
Supplementary Materials

#####pdf plots#####

```
x=seq(0,15,0.0008)
```

```
omega=1.2
```

```
psi=1.5
```

```
alpha=0.5
```

```
rho=1.3
```

```
sigma=1.4
```

```
f=2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1
```

```
lower.X=0 ;upper.X=8 ; lower.Y=0 ; upper.Y=0.47
```

```
windows()
```

```
par(new=TRUE)
```

```
plot(x,f,lty=1,type="l", lwd = 2.5,xlim =c(lower.X, upper.X), ylim = c(lower.Y, upper.Y),xlab="x",ylab="pdf",col="l")
```

```
x=seq(0,15,0.0008)
```

```
omega=0.5
```

```
psi=4.5
```

```
alpha=2.2
```

```
rho=1.4
```

```
sigma=1.5
```

```
f=2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1
```

```
lines(x,f,lty=3,lwd = 2.5, type="l",col="2")
```

```
x=seq(0,15,0.0008)
```

```
omega=0.8
```

```
psi=0.5
```

```
alpha=1.2
```

```
rho=4.2
```

```
sigma=1.5
```

```
f=2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1
```

```
lines(x,f,lty=3,lwd = 2.5, type="l",col="3")
```

```
x=seq(0,15,0.0008)
```

```
omega=0.9
```

```
psi=2.5
```

```
alpha=2.2
```

```
rho=3.8
```

```
sigma=2.5
```

```

f=2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1)
lines(x,f,lty=4,lwd = 2.5, type="l",col="4")
x=seq(0,15,0.0008)
omega=0.7
psi=2.8
alpha=1.2
rho=4.2
sigma=0.2
f=2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1)
lines(x,f,lty=5,lwd = 2.5, type="l",col="5")
x=seq(0,15,0.0008)
omega=0.9
psi=2.9
alpha=1.3
rho=2.2
sigma=1.5
f=2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1)
lines(x,f,lty=6,lwd = 2.5, type="l",col="6")
legend("topright"),legend=c(expression (omega == 1.2 ~ psi == 1.5 ~ alpha == 0.5 ~ rho == 1.3 ~ sigma ==
1.4),expression (omega == 0.5 ~ psi == 4.5 ~ alpha == 2.2 ~ rho == 1.4 ~ sigma == 1.5),expression (omega
== 0.8 ~ psi == 0.5 ~ alpha == 1.2 ~ rho == 4.2 ~ sigma == 1.5),expression (omega == 0.9 ~ psi == 2.5 ~
alpha == 2.2 ~ rho == 3.8 ~ sigma == 2.5),expression (omega == 0.7 ~ psi == 2.8 ~ alpha == 1.2 ~ rho ==
4.2 ~ sigma == 0.2),expression (omega == 0.9 ~ psi == 2.9 ~ alpha == 1.3 ~ rho == 2.2 ~ sigma == 1.5)),cex
= 1,lty=c(1,2,3,4,5,6),col=( "1":"2":"3":"4":"5":"6"), lwd = 2)

```

#####Hazard Function Plot#####

```

x=seq(0,15,0.0008)
omega=1.2
psi=1.5
alpha=0.5
rho=1.3
sigma=1.4
f=(2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1)
(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi))))
lower.X=0 ; upper.X=10 ; lower.Y=0 ; upper.Y=1
windows()
par(new=TRUE)

```

```

plot(x,f,lty=1,type="l", lwd = 2.5,xlim =c(lower.X, upper.X), ylim = c(lower.Y, upper.Y),xlab="x",ylab="Hazard
function",col="l")
x=seq(0,15,0.0008)
omega=0.5
psi=4.5
alpha=2.2
rho=1.4
sigma=1.5
f=(2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1))/(1-(1-(1-
(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi))
lines(x,f,lty=3,lwd = 2.5, type="l",col="2")
x=seq(0,15,0.0008)
omega=0.8
psi=0.5
alpha=1.2
rho=4.2
sigma=1.5
f=(2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1))/(1-(1-(1-
(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi))
lines(x,f,lty=3,lwd = 2.5, type="l",col="3")
x=seq(0,15,0.0008)
omega=0.9
psi=2.5
alpha=2.2
rho=3.8
sigma=2.5
f=(2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1))/(1-(1-(1-
(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi))
lines(x,f,lty=4,lwd = 2.5, type="l",col="4")
x=seq(0,15,0.0008)
omega=0.7
psi=2.8
alpha=1.2
rho=4.2
sigma=0.2
f=(2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega)**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1))/(1-(1-(1-
(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi))

```

```

lines(x,f,lty=5,lwd = 2.5, type="l",col="5")
x=seq(0,15,0.0008)
omega=0.9
psi=2.9
alpha=1.3
rho=2.2
sigma=1.5
f=(2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho-1)*(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi-1))/(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi))))))
lines(x,f,lty=6,lwd = 2.5, type="l",col="6")
legend("topright"),legend=c(expression (omega == 1.2 ~ psi == 1.5 ~ alpha == 0.5 ~ rho == 1.3 ~ sigma == 1.4),expression (omega == 0.5 ~ psi == 4.5 ~ alpha == 2.2 ~ rho == 1.4 ~ sigma == 1.5),expression (omega == 0.8 ~ psi == 0.5 ~ alpha == 1.2 ~ rho == 4.2 ~ sigma == 1.5),expression (omega == 0.9 ~ psi == 2.5 ~ alpha == 2.2 ~ rho == 3.8 ~ sigma == 2.5),expression (omega == 0.7 ~ psi == 2.8 ~ alpha == 1.2 ~ rho == 4.2 ~ sigma == 0.2),expression (omega == 0.9 ~ psi == 2.9 ~ alpha == 1.3 ~ rho == 2.2 ~ sigma == 1.5)),cex = 1,lty=c(1,2,3,4,5,6),col=("1":"2":"3":"4":"5":"6"), lwd = 2)

```

#####Survival Function Plot#####

```

x=seq(0,15,0.0008)
omega=1.2
psi=1.5
alpha=0.5
rho=1.3
sigma=1.4
f=1-(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi)
lower.X=0 ;upper.X=10 ; lower.Y=0 ; upper.Y=1
windows()
par(new=TRUE)
plot(x,f,lty=1,type="l", lwd = 2.5,xlim =c(lower.X, upper.X), ylim = c(lower.Y, upper.Y),xlab="x",ylab="Survival function",col="1")
x=seq(0,15,0.0008)
omega=0.5
psi=4.5
alpha=2.2
rho=1.4
sigma=1.5
f=1-(1-(1-(1-(1-exp(-sigma*x**(-alpha)))**((omega)**(rho))**2))**((psi)
lines(x,f,lty=3,lwd = 2.5, type="l",col="2")
x=seq(0,15,0.0008)
omega=0.8
psi=0.5
alpha=1.2
rho=4.2

```

```

sigma=1.5
f=1-(1-(1-(1-(1-exp(-sigma*x**(-alpha))))*(omega))**(rho))**(2))*(psi)
lines(x,f,lty=3,lwd = 2.5, type="l",col="3")
x=seq(0,15,0.0008)
omega=0.9
psi=2.5
alpha=2.2
rho=3.8
sigma=2.5
f=1-(1-(1-(1-(1-exp(-sigma*x**(-alpha))))*(omega))**(rho))**(2))*(psi)
lines(x,f,lty=4,lwd = 2.5, type="l",col="4")
x=seq(0,15,0.0008)
omega=0.7
psi=2.8
alpha=1.2
rho=4.2
sigma=0.2
f=1-(1-(1-(1-(1-exp(-sigma*x**(-alpha))))*(omega))**(rho))**(2))*(psi)
lines(x,f,lty=5,lwd = 2.5, type="l",col="5")
x=seq(0,15,0.0008)
omega=0.9
psi=2.9
alpha=1.3
rho=2.2
sigma=1.5
f=1-(1-(1-(1-(1-exp(-sigma*x**(-alpha))))*(omega))**(rho))**(2))*(psi)
lines(x,f,lty=6,lwd = 2.5, type="l",col="6")
legend("topright"),legend=c(expression (omega == 1.2 ~ psi == 1.5 ~ alpha == 0.5 ~ rho == 1.3 ~ sigma ==
1.4),expression (omega == 0.5 ~ psi == 4.5 ~ alpha == 2.2 ~ rho == 1.4 ~ sigma == 1.5),expression (omega
== 0.8 ~ psi == 0.5 ~ alpha == 1.2 ~ rho == 4.2 ~ sigma == 1.5),expression (omega == 0.9 ~ psi == 2.5 ~
alpha == 2.2 ~ rho == 3.8 ~ sigma == 2.5),expression (omega == 0.7 ~ psi == 2.8 ~ alpha == 1.2 ~ rho ==
4.2 ~ sigma == 0.2),expression (omega == 0.9 ~ psi == 2.9 ~ alpha == 1.3 ~ rho == 2.2 ~ sigma == 1.5)),cex
= 1,lty=c(1,2,3,4,5,6),col=("1":"2":"3":"4":"5":"6"), lwd = 2)

#####Cumulative Distribution Function#####
x=seq(0,15,0.0008)
omega=1.2
psi=1.5
alpha=0.5
rho=1.3
sigma=1.4
f=(1-(1-(1-(1-exp(-sigma*x**(-alpha))))*(omega))**(rho))**(2))*(psi)
lower.X=0 ;upper.X=8 ; lower.Y=0 ; upper.Y=1
windows()
par(new=TRUE)

```

```

plot(x,f,lty=1,type="l", lwd = 2.5,xlim =c(lower.X, upper.X), ylim = c(lower.Y,
upper.Y),xlab="x",ylab="cdf",col="l")
x=seq(0,15,0.0008)
omega=0.5
psi=4.5
alpha=2.2
rho=1.4
sigma=1.5
f=(1-(1-(1-(1-exp(-sigma*x**(-alpha))))**(omega))**(rho))**(2))**(psi)
lines(x,f,lty=3,lwd = 2.5, type="l",col="2")
x=seq(0,15,0.0008)
omega=0.8
psi=0.5
alpha=1.2
rho=4.2
sigma=1.5
f=(1-(1-(1-(1-exp(-sigma*x**(-alpha))))**(omega))**(rho))**(2))**(psi)
lines(x,f,lty=3,lwd = 2.5, type="l",col="3")
x=seq(0,15,0.0008)
omega=0.9
psi=2.5
alpha=2.2
rho=3.8
sigma=2.5
f=(1-(1-(1-(1-exp(-sigma*x**(-alpha))))**(omega))**(rho))**(2))**(psi)
lines(x,f,lty=4,lwd = 2.5, type="l",col="4")
x=seq(0,15,0.0008)
omega=0.7
psi=2.8
alpha=1.2
rho=4.2
sigma=0.2
f=(1-(1-(1-(1-exp(-sigma*x**(-alpha))))**(omega))**(rho))**(2))**(psi)
lines(x,f,lty=5,lwd = 2.5, type="l",col="5")
x=seq(0,15,0.0008)
omega=0.9
psi=2.9
alpha=1.3
rho=2.2
sigma=1.5
f=(1-(1-(1-(1-exp(-sigma*x**(-alpha))))**(omega))**(rho))**(2))**(psi)
lines(x,f,lty=6,lwd = 2.5, type="l",col="6")
legend("topright"),legend=c(expression (omega == 1.2 ~ psi == 1.5 ~ alpha == 0.5 ~ rho == 1.3 ~ sigma ==
1.4),expression (omega == 0.5 ~ psi == 4.5 ~ alpha == 2.2 ~ rho == 1.4 ~ sigma == 1.5),expression (omega
== 0.8 ~ psi == 0.5 ~ alpha == 1.2 ~ rho == 4.2 ~ sigma == 1.5),expression (omega == 0.9 ~ psi == 2.5 ~

```

```
alpha == 2.2 ~ rho == 3.8 ~ sigma == 2.5),expression (omega == 0.7 ~ psi == 2.8 ~ alpha == 1.2 ~ rho ==
4.2 ~ sigma == 0.2),expression (omega == 0.9 ~ psi == 2.9 ~ alpha == 1.3 ~ rho == 2.2 ~ sigma == 1.5)),cex
= 1,lty=c(1,2,3,4,5,6),col=( "1":"2":"3":"4":"5":"6"), lwd = 2)
```

#####Estimation of parameters for dist data set 1

#####MLE for EGFr distribution

```
dat=c(0.85,0.39,1.25,1.08,1.47,11.57,1.61,1.69,1.61,1.80,1.87,1.84,2.03,1.89,2.03,2.05,2.35,2.41,2.12,2.43,2.50,
2.48,2.55,2.53,2.55,2.56,2.67,2.73,2.59,2.75,1.81,2.79,2.85,2.82,2.87,2.88,2.95,2.96,2.93,2.97,3.11,3.09,3.15,3.1
1,3.15,3.19,3.22,3.27,3.22,3.28,3.31,3.31,3.39,3.33,3.39,3.56,3.65,3.68,3.60,3.70,4.20,3.75,4.42,4.38,4.70,4.90)
pdf_egfr<- function(x, omega,psi,alpha,rho,sigma){
```

```
  2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))*(omega-1)*(1-(1-exp(-sigma*x**(-alpha)))*(omega))**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))*(omega))**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))*(omega))**(rho))**(2))**(psi-1)
}
```

```
L_egfr<- function(par){
```

```
  omega=par[1]
```

```
  psi=par[2]
```

```
  alpha=par[3]
```

```
  rho=par[4]
```

```
  sigma=par[5]
```

```
  R=pdf_egfr(x=dat,omega,psi,alpha,rho,sigma)
```

```
  -sum(log(R))
```

```
}
```

```
length(dat)
```

```
summary(dat)
```

```
result_1 <- optim(c(1,1,1,1,0.01), L_egfr)
```

```
result_1
```

```
AIC = 2*result_1$value+2*5
```

```
AIC
```

```
BIC=2*result_1$value+5*log(66)
```

```
BIC
```

#####FrW

```
dat=c(0.85,0.39,1.25,1.08,1.47,11.57,1.61,1.69,1.61,1.80,1.87,1.84,2.03,1.89,2.03,2.05,2.35,2.41,2.12,2.43,2.50,
2.48,2.55,2.53,2.55,2.56,2.67,2.73,2.59,2.75,1.81,2.79,2.85,2.82,2.87,2.88,2.95,2.96,2.93,2.97,3.11,3.09,3.15,3.1
1,3.15,3.19,3.22,3.27,3.22,3.28,3.31,3.31,3.39,3.33,3.39,3.56,3.65,3.68,3.60,3.70,4.20,3.75,4.42,4.38,4.70,4.90)
```

```
pdf_frw<- function(x,alpha,psi,sigma,rho){
```

```
  alpha*psi*sigma**(alpha)*rho**(alpha*psi)*x**(-1-alpha*psi)*exp(-sigma**(alpha)*(rho/x)**(alpha*psi))
```

```
}
```

```
L_frw<- function(par){
```

```
  alpha=par[1]
```

```
  psi=par[2]
```

```
  sigma=par[3]
```

```
  rho=par[4]
```

```
  R=pdf_frw(x=dat,alpha,psi,sigma,rho)
```

```
  -sum(log(R))
```

```

}
result_2<- optim(c(0.0001,0.01,0.0001,0.0001), L_frw)
result_2
AIC = 2*result_2$value+4*2
AIC
BIC=2*result_2$value+4*log(66)
BIC
#####2Fr #####
dat=c(0.85,0.39,1.25,1.08,1.47,11.57,1.61,1.69,1.61,1.80,1.87,1.84,2.03,1.89,2.03,2.05,2.35,2.41,2.12,2.43,2.50,
2.48,2.55,2.53,2.55,2.56,2.67,2.73,2.59,2.75,1.81,2.79,2.85,2.82,2.87,2.88,2.95,2.96,2.93,2.97,3.11,3.09,3.15,3.1
1,3.15,3.19,3.22,3.27,3.22,3.28,3.31,3.31,3.39,3.33,3.39,3.56,3.65,3.68,3.60,3.70,4.20,3.75,4.42,4.38,4.70,4.90)
pdf_frii<- function(x,alpha,psi){
alpha*psi**(alpha)*x**(-(alpha+1))*exp(-(psi/x)**(alpha))
}
L_frii<- function(par){
alpha=par[1]
psi=par[2]
R=pdf_frii(x=dat,alpha,psi)
-sum(log(R))
}
result_3<- optim(c(1,0.01), L_frii)
result_3
AIC = 2*result_3$value+2*2
AIC
BIC=2*result_3$value+2*log(66)
BIC

##### 1Fr #####
dat=c(0.85,0.39,1.25,1.08,1.47,11.57,1.61,1.69,1.61,1.80,1.87,1.84,2.03,1.89,2.03,2.05,2.35,2.41,2.12,2.43,2.50,
2.48,2.55,2.53,2.55,2.56,2.67,2.73,2.59,2.75,1.81,2.79,2.85,2.82,2.87,2.88,2.95,2.96,2.93,2.97,3.11,3.09,3.15,3.1
1,3.15,3.19,3.22,3.27,3.22,3.28,3.31,3.31,3.39,3.33,3.39,3.56,3.65,3.68,3.60,3.70,4.20,3.75,4.42,4.38,4.70,4.90)
pdf_fri<- function(x,alpha){
alpha*x**(-(alpha+1))*exp(-(x)**(alpha))
}
L_fri<- function(par){
alpha=par[1]
R=pdf_fri(x=dat,alpha)
-sum(log(R))
}
result_4<- optim(c(1), L_fri,method = "SANN")
result_4
AIC = 2*result_4$value+2*1
AIC
BIC=2*result_4$value+1*log(66)
BIC

```

#####GoIRa

```
dat=c(0.85,0.39,1.25,1.08,1.47,11.57,1.61,1.69,1.61,1.80,1.87,1.84,2.03,1.89,2.03,2.05,2.35,2.41,2.12,2.43,2.50,
2.48,2.55,2.53,2.55,2.56,2.67,2.73,2.59,2.75,1.81,2.79,2.85,2.82,2.87,2.88,2.95,2.96,2.93,2.97,3.11,3.09,3.15,3.1
1,3.15,3.19,3.22,3.27,3.22,3.28,3.31,3.31,3.39,3.33,3.39,3.56,3.65,3.68,3.60,3.70,4.20,3.75,4.42,4.38,4.70,4.90)
pdf_goira<- function(x,alpha,psi,sigma){
  alpha*(2*psi**(2)*x**(-3)*exp(-(psi/x)**(2)))*(1-(exp(-(psi/x)**(2))))**(-sigma-1)*exp((alpha/sigma)*(1-
(1-(exp(-(psi/x)**(2))))**(-sigma)))
}
L_goira<- function(par){
alpha=par[1]
psi=par[2]
sigma=par[3]
  R=pdf_goira(x=dat,alpha,psi,sigma)
  -sum(log(R))
}
result_5<- optim(c(0.1,0.1,0.001), L_goira)
result_5
AIC = 2*result_5$value+2*3
AIC
BIC=2*result_5$value+3*log(66)
BIC
```

```
windows()
hist(dat, freq = FALSE, ylim = c(0, 0.6), ylab = "Density",xlab = "Data",main = "")
x <- seq(0, 2000, 0.005)
lines(x, pdf_egfr(x, omega=result_1$par[1], psi=result_1$par[2], alpha=result_1$par[3], rho=result_1$par[4],
sigma=result_1$par[5]), xlim = c(0, 200), ylab = "Density", xlab = "data", lwd=2.5, lty=1,col="1")
lines(x, pdf_frW(x, alpha=result_2$par[1], psi=result_2$par[2], sigma=result_2$par[3], rho=result_2$par[3]), lty
= 2, lwd=2.5,col="2")
lines(x, pdf_frii(x, alpha=result_3$par[1],psi=result_3$par[2]), lty = 3, lwd=2.5,col="3")
lines(x, pdf_fri(x, alpha=result_4$par[1]), lty = 4, lwd=2.5,col="4")
lines(x, pdf_goira(x, alpha=result_5$par[1],psi=result_5$par[2],sigma=result_5$par[3]), lty = 5, lwd=2.5,col="5")
legend("topright", legend=c("EGFr","FrW","2Fr","1Fr","GoIRa"), lty = c(1,2,3,4,5), lwd=2.5,border =
NULL,col=(c("1":"2":"3":"4":"5")))
```

#####Estimation of parameters for dist data set 2

#####MLE for EGFr distribution

```
dat=c(1.901,2.132,2.203,2.228,2.257,2.35,2.361,2.396,2.397,2.445,2.454,2.474,2.518,2.522,2.525,2.532,2.575,2.
614,2.616,2.618,2.624,2.659,2.675,2.738,2.74,2.856,2.917,2.928,2.937,2.937,2.977,2.996,3.03,3.125,3.139,3.14
5,3.22,3.223,3.235,3.243,3.264,3.272,3.294,3.332,3.346,3.377,3.408,3.435,3.493,3.501,3.539,3.554,3.562,3.628,
3.852,3.871,3.586,3.971,4.024,4.027,4.225,5.395,5.02)
pdf_egfr<- function(x, omega,psi,alpha,rho,sigma){
  2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-alpha))*(1-exp(-sigma*x**(-
alpha)))*(omega-1)*(1-(1-exp(-sigma*x**(-alpha)))*(omega))**(rho-1)*(1-(1-(1-exp(-sigma*x**(-
alpha)))*(omega))**(rho))*(1-(1-(1-(1-exp(-sigma*x**(-alpha)))*(omega))**(rho))**(2))**(psi-1)
```

```

}
L_egfr<- function(par){
omega=par[1]
psi=par[2]
alpha=par[3]
rho=par[4]
sigma=par[5]
  R=pdf_egfr(x=dat,omega,psi,alpha,rho,sigma)
  -sum(log(R))
}
length(dat)
summary(dat)
result_1 <- optim(c(1,1,0.01,0.01,0.001), L_egfr)
result_1
AIC = 2*result_1$value+2*5
AIC
BIC=2*result_1$value+5*log(63)
BIC
#####FrW #####
dat=c(1.901,2.132,2.203,2.228,2.257,2.35,2.361,2.396,2.397,2.445,2.454,2.474,2.518,2.522,2.525,2.532,2.575,2.
614,2.616,2.618,2.624,2.659,2.675,2.738,2.74,2.856,2.917,2.928,2.937,2.937,2.977,2.996,3.03,3.125,3.139,3.14
5,3.22,3.223,3.235,3.243,3.264,3.272,3.294,3.332,3.346,3.377,3.408,3.435,3.493,3.501,3.539,3.554,3.562,3.628,
3.852,3.871,3.586,3.971,4.024,4.027,4.225,5.395,5.02)
pdf_frw<- function(x,alpha,psi,sigma,rho){
alpha*psi*sigma**((alpha)*rho**((alpha*psi)*x**(-1-alpha*psi)*exp(-sigma**((alpha)*(rho/x)**(alpha*psi))
}
L_frw<- function(par){
alpha=par[1]
psi=par[2]
sigma=par[3]
rho=par[4]
  R=pdf_frw(x=dat,alpha,psi,sigma,rho)
  -sum(log(R))
}
result_2<- optim(c(1,0.001,0.001,0.01), L_frw)
result_2
AIC = 2*result_2$value+2*2
AIC
BIC=2*result_2$value+2*log(63)
BIC
#####2Fr #####
dat=c(1.901,2.132,2.203,2.228,2.257,2.35,2.361,2.396,2.397,2.445,2.454,2.474,2.518,2.522,2.525,2.532,2.575,2.
614,2.616,2.618,2.624,2.659,2.675,2.738,2.74,2.856,2.917,2.928,2.937,2.937,2.977,2.996,3.03,3.125,3.139,3.14
5,3.22,3.223,3.235,3.243,3.264,3.272,3.294,3.332,3.346,3.377,3.408,3.435,3.493,3.501,3.539,3.554,3.562,3.628,
3.852,3.871,3.586,3.971,4.024,4.027,4.225,5.395,5.02)

```

```
pdf_frii<- function(x,alpha,psi){
alpha*psi**(alpha)*x**(-(alpha+1))*exp(-(psi/x)**(alpha))
}
```

```
L_frii<- function(par){
alpha=par[1]
psi=par[2]
R=pdf_frii(x=dat,alpha,psi)
-sum(log(R))
}
```

```
result_3<- optim(c(1,0.01), L_frii)
```

```
result_3
```

```
AIC = 2*result_3$value+2*2
```

```
AIC
```

```
BIC=2*result_3$value+2*log(63)
```

```
BIC
```

```
#####1Fr #####
```

```
dat=c(1.901,2.132,2.203,2.228,2.257,2.35,2.361,2.396,2.397,2.445,2.454,2.474,2.518,2.522,2.525,2.532,2.575,2.
614,2.616,2.618,2.624,2.659,2.675,2.738,2.74,2.856,2.917,2.928,2.937,2.937,2.977,2.996,3.03,3.125,3.139,3.14
5,3.22,3.223,3.235,3.243,3.264,3.272,3.294,3.332,3.346,3.377,3.408,3.435,3.493,3.501,3.539,3.554,3.562,3.628,
3.852,3.871,3.586,3.971,4.024,4.027,4.225,5.395,5.02)
```

```
pdf_fri<- function(x,alpha){
alpha*x**(-(alpha+1))*exp(-(x)**(alpha))
}
```

```
L_fri<- function(par){
alpha=par[1]
R=pdf_fri(x=dat,alpha)
-sum(log(R))
}
```

```
result_4<- optim(c(1), L_fri,method = "SANN")
```

```
result_4
```

```
AIC = 2*result_4$value+2*1
```

```
AIC
```

```
BIC=2*result_4$value+1*log(63)
```

```
BIC
```

```
#####GoIRa #####
```

```
dat=c(1.901,2.132,2.203,2.228,2.257,2.35,2.361,2.396,2.397,2.445,2.454,2.474,2.518,2.522,2.525,2.532,2.575,2.
614,2.616,2.618,2.624,2.659,2.675,2.738,2.74,2.856,2.917,2.928,2.937,2.937,2.977,2.996,3.03,3.125,3.139,3.14
5,3.22,3.223,3.235,3.243,3.264,3.272,3.294,3.332,3.346,3.377,3.408,3.435,3.493,3.501,3.539,3.554,3.562,3.628,
3.852,3.871,3.586,3.971,4.024,4.027,4.225,5.395,5.02)
```

```
pdf_goiri<- function(x,alpha,psi,sigma){
alpha*(2*psi**(2)*x**(-3)*exp(-(psi/x)**(2)))*(1-(exp(-(psi/x)**(2))))*(-sigma-1)*exp((alpha/sigma)*(1-
(1-(exp(-(psi/x)**(2))))*(-sigma)))
}
```

```
L_goiri<- function(par){
```

```

alpha=par[1]
psi=par[2]
sigma=par[3]
  R=pdf_goira(x=dat,alpha,psi,sigma)
  -sum(log(R))
}
result_5<- optim(c(0.0001,0.0001,0.00001), L_goira)
result_5
AIC = 2*result_5$value+2*3
AIC
BIC=2*result_5$value+3*log(63)
BIC

windows()
hist(dat, freq = FALSE, ylim = c(0, 0.8), ylab = "Density",xlab = "Data",main = "")
x <- seq(0, 2000, 0.005)
lines(x, pdf_egfr(x, omega=result_1$par[1], psi=result_1$par[2], alpha=result_1$par[3], rho=result_1$par[4],
sigma=result_1$par[5]), xlim = c(0, 200), ylab = "Density", xlab = "data", lwd=2.5, lty=1,col="1")
lines(x, pdf_frwx(x, alpha=result_2$par[1], psi=result_2$par[2], sigma=result_2$par[3], rho=result_2$par[3]), lty
= 2, lwd=2.5,col="2")
lines(x, pdf_frii(x, alpha=result_3$par[1],psi=result_3$par[2]), lty = 3, lwd=2.5,col="3")
lines(x, pdf_fri(x, alpha=result_4$par[1]), lty = 4, lwd=2.5,col="4")
lines(x, pdf_goira(x, alpha=result_5$par[1],psi=result_5$par[2],sigma=result_5$par[3]), lty = 5, lwd=2.5,col="5")
legend("topright", legend=c("EGFr","FrW","2Fr","1Fr","GoIRa"), lty = c(1,2,3,4,5), lwd=2.5,border =
NULL,col=("1":"2":"3":"4":"5"))

#####MLE EGFr distribution data 1
x=c(0.85,0.39,1.25,1.08,1.47,11.57,1.61,1.69,1.61,1.80,1.87,1.84,2.03,1.89,2.03,2.05,2.35,2.41,2.12,2.43,2.50,2.
48,2.55,2.53,2.55,2.56,2.67,2.73,2.59,2.75,1.81,2.79,2.85,2.82,2.87,2.88,2.95,2.96,2.93,2.97,3.11,3.09,3.15,3.11,
3.15,3.19,3.22,3.27,3.22,3.28,3.31,3.31,3.39,3.33,3.39,3.56,3.65,3.68,3.60,3.70,4.20,3.75,4.42,4.38,4.70,4.90)
degfr<- function(x,omega,psi,alpha,rho,sigma) 2*sigma*omega*psi*alpha*rho*x**(-alpha-1)*exp(-sigma*x**(-
alpha))*(1-exp(-sigma*x**(-alpha)))**((omega-1)*(1-(1-exp(-sigma*x**(-alpha)))**((omega))**((rho-1)*(1-(1-
(1-exp(-sigma*x**(-alpha)))**((omega))**((rho))*(1-(1-(1-(1-exp(-sigma*x**(-
alpha)))**((omega))**((rho))**((2))**((psi-1)
pegfr<- function(q,omega,psi,alpha,rho,sigma) (1-(1-(1-(1-exp(-sigma*q**(-
alpha)))**((omega))**((rho))**((2))**((psi)
qegfr<- function(p,omega,psi,alpha,rho,sigma) ((1/sigma)*(-log(1-(1-(1-(1-
p**((1/psi))**((1/2))**((1/rho))**((1/omega))))*(-1/alpha)
egfrMLE<-fitdist(x,"egfr",start=list(omega=0.5,psi=0.05,alpha=0.05,rho=0.005,sigma=0.005),lower = 0, upper =
Inf,method="mle")

```
