Journal of Modern Applied Statistical Methods

Volume 23 | Issue 2 Article 1

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Neha Garg
School of Sciences, Indira Gandhi National Open University, India.

Ambika C
School of Sciences, Indira Gandhi National Open University, India

Menakshi Pachori*
Sanskriti Ayurvedic Medical College & Hospital, Sanskriti University, India,
pachorim07@gmail.com

Recommended Citation

Neha Garg, Ambika C, Menakshi Pachori (2024). Proportionality Adjusted Ratio-Type Calibration Estimators of Population Mean Under Stratified Sampling. Journal of Modern Applied Statistical Methods, 23(2), https://doi.org/10.56801/Jmasm.V23.i2.1

Journal of Modern Applied Statistical Methods
2024, Vol. 23, No. 2,
Doi: 10.56801/Jmasm.V23.i2.1
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ISSN 1538 – 9472

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Neha Garg
School of Sciences, Indira Gandhi National Open University, India.

Ambika C
School of Sciences, Indira Gandhi National Open University, India

Menakshi Pachori
Sanskriti Ayurvedic Medical College & Hospital, Sanskriti University, India

The estimation accuracy of estimators of finite population parameters in sample surveys can be enhanced by taking advantage of existing auxiliary information using the calibration approach. The present study developed the calibration estimators of population mean utilizing known auxiliary information. The properties of the recommended estimators have been studied. The performance of the suggested estimators has also been investigated with the help of a simulation study and compared with the estimators given by Clement [2] and Khare et al. [11].

Keywords: Auxiliary variable, Calibration estimation, Mean, Mean Squared Error, Stratified sampling.

1. Introduction

The main interest in sampling is to obtain estimators of the population parameters which should be precise. Additional information is used to acquire such estimators which is available in the form of auxiliary variables at the planning or at the design stage. The known auxiliary information is used in ratio and regression method of estimation to enhance the precision of estimators.

The calibration approach used to develop estimators of various population parameters like population mean, total, proportion, etc. has gained popularity using the available auxiliary information since the early 1990's. The calibration technique introduced by Deville and Sarndal [3] has turned into a relevant topic for adjusting weights in sample survey. They opted the calibration weights which minimize the given distance functions subject to calibration constraints associated with available auxiliary variables. A realistic fascination about this calibration estimation is to attain it computationally. Thus, there are many situations where the procedure of the calibration method was considered as an effective way to ameliorate the precision of the various estimator of the population parameters.

Several researchers have penned down various estimators for the population parameters under different sampling techniques using the calibration method. When population under consideration is heterogenous then stratified sampling is applied, by dividing the population into homogeneous sub groups called as strata. Tracy et al. [16], Koyuncu and Kadilar ([8], [9]), Nidhi et al. ([12], [13]), Koyuncu [10], Garg and Pachori ([4], [5], [6], [7]), Ozgul [14], Audu et al. [1], etc., have recommended calibration estimators of the population mean for stratified random sampling scheme using one or more auxiliary variables with different calibrated weights. Clement [2] and Khare et al. [11] suggested a modified separate ratio estimator of the finite population mean using calibration approach for stratified random sampling.

Singh and Sedory [15] have suggested a two-step calibration estimator under probability proportional to size sampling scheme. In this paper, we present ratio-type calibration estimators by adjusting the proportionality of stratum mean of the auxiliary variable in case of stratified random sampling. We suggested two ratio-type calibration estimators using available information on the auxiliary variable. The biases and mean squared errors of the developed calibration estimators have been determined up to first order approximation. The simulation study has also been executed in R software on a real dataset for comparing the proposed estimators with other estimators.

2. Calibration Estimation in Sample Survey

The stratified random sampling technique is employed in case of heterogeneous population. In this sampling scheme we first divide the given population consisting of N units into C homogeneous strata of N_k units so that $\sum_{k=1}^{C} N_k = N$. Then sample of size n_k is selected from each stratum applying simple random sampling without replacement (SRSWOR) technique such that $\sum_{k=1}^{C} n_k = n$. Let X denotes the available auxiliary variable which is positively correlated with the study variable Y.

Clement [2] proposed a separate ratio calibration estimator in stratified sampling given as:

$$\bar{y}_{cl} = \sum_{k=1}^{C} \Delta_k \hat{R}_k \bar{x}_k \tag{1}$$

the calibrated weights were obtained subject to the following calibration constraint:

$$\sum_{k=1}^{C} \Delta_k \bar{x}_k = \bar{X} \tag{2}$$

Then calibration separate ratio estimator given by Clement [2] becomes

$$\bar{y}_{cl} = \sum_{k=1}^{C} W_k \hat{R}_k \bar{x}_k + \frac{\sum_{k=1}^{C} W_k q_k \hat{R}_k \bar{x}_k^2}{\sum_{k=1}^{C} W_k q_k \bar{x}_k^2} (\bar{X} - \sum_{k=1}^{C} W_k \bar{x}_k) \tag{3}$$

where, q_k is a constant.

Khare et al. [11] proposed separate ratio calibration estimator for stratified sampling as:

$$\bar{y}_{kh} = \sum_{k=1}^{C} \Delta_k \frac{\bar{y}_k}{\bar{x}_k} \bar{X}_k \tag{4}$$

The calibrated weights were obtained subject to the calibration constraints

$$\sum_{k=1}^{C} \Delta_k \bar{x}_k = \sum_{k=1}^{C} W_k \bar{X}_k \tag{5}$$

and

$$\sum_{k=1}^{C} W_k = 1 \tag{6}$$

Then calibration separate ratio estimator given by Khare et al. [11] becomes

$$\bar{y}_{kh} = \sum_{k=1}^{C} W_k \frac{\bar{y}_k}{\bar{x}_k} \bar{X}_k + \hat{\beta}\left(\bar{X} - \sum_{k=1}^{C} W_k \bar{x}_k \right) \tag{7}$$

where $$\hat{\beta} = \frac{\sum_{k=1}^{C} q_k W_k \sum_{k=1}^{C} q_k W_k \bar{y}_k \bar{X}_k - \sum_{k=1}^{C} q_k W_k \frac{\bar{y}_k}{\bar{x}_k} \bar{X}_k \sum_{k=1}^{C} q_k W_k \bar{x}_k}{\sum_{k=1}^{C} q_k W_k \sum_{k=1}^{C} q_k W_k \bar{x}_k^2 - \left(\sum_{k=1}^{C} q_k W_k \bar{x}_k \right)^2} \tag{8}$$

3. Proposed Calibration Estimator in Stratified Sampling

We propose two ratio-type calibration estimators by adjusting proportionality of calibrated mean to design mean of the auxiliary variable and obtain the calibration weights by minimizing the chi-square type distance measure satisfying calibration constraints using Lagrange's method.

The suggested ratio-type calibration estimator of finite population mean under stratified random sampling is stated as:

$$\bar{y}_p = \sum_{k=1}^{C} \Psi_k \bar{y}_{Rk} \tag{9}$$

where $\bar{y}_{Rk} = \frac{\bar{y}_k}{\bar{x}_k} \bar{X}_k$ is the k^th^ stratum ratio estimator.

We consider two different sets of constraints to obtain the calibration weights given as follows:

3.1 First Approach

In this approach, we assume that the sum of the calibrated weights is equal to the sum of the

design weights.

$$\sum_{k=1}^{C} \Psi_k = \sum_{k=1}^{C} W_k \tag{10}$$

where $W_k = \frac{N_k}{N}$ is the design weight of k^th^ stratum.

Assuming known population mean of the auxiliary variable X, we use another calibration constraint in such a way that the product of calibrated weight and sample mean of auxiliary variable in the k^th^ stratum is proportional to the product of design weight and population mean of that auxiliary variable in the k^th^ stratum, i.e.,

$$\Psi_k \bar{x}_k \propto W_k \bar{X}_k \tag{11}$$

this implies that

$$\Psi_k \bar{x}_k = \xi W_k \bar{X}_k$$

where λ is the constant of proportionality.

After taking summation on both sides, it reduces to

$$\sum_{k=1}^{C} \Psi_k \bar{x}_k = \xi \bar{X} \tag{12}$$

The calibrated weights Ψ_k are obtained by minimizing the following chi-square type distance function:

$$\sum_{k=1}^{C} \frac{(\Psi_k - W_k)^2}{W_k q_k} \tag{13}$$

Using the method of Lagrange's multiplier, we optimize the following function to obtain the calibration estimator:

$$\Phi_1 = \frac{1}{2}\sum_{k=1}^{C} \frac{(\Psi_k - W_k)^2}{W_k q_k} - \lambda_1 \left(\sum_{k=1}^{C} \Psi_k - \sum_{k=1}^{C} W_k \right) - \lambda_2 \left(\sum_{k=1}^{C} \Psi_k \bar{x}_k - \xi \bar{X} \right) \tag{14}$$

After differentiating Φ_1 with respect to Ψ_k and equating to zero, we obtain the calibrated weights given as:

$$\Psi_k = W_k + \lambda_1 W_k q_k + \lambda_2 W_k q_k \bar{x}_k \tag{15}$$

the values of Lagrange's Multipliers λ_1 and λ_2 are determined as:

$$\lambda_1 = \frac{\left(\sum_{k=1}^{C} W_k \bar{x}_k - \xi \bar{X}\right) \sum_{k=1}^{C} W_k q_k \bar{x}_k}{\sum_{k=1}^{C} W_k q_k . \sum_{k=1}^{C} W_k q_k \bar{x}_k^2 - \left(\sum_{k=1}^{C} W_k q_k \bar{x}_k\right)^2} \tag{16}$$

$$\lambda_2 = \frac{\left(\xi \bar{X} - \sum_{k=1}^{C} W_k \bar{x}_k\right) \sum_{k=1}^{C} W_k q_k}{\sum_{k=1}^{C} W_k q_k . \sum_{k=1}^{C} W_k q_k \bar{x}_k^2 - \left(\sum_{k=1}^{C} W_k q_k \bar{x}_k\right)^2} \tag{17}$$

By substituting the calibrated weights in equation (9), we obtain the proposed estimator as:

$$\bar{y}_{p'} = \sum_{k=1}^{C} W_k \bar{y}_{Rk} + \hat{\beta}_{p'} \left(\xi \bar{X} - \sum_{k=1}^{C} W_k \bar{x}_k\right) \tag{18}$$

i.e., $\bar{y}_{p'} = \bar{y}_{Rst} + \hat{\beta}_{p'} (\xi \bar{X} - \bar{x}_{st})$

where $\hat{\beta}_{p'} = \dfrac{\sum_{k=1}^{C} W_k q_k \sum_{k=1}^{C} W_k q_k \bar{x}_k \bar{y}_{Rk} - \sum_{k=1}^{C} W_k q_k \bar{x}_k \sum_{k=1}^{C} W_k q_k \bar{y}_{Rk}}{\sum_{k=1}^{C} W_k q_k \sum_{k=1}^{C} W_k q_k \bar{x}_k^2 - \left(\sum_{k=1}^{C} W_k q_k \bar{x}_k\right)^2}$

q_k is the constant, we can choose different values of it to acquire the different forms of the suggested estimator.

3.2 Second Approach

Similarly, the first calibration constraint is defined based on the sums of the calibrated weights and design weights as:

$$\sum_{k=1}^{C} \Psi_k^* = \sum_{k=1}^{C} W_k \tag{19}$$

Here we consider the product of calibrated weight and log of sample mean of auxiliary variable proportional to the product of design weight and log of population mean of that auxiliary variable in the kth stratum which is given as:

$$\Psi_k^* \log \bar{x}_k \propto W_k \log \bar{X}_k \tag{20}$$

i.e., $\Psi_k^* \log \bar{x}_k = \tau W_k \log \bar{X}_k$

Taking summation on both sides, it will give the following calibration constraint:

$$\sum_{k=1}^{C} \Psi_k^* \log \bar{x}_k = \tau \sum_{k=1}^{C} W_k \log \bar{X}_k \tag{21}$$

where τ is the constant of proportionality.

The weights Ψ_k^* are obtained by minimizing the following chi-square type distance function:

$$\sum_{k=1}^{C} \frac{(\Psi_k^* - W_k)^2}{W_k q_k} \tag{22}$$

For obtaining the calibration estimator, we define the Lagrange's function given as:

$$\Phi_2 = \frac{1}{2} \sum_{k=1}^{C} \frac{(\Psi_k^* - W_k)^2}{W_k q_k} - \eta_1 \left(\sum_{k=1}^{C} \Psi_k^* - \sum_{k=1}^{C} W_k \right) - \eta_2 \left(\sum_{k=1}^{C} \Psi_k^* \log \bar{x}_k - \tau \sum_{k=1}^{C} W_k \log \bar{X}_k \right) \tag{23}$$

The calibration weights can be obtained after differentiating equation (23) with respect

to Ψ_k^* and equating to zero, which are given as:

$$\Psi_k^* = W_k + \eta_1 W_k q_k + \eta_2 W_k q_k \bar{x}_k \tag{24}$$

where

$$\eta_1 = \frac{\left(\sum_{k=1}^{C} W_k \log \bar{x}_k - \tau \sum_{k=1}^{C} W_k \log \bar{X}_k \right) \sum_{k=1}^{C} W_k q_k \log \bar{x}_k}{\sum_{k=1}^{C} W_k q_k . \sum_{k=1}^{C} W_k q_k (\log \bar{x}_k)^2 - \left(\sum_{k=1}^{C} W_k q_k \log \bar{x}_k \right)^2} \tag{25}$$

$$\eta_2 = \frac{\left(\tau \sum_{k=1}^{C} W_k \log \bar{X}_k - \sum_{k=1}^{C} W_k \log \bar{x}_k \right) \sum_{k=1}^{C} W_k q_k}{\sum_{k=1}^{C} W_k q_k . \sum_{k=1}^{C} W_k q_k (\log \bar{x}_k)^2 - \left(\sum_{k=1}^{C} W_k q_k \log \bar{x}_k \right)^2} \tag{26}$$

After substituting the values from the equations (24), (25) and (26) in equation (9), the proposed calibration estimator will become

$$\bar{y}_{p''} = \bar{y}_{Rst} + \hat{\beta}_{p''} \left(\tau \sum_{k=1}^{C} W_k \log \bar{X}_k - \sum_{k=1}^{C} W_k \log \bar{x}_k \right) \tag{27}$$

where

$$\bar{y}_{Rst} = \sum_{k=1}^{C} W_k \bar{y}_{Rk} \text{ and } \hat{\beta}_{p''} = \frac{\sum_{k=1}^{C} W_k q_k \sum_{k=1}^{C} W_k q_k \bar{y}_{Rk} \log \bar{x}_k - \left(\sum_{k=1}^{C} W_k q_k \log \bar{x}_k \sum_{k=1}^{C} W_k q_k \bar{y}_{Rk} \right)}{\sum_{k=1}^{C} W_k q_k \sum_{k=1}^{C} W_k q_k (\log \bar{x}_k)^2 - \left(\sum_{k=1}^{C} W_k q_k \log \bar{x}_k \right)^2}$$

4. Properties of the Proposed Estimators

The properties of the suggested estimators are derived under both cases considered in section 3 one by one.

4.1 Bias and Mean Squared Error of $\bar{y}_{p'}$

Let us first consider the calibration estimator $\bar{y}_{p'}$ obtained in equation (18) to study the properties of it.

Let us define

$$\bar{y}_{Rst} = \bar{Y}(1+\in_0),\ \bar{x}_{st} = \bar{X}(1+\in_1),\ \hat{\beta}_{p'} = \beta_{p'}(1+\in_2) \text{ and } E(\in_0) = E(\in_1) = E(\in_2) = 0 \tag{28}$$

After substituting these values in equation (18), we obtain the proposed estimator as:

$$\bar{y}_{p'} = \bar{Y}(1+\in_0) + \beta_{p'}(1+\in_2)[\xi \bar{X} - \bar{X}(1+\in_1)] \tag{29}$$

The bias of the developed estimator is

$$Bias(\bar{y}_{p'}) = E(\bar{y}_{p'} - \bar{Y}) = E[(\bar{Y}(1+\in_0) + \beta_{p'}(1+\in_2)(\xi \bar{X} - \bar{X}(1+\in_1)) - \bar{Y}] \tag{30}$$

By taking the expectation, we have

$$Bias(\bar{y}_{p'}) = \beta_{p'}(\xi - 1)\bar{X} - \text{cov}(\bar{x}_{st}, \hat{\beta}_{p'}) \tag{31}$$

The mean squared error of the proposed estimator up to first order of approximation is

$$MSE(\bar{y}_{p'}) = E(\bar{y}_{p'} - \bar{Y})^2 = E[\bar{Y} \in_0 + (\beta_{p'} + \beta_{p'} \in_2)(\xi \bar{X} - \bar{X} - \bar{X} \in_1)]^2 \tag{32}$$

By taking the expectation and neglecting the higher order terms, we have

$$\begin{aligned} MSE(\bar{y}_{p'}) = V(\bar{y}_{Rst}) + \beta_{p'}^2 V(\bar{x}_{st}) + (\bar{X})^2(\xi - 1)^2(\beta_{p'}^2 + V(\hat{\beta}_{p'})) - 2\beta_{p'} \text{cov}(\bar{y}_{Rst}, \bar{x}_{st}) \\ + 2(\xi - 1)\bar{X}(\text{cov}(\bar{y}_{Rst}, \hat{\beta}_{p'}) - 2\beta_{p'} \text{cov}(\bar{x}_{st}, \hat{\beta}_{p'})) \end{aligned} \tag{33}$$

The constant of proportionality ξ is obtained by equating the bias of the estimator given in equation (31) equal to zero which is given as:

$$\xi = 1 + \frac{\operatorname{cov}(\bar{x}_{st}, \hat{\beta}_{p'})}{\beta_{p'} \bar{X}} \tag{34}$$

We can also obtain the constant of proportionality λ by minimizing the mean squared error (MSE) of the recommended estimator given in equation (33) given as follows:

$$\xi = 1 + \frac{2\beta_{p'} \operatorname{cov}(\bar{x}_{st}, \hat{\beta}_{p'}) - \operatorname{cov}(\bar{y}_{Rst}, \hat{\beta}_{p'})}{\bar{X}(\beta_{p'}^2 + V(\hat{\beta}_{p'}))} \tag{35}$$

4.2 Bias and Mean Squared Error of $\bar{y}_{p''}$

Here we obtain the properties of the proposed estimator $\bar{y}_{p''}$ for which we define

$$\bar{y}_{Rst} = \bar{Y}(1+\in_0),\ \bar{x}_{lst} = \bar{X}_l(1+\in_1),\ \hat{\beta}_{p''} = \beta_{p''}(1+\in_2) \text{ and } E(\in_0) = E(\in_1) = E(\in_2) = 0 \tag{36}$$

where $\bar{X}_l = \sum_{k=1}^{C} W_k \log \bar{X}_k$ *and* $\bar{x}_{lst} = \sum_{k=1}^{C} W_k \log \bar{x}_k$

Using these notations in the proposed estimator, we get

$$\bar{y}_{p''} = \bar{Y}(1+\in_0) + \beta_{p''}(1+\in_2)(\tau \bar{X}_l - \bar{X}_l(1+\in_1)) \tag{37}$$

The bias of the suggested estimator $\bar{y}_{p''}$ is obtained as:

$$B(\bar{y}_{p''}) = \beta_{p''}(\tau - 1)\bar{X}_l - \operatorname{cov}(\bar{x}_{lst}, \hat{\beta}_{p''}) \tag{38}$$

The MSE up to first order of approximation can be derived for the proposed estimator as:

$$\begin{aligned} MSE(\bar{y}_{p''}) = {} & V(\bar{y}_{Rst}) + \beta_{p''}^2 V(\bar{x}_{lst}) + (\bar{X}_l)^2 (\tau - 1)^2 (\beta_{p''}^2 + V(\hat{\beta}_{p''})) - 2\beta_{p''} \operatorname{cov}(\bar{y}_{Rst}, \bar{x}_{lst}) \\ & + 2(\tau - 1)\bar{X}_l((\operatorname{cov}(\bar{y}_{Rst}, \hat{\beta}_{p''}) - 2\beta_{p''} \operatorname{cov}(\bar{x}_{lst}, \hat{\beta}_{p''})) \end{aligned} \tag{39}$$

Taking the bias of the estimator obtained in equation (38) equal to zero, the constant of proportionality c is obtained as:

$$\tau = 1 + \frac{\operatorname{cov}(\bar{x}_{lst}, \hat{\beta}_{p''})}{\beta_{p''} \bar{X}_l} \tag{40}$$

By minimizing the MSE of the proposed estimator, the constant of proportionality can also be determined as:

$$\tau = 1 + \frac{2\beta_{p''} \operatorname{cov}(\bar{x}_{lst}, \hat{\beta}_{p''}) - \operatorname{cov}(\bar{y}_{Rst}, \hat{\beta}_{p''})}{\bar{X}_l(\beta_{p''}^2 + V(\hat{\beta}_{p''}))} \tag{41}$$

5. Simulation Study

In the simulation study we compare the performance of our developed estimators with other estimators given by Clement [2] and Khare et al. [11]. We have used real data set from MU284 population given in Sarndal et al. (2003). Here the population has 284 units, which is divided into 8 strata of different sizes. The study variable Y considered here is RMT85 and the auxiliary variable X is ME84. We have generated 25000 random samples of sizes 35, 40, 45 and 50 with the help of R software from each stratum using proportional allocation under SRSWOR. The percentage relative root mean squared error (%RRMSE) and percentage relative efficiency (%RE) are calculated to compare the proposed estimators using the following formulae:

$$\%RRMSE(\bar{y}_\delta) = \sqrt{\frac{1}{25000}\sum_{i=1}^{25000}\left(\frac{\bar{y}_{i\delta} - \bar{Y}}{\bar{Y}}\right)^2} X100; \quad \delta = cl, kh, p', p'' \tag{42}$$

$$\%RE(\bar{y}_\delta) = \frac{\bar{y}_{cl}}{\bar{y}_\delta} \times 100; \quad \delta = kh, p', p'' \tag{43}$$

We have obtained %RRMSE and %RE are given in Tables 1and 2, respectively, for the proposed calibration estimator as well as estimators given by Clement [2] and Khare et al. [11]. Figure 1 shows the percentage of RRMSE of the Estimators.

Table 1. Percentage Relative Root Mean Squared Error (%RRMSE) of the Estimators

Sample Size (n)	%RRMSE $(\bar{y}_{cl})$	%RRMSE $(\bar{y}_{kh})$	%RRMSE $(\bar{y}_{p'})$	%RRMSE $(\bar{y}_{p''})$
35	30.0717	18.1759	17.0028	10.8577
40	28.3030	17.2548	16.0906	10.3836
45	27.7657	17.4242	16.2301	10.5918
50	26.8765	17.5643	16.3389	10.7467

Table 2. Percentage Relative Efficiency (%RE) of the Estimators

Sample Size (n)	%RRMSE $(\bar{y}_{cl})$	%RRMSE $(\bar{y}_{kh})$	%RRMSE $(\bar{y}_{p'})$	%RRMSE $(\bar{y}_{p''})$
35	100.00	165.45	176.86	276.96
40	100.00	164.03	175.90	272.57
45	100.00	159.35	171.08	262.14
50	100.00	153.02	164.49	250.09

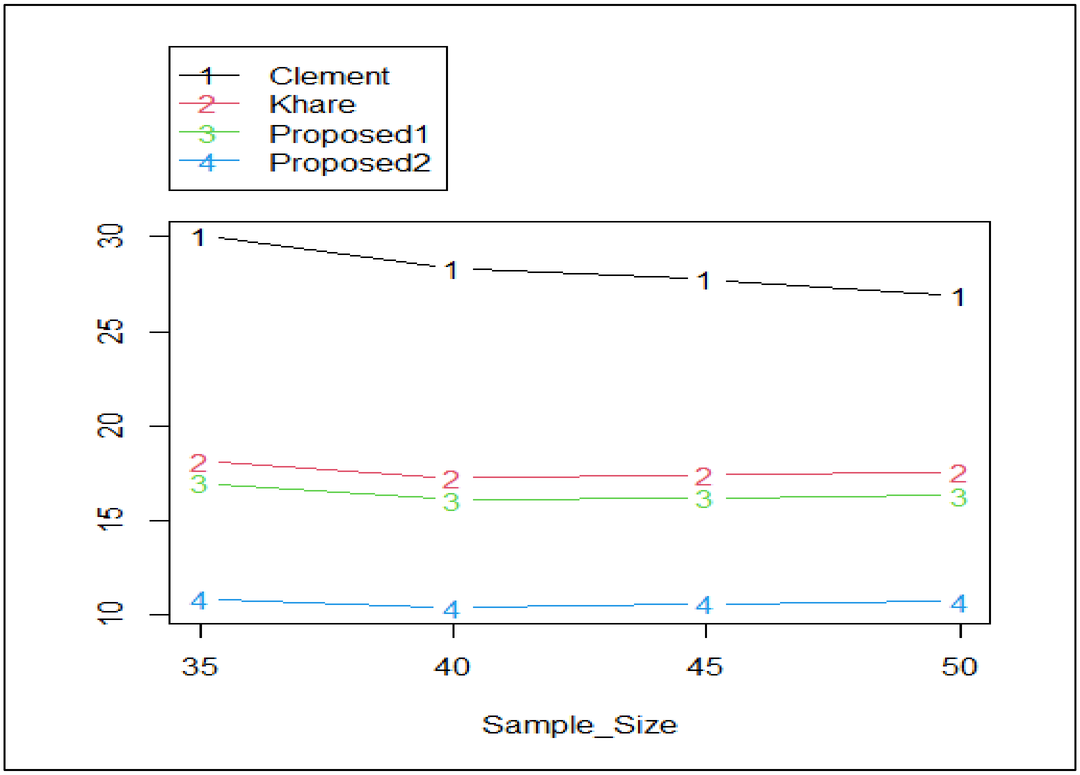

Figure 1. %RRMSE of the Estimators

The summary of %RRMSE of the suggested estimators $\bar{y}_{p'}$ *and* $\bar{y}_{p''}$ for the different ranges of the proportionality constant values (ξ in case of $\bar{y}_{p'}$ and τ in case of $\bar{y}_{p''}$) for the different sample sizes n are given in Tables 3 and 4 respectively. By simulation study we have observed that the suggested estimators are performing well for the proportionality constant lying between 0.951 to 0.999.

Table 3. Summary of %RRMSE of $\bar{y}_{p'}$

A. ‘k’ ranges from ‘0.951 - 0.959’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	16.08107	15.17969	15.29583	15.38151
Standard Error	0.04112	0.04042	0.04145	0.04241
Median	16.08082	15.17938	15.29552	15.38116
Standard Deviation	0.12337	0.12126	0.12436	0.12722
Range	0.36039	0.35422	0.36326	0.37162
Minimum	15.90123	15.00301	15.11465	15.19619
Maximum	16.26162	15.35723	15.47791	15.56781
Confidence Level (95.0%)	0.09483	0.09321	0.09559	0.09779

B. ‘k’ ranges from ‘0.961 - 0.969’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	16.53524	15.62696	15.75456	15.85109
Standard Error	0.04179	0.04123	0.04229	0.04332
Median	16.53501	15.62668	15.75426	15.85077
Standard Deviation	0.12536	0.12369	0.12686	0.12995
Range	0.36620	0.36133	0.37059	0.37959
Minimum	16.35247	15.44670	15.56967	15.66174
Maximum	16.71866	15.80802	15.94026	16.04133
Confidence Level (95.0%)	0.09636	0.09508	0.09752	0.09989
C. ‘k’ ranges from ‘0.971 - 0.979’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	16.99636	16.08275	16.22204	16.33020
Standard Error	0.04240	0.04197	0.04305	0.04415
Median	16.99615	16.08249	16.22178	16.32991
Standard Deviation	0.12719	0.12592	0.12916	0.13244
Range	0.37153	0.36784	0.37730	0.38688
Minimum	16.81089	15.89919	16.03377	16.13716
Maximum	17.18243	16.26703	16.41107	16.52405
Confidence Level (95.0%)	0.09776	0.09679	0.09928	0.10180
D. ‘k’ ranges from ‘0.981 - 0.989’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	17.4639	16.54634	16.69756	16.81803
Standard Error	0.0430	0.04266	0.04376	0.04491
Median	17.4637	16.54610	16.69731	16.81776
Standard Deviation	0.1289	0.12797	0.13127	0.13473
Range	0.3765	0.37382	0.38345	0.39355
Minimum	17.2759	16.35976	16.50618	16.62162
Maximum	17.6524	16.73358	16.88963	17.01518
Confidence Level (95.0%)	0.0991	0.09837	0.10090	0.10356
E. ‘k’ ranges from ‘0.991 - 0.999’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	17.93732	17.01709	17.18044	17.31384
Standard Error	0.04347	0.04328	0.04440	0.04561
Median	17.93714	17.01688	17.18021	17.31360
Standard Deviation	0.13042	0.12985	0.13320	0.13682
Range	0.38099	0.37931	0.38909	0.39967
Minimum	17.74708	16.82774	16.98621	17.11435
Maximum	18.12807	17.20706	17.37530	17.51401
Confidence Level (95.0%)	0.10025	0.09981	0.10238	0.10517

Table 4. Summary of %RRMSE of $\bar{y}_{p''}$

A. ‘c’ ranges from ‘0.951 - 0.959’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	9.64717	9.99970	10.39146	11.03484
Standard Error	0.07397	0.12433	0.13958	0.16677
Median	9.61566	9.97032	10.36185	11.00680
Standard Deviation	0.22190	0.37298	0.41875	0.50030
Range	0.64122	1.08460	1.21839	1.45715
Minimum	9.37061	9.49851	9.82372	10.34554
Maximum	10.01183	10.58311	11.04211	11.80270
Confidence Level (95.0%)	0.17057	0.28670	0.32188	0.38457
B. ‘c’ ranges from ‘0.961 - 0.969’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	9.34514	9.13990	9.37552	9.70641
Standard Error	0.02218	0.03178	0.04387	0.07204
Median	9.31047	9.10141	9.33517	9.66517
Standard Deviation	0.06653	0.09534	0.13162	0.21613
Range	0.18976	0.26596	0.36754	0.61970
Minimum	9.28863	9.05645	9.24813	9.45420
Maximum	9.47839	9.32241	9.61566	10.07390
Confidence Level (95.0%)	0.05114	0.07328	0.10117	0.16614
C. ‘c’ ranges from ‘0.971 - 0.979’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	10.04924	9.39345	9.52713	9.57688
Standard Error	0.10687	0.07537	0.07002	0.04973
Median	10.02137	9.35799	9.48868	9.53394
Standard Deviation	0.32062	0.22612	0.21005	0.14918
Range	0.93188	0.65193	0.60310	0.41872
Minimum	9.62229	9.11705	9.27932	9.42751
Maximum	10.55417	9.76898	9.88242	9.84623
Confidence Level (95.0%)	0.24645	0.17381	0.16146	0.11467
D. ‘c’ ranges from ‘0.981 - 0.989’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	11.58063	10.68319	10.79840	10.68878
Standard Error	0.16945	0.15670	0.15866	0.15072
Median	11.56244	10.65911	10.77202	10.65792
Standard Deviation	0.50836	0.47010	0.47599	0.45217
Range	1.48272	1.36975	1.38648	1.31580
Minimum	10.86476	10.03204	10.14211	10.07410
Maximum	12.34748	11.40179	11.52859	11.38989
Confidence Level (95.0%)	0.39076	0.36135	0.36588	0.34757
E. ‘c’ ranges from ‘0.991 - 0.999’				
	n=35	**n=40**	**n=45**	**n=50**
Mean	13.66650	12.70167	12.86663	12.72658
Standard Error	0.20877	0.20831	0.21507	0.21688
Median	13.65545	12.68737	12.85107	12.70833
Standard Deviation	0.62632	0.62494	0.64522	0.65064

Range	1.82841	1.82386	1.88291	1.89827
Minimum	12.76779	11.80979	11.94701	11.80304
Maximum	14.59620	13.63366	13.82992	13.70131
Confidence Level (95.0%)	0.48143	0.48037	0.49596	0.50013

We observe that our proposed estimators have lesser %RRMSE values and greater %RE values than the other estimators given by Clement [2] and Khare et al. [11], so we can state that our proposed estimators are more precise and efficient than these existing estimators.

6. Conclusion

In this study, two calibration estimators for finite population mean were developed under stratified random sampling. We have acquired the expressions for bias and MSE of the recommended estimators as well as attained the optimal values of proportionality constant for both estimators. A simulation study has also been presented in R SOFTWARE to compare the developed estimators with other estimators given by Clement [2] and Khare et al. [11], in terms of percentage RRMSE and percentage RE. The results show that the proposed calibration estimators are more efficient estimators of the finite population mean under stratified random sampling scheme.

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