{x^{(2-\delta)}}{\theta^{2(2-\delta)}} e^{-\frac{(2-\delta+u)x^2}{2\theta^2}} \end{array} \right\}.$$

$$g_{MH}(x) = \frac{1}{1-\delta} log \left\{ \begin{array}{c} \sum_{p=0}^{\infty} (\alpha\beta\gamma)^{2-\delta} \sum_{q=0}^{u}\sum_{u=0}^{\infty} \binom{(2-\delta)(\beta-1)}{p}\binom{(2-\delta)(\alpha-1)+\alpha p}{q} \\ \binom{(2-\delta)(\gamma-1)+\gamma q}{u} \frac{2^{\frac{1}{2}(1-\delta)}\theta^{\frac{3}{2}(\delta-1)}\Gamma\left(\frac{3}{2}-\frac{\delta}{2}\right)}{(2-\delta+u)^{\frac{3}{2}(1-\delta)}} \end{array} \right\}.$$

2.5 Parameter Estimation

Let $\zeta = (\alpha, \beta, \gamma, \theta)^T$ be the vector of parameters of GIKw-RD. Let l be the log-likelihood function computed from a random sample of size n drawn from GIKw-RD.

$$l = nlog\alpha + nlog\beta + nlog\gamma + \frac{1}{\theta^{2n}}\sum_{i=1}^{n} logx_i - \frac{1}{2\theta^2}\sum_{i=1}^{n} x_i^2 + (\gamma-1)log\left(1 - e^{-\frac{x^2}{2\theta^2}}\right) + (\alpha-1)log\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^\gamma\right) + (\beta-1)log\left\{1-\left(1-\left(1-e^{-\frac{x^2}{2\theta^2}}\right)^\gamma\right)^\alpha\right\}.$$

The elements of the score vector $U(\zeta) = \frac{\partial l}{\partial \zeta} = \left(\frac{\partial l}{\partial \alpha}, \frac{\partial l}{\partial \beta}, \frac{\partial l}{\partial \gamma}, \frac{\partial l}{\partial \theta}\right)^T$ upon equating to zero yield the ML estimates $\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$ and $\widehat{\theta}$ respectively.

3. Application

In this section, the applicability of GIKw-RD is established by comparing its performance with two model namely RD and IKwD using two real life data sets. The criterion such as -2logl, AIC and SIC are used as performance comparing tools. Also the value of KS- Statistic and associated p value is computed. The R software has been used to carry out all the computations.

The MLEs of the parameters, the associated values of comparison criterions are for data set 1 and 2 are presented in Table 1 and 2 respectively. Also, the relative histogram and plots of the fitted GIKw-RD, RD and IKwD are displayed in Figure 3 for the two data sets.

Data set 1: This data set corresponds to the Average Annual Percent Change in Private Health Insurance Premiums (All Benefits: Health Services and Supplies), Calendar Years 1969-2007 (SOURCE: Centres for Medicare & Medicaid Services, Office of the Actuary, National Health Statistics Group).

Table 1. Ml estimates and values of comparison criterions for data set 1.

Model	MLE's				-2log l	AIC	SIC	KS-statistic	P-value
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\theta}$					
GIKw-RD	2.210 (1.688)	2.504 (3.180)	0.2303 (0.224)	3.477 (2.41)	115.4	123.4	130.6	0.1077	0.63
RD	-	-	-	1.2882 (0.096)	155.8	157.8	159.6	0.3530	1.5e-05
IKwD	1.978 (0.328)	1.861 (0.403)	-	-	119.0	806.9	123.0	0.1277	0.41

Data set 2: This data set consists of the strength data reported by Badar and Priest (1982).

Table 2. Ml estimates and values of comparison criterions for data set 2.

Model	MLE's				-2log l	AIC	SIC	KS-statistic	P-value
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\theta}$					
GIKw-RD	2.902 (77.46)	1.205 (1.751)	0.977 (1.048)	1.6177 (22.01)	111.50	119.50	128.07	0.0621	0.96
RD				0.9932 (0.063)		114.34	136.48	0.068	0.92
IKwD	3.450 (0.361)	8.300 (1.974)	-	-	126.65	130.65	143.22	0.261	0.000 357

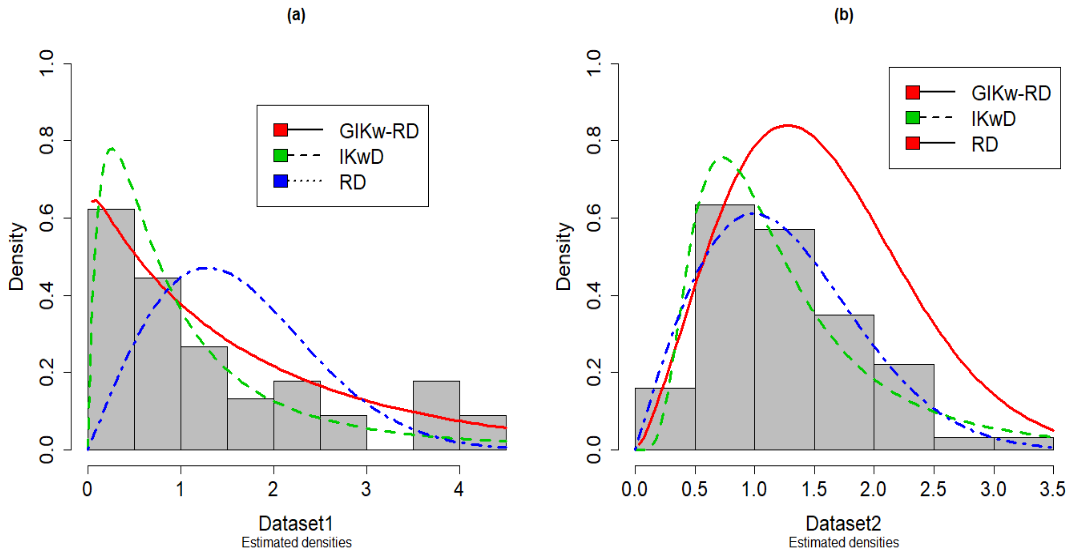

Figure 3. (a) Plots of estimated pdf of GIKw-RD and other competitive models for data set 1. (b) Plots of estimated pdf of GIKw-RD and other competitive models for data set 2.

4. Conclusion

In this paper, a new four parameter lifetime distribution namely Generalized Inverted Kumaraswamy- Rayleigh Distribution (GIKw-RD) and its properties have been studied. The new distribution is compared with two well-known models using two real life data sets and the results are presented in Table 1 and Table 2. From Table 1 and Table 2, it can clearly be seen that GIKw-RD has least value of -2logl, AIC and SIC. Hence, we can conclude that the proposed models provides a better fit than the models used for comparison for the given data sets. We hope that the proposed model entices wider application in diverse fields.

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