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Abstract:

In several scientific fields, the inverse Weibull distribution (IWD) is widely used as a model for dependability analysis. This paper's main objective is to develop a linear, exponential loss function based on lowest record values for the Inverse Weibull Distribution's dependability value and variables. After weighting the LXLF in the construction of the current function loss, the modified linear, exponential loss function (WLXLF) was given its name. Next, estimate the Inverse Weibull distribution's parameters and reliability functions using WLXLF. This paper uses real-time data to estimate the parameter intervals for the inverse Weibull distribution using two estimation methods. The maximum likelihood approach and the relative maximum likelihood method are the foundations of these two tactics. Additionally, we assess how relative maximum likelihood intervals for actual datasets compare to maximum likelihood intervals. Consequently, determining the values of the IWD's maximum likelihood intervals using fuzzy parameters inside the Corresponding real-time problem's interval. Using a fuzzy range of parameter values for real-time data, the present study investigated the probability density function, cumulative density function, and hazard rate function of the inverse Weibull distribution. We examined Tamil Nadu's agricultural output in 2018 in this research. It was thus shown from the comparison that the fuzziness interval estimate performs better than the real-time data.

Key Words: Agriculture data, Inverse Weibull distribution, Fuzzy Parameter Value, Maximum likelihood.

Mathematical Subject Classification (2020): 03B52, 62A86.

1. Introduction

Fuzzy sets of numbers can improve the accuracy of real-life models, particularly for uncertainty models like statistical fluctuation in observed lifespan. Voids are another kind of uncertainty that arises when an observation is not a precise number but rather a fuzzy one. Zadeh used the term "fuzzy variable" in 1965 [1] to refer to incorrect linguistic idiom and vernacular. Fuzzy set theory got its start with this. A fuzzy set is made up of elements with different membership levels. Each item in the set is distinguished by its membership function, which has degrees ranging from zero to one. There are two approaches to parameter estimate in statistical inference: interval estimation, which allocates a range of values to the parameters, and point estimation, which gives the parameters a single value. The ideal solution of the inventory model is evaluated in both clear-cut and fuzzy scenarios. The inventory model that has been proposed uses triangular fuzzy symmetric parameters. The whole value of the λ-integral is used to defuzzify the answer. Numerical examples are used to demonstrate the model, and a sensitivity analysis of the decision variables was carried out to look at how changes in parameter values affected the best inventory plan in the presence of fuzziness [2]. The reliability function, Bayes estimation with square error loss and quadratic loss functions for the unknown parameters in the distribution, and the Maximum Likelihood Estimator (MLE) using the Newton Raphson (NR) method. Through a Monte-Carlo simulation analysis, the derived estimates of the unknown parameters and reliability function are quantitatively compared in terms of the mean square error (MSE) values and (IMSE) by [3]. Three different methods were used to estimate the parameters: MLE, MOM, and PEC(Regression). The values of ti were derived via inverse transformation from the C.D.F. As shown in the Tables [4], we then selected a set of five values of ti for application and estimate in order to determine which estimators had the lowest mean square error. The method used to estimate the coefficients of Bayesian regression is called iteratively reweighted least squares (IRLS). Additionally, we use performance measures like deviation (D), mean squared error (MSE), and Akaike's information criterion (AIC) to compare IW-Reg with IW-BReg. Lastly, we validate the theoretical conclusions using real datasets that were collected in Saudi Arabia and the associated explanatory factors. The Bayesian methodology fared better than the conventional method in terms of the performance criteria that were examined [5]. [6] used the maximum likelihood approach to investigate the reliability function and estimate parameters of the Weibull-Pareto distribution. We used the "Newton-Raphson (NR)" and "Fisher's Scoring (FS) method" iterative approximation techniques when the available data was in the form of fuzzy integers. The stress-strength reliability function's Bayesian estimator is obtained. A real-data application is shown to illustrate the stress-strength model's results based on actual data and to contrast current distributions with the exponentiated power generalized Weibull distribution [7]. The goal of the research is to learn more about how maximum likelihood estimates behave. We contrast current distributions in classical statistics with the proposed DUS-neutrosophic multivariate inverse Weibull distribution [8]. In contrast to the known distributions under classical statistics, the proposed DUS-neutrosophic multivariate inverse Weibull distribution exhibits lower values of Akaike's information criterion and Bayesian information criteria. Both maximum likelihood and Bayesian approaches are used to find the parameters of the inverse Weibull populations [9]. The observed Fisher information matrix and bootstrap confidence intervals are used to compute asymptotic confidence intervals. Our recommendation is to build credible intervals with separate gamma priors by using Markov chain Monte Carlo (MCMC) methods. All theoretical findings are verified and contrasted using Monte Carlo simulations. An real sample from a comparable population is investigated to illustrate. Using real-time data and a fuzzy range of parameter values, the present research examined the probability density function, cumulative density function, and hazard rate function of the inverse Weibull distribution. We examined Tamil Nadu's agricultural sector in this research in 2018.

2. Fuzzy Mathematical Model.

Fuzzy Set: Each element in this set has a certain level of membership. This degree of membership falls inside the interval [0,1]. An element or object may belong to a partial membership. Let X denote the universe set. The fuzzy subset $\overline{A}$ is distinguished by a membership function $\mu_{A^-}(x)$, such that:

$$\tilde{A} = \{(x_i, \mu_{A^-}(x_i)), x_i \in X, i = 1,2,3 \dots \dots n, 0 < \mu_{A^-} < 1\}$$

If $\mu_{A^-}(x) = 1$, then x entirely belongs to $\tilde{A}$. $\mu_{\overline{A}}(x) = 0$, then x does not belong to Ã. If $\mu_{\overline{A}}(x) = $ =0.6, then x has a degree of belonging to Ã of 0.6.

Fuzzy numbers are a method for characterizing uncertainty, whether it is triangular, trapezoidal, or any other formula. Ã represents a partial number to a set of real numbers, with a membership function. The fuzzy numbers constitute a fuzzy set under the following conditions [10]:

Triangular fuzzy numbers are the most well-known type since they may be represented by three points (a_1, a_2, a_3) with a_$a_1 < a_2 < a_3$ forming a triangle within the interval $[a_1, a_2]$. The head of the triangle at $x = a_2$ is expressed as $\overline{A} = (a_1/a_2/a_3\ )$. This number contains a membership function.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

Fuzzy Sample Space

It is fuzzy parts $\tilde{x} = (\tilde{x}, \tilde{x}, \tilde{x}, \dots \dots, \tilde{x})$ from $= (X_1, X_2, X_3, \dots \dots X_n)$. Fuzzy sets for X with membershipfunctions has Borel measure, with orthogonal constraint:

$$\sum \tilde{x} \in X\ \mu_{\tilde{x}}(x) = 1, \text{for each } x \in X.$$

Furthermore, this area is referred to as a fuzzy information system (FIS).

Fuzzy Event

Let Consider $X = (X_1, X_2, X_3, \dots \dots X_n)$ in space, where B_x is the smallest Borel field in X. Fuzzy events are fuzzy subsets $\widetilde{A}$ with measurable Borel fields .

2.1. Inverse Weibull Distribution

A continuous lifespan probability distribution is the inverse Weibull distribution. Maurice Frechet (1828–1973) was the first to distribute this way. This distribution may be used to simulate and analyze many different types of natural events, such as sea currents, earthquakes, floods, rainfall, wind speeds, and life tests. Additionally, it is utilized to mimic failure rates, which are often used in reliable biological investigations and modeling of infant mortality [11].

The inverse of a random variable having a Weibull distribution, denoted as (1/y), has a probability distribution known as the Inverse Weibull distribution with the following density if x is the random variable:

$$f(x,\alpha,\beta) = \alpha\beta^{\alpha}x^{-(\alpha+1)}\exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right),x > 0,\alpha,\beta > 0$$

where α is the shape parameter, and β is the scale parameter. The cumulative distribution function and hazard rate is defined as follows:

$$F(x,\alpha,\beta) = p(X \le x) = \int_0^x f(u)du = \exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right),x > 0$$

$$S(x) = 1 - F(x) = 1 - \exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right),x > 0$$

$$H(x) = \frac{\alpha\beta^{\alpha}x^{-(\alpha+1)}\exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right)}{1-\exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right),}\ x > 0,\alpha,\beta > 0.$$

2.2 Interval Estimators of the Parameters of IW Distribution

Since estimation provides parameter values from the population's tested models to statistics ordered by drawn sample, it is considered a key notion in statistical inference. An overview of several estimate techniques for using fuzzy data to determine interval estimators of the IW distribution's parameters is given in this section. Since estimation provides parameter values from the population's tested models to statistics ordered by drawn sample, it is considered a key notion in statistical inference. An overview of several estimate techniques for using fuzzy data to determine interval estimators of the IW distribution's parameters is given in this section.

Fuzzy Maximum Likelihood Estimation Method.

A random sample of size $n, x = (x_1, \dots\dots\dots., x_n)$, follows the inverse Weibull distribution with a density function.

$$f(x,\alpha,\beta) = \alpha\beta^{\alpha}x^{-(\alpha+1)}\exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right)$$

Let $X = (X_1, \dots \dots, X_n)$ be a random vector that represents the sample space. The probability function of complete data (Crispset) is

$$Q(\alpha, \beta; x) = \prod_{i=1}^{n} f\,(\alpha, \beta; x) = \alpha^n \beta^{n\alpha} \prod_{i=1}^{n} x^{-(\alpha+1)} \text{ exp } (-\sum_{i=1}^{n} (\frac{\beta}{x})^{\alpha})$$

where x is purposefully visible and provides complete information about the sharp vector. If x is only partially observable and available in a fuzzy subset form with a membership function $\mu_{\tilde{A}} x$ with Borel measurement. The fuzzy observation $\tilde{x}$ represents the partial observation x from the random vector X. The membership function $\mu_{\tilde{A}}$ is a probability distribution that describes the limits of a partial observation $\tilde{x}$. The fuzzy set X possesses two characteristics.

The information on x can be considered as the following probability distribution:

$$\mu_{\tilde{x}}(x) = \mu_{\tilde{x}}(x) \times \dots \dots \dots \times \mu_{\tilde{x}}(x)$$

So, if x is given, its function is expected to have Boral measurement. The likelihood of this can then be calculated using the fuzzy probability definition. Furthermore, we can define the maximum likelihood function as follows:

$$L(\alpha, \beta; \tilde{x}) = p(\tilde{x}; \alpha, \beta) = \int f(\tilde{x}; \alpha, \beta) x\, x\, dx$$

The fuzzy maximum likelihood function of the fuzzy inverse Weibull distribution may be expressed as follows because the data vector x is identically and independently distributed and the membership function is analytic:

$$\begin{aligned} Q^{\cdot} &= \log\,(L_0(\alpha, \beta; \tilde{x})) \\ &= n \log\,(\alpha) + n\alpha \log\,(\beta) + \sum_{i=1}^{n} \log\,\left(\int_0^{\infty} x^{-(\alpha+1)} \exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right) u_{x_i}(x) dx\,\right) \end{aligned}$$

To derive ML estimators for α and β, maximise Q^* and use a partial derivation with the equivalence of zero, as follows:

$$= \frac{n}{\hat{\alpha}} + n\log(\hat{\beta}) - \left[\sum_{i=1}^{n} \frac{\int_0^\infty [\left(x^{-(\hat{\alpha}+1)} \cdot \ln(x) + x^{-2\hat{\alpha}-1}\left((\hat{\beta})^{\hat{\alpha}} \cdot \ln\left(\frac{\hat{\beta}}{x}\right)\right)\right)\exp\left(-\left(\frac{\hat{\beta}}{x}\right)^{\hat{\alpha}}\right) \cdot \mu_{\tilde{x}}(x)dx]}{\int_0^\infty x^{-(\hat{\alpha}+1)} \exp(-(\frac{\hat{\beta}}{x})^{\hat{\alpha}})\mu_{\tilde{x}}(x)dx}\right] = 0$$

$$\frac{\partial Q^*}{\partial \beta} = \frac{n\hat{\alpha}}{\hat{\beta}} - \sum_{i=1}^{n} \frac{\int_0^\infty x^{-\alpha-1}(\frac{\hat{\beta}}{x})^{\hat{\alpha}}\frac{\hat{\alpha}}{\hat{\beta}} \exp(-(\frac{\hat{\beta}}{x})^{\hat{\alpha}})\mu_{\tilde{x}}(x)dx}{\int_0^\infty x^{-(\hat{\alpha}+1)} \exp(-(\frac{\hat{\beta}}{x})^{\hat{\alpha}})\mu_{\tilde{x}}(x)dx} = 0$$

There is no closed formula for the solutions to above equations. We will use the Newton-Raphson approach to obtain the ML estimators $\hat{\alpha}$ and $\hat{\beta}$. Let $\theta = (\alpha, \beta)^T$ be the parameters vector. After $(h+1)$ steps of iterative steps, we have the parameters.

$$\theta^{h+1} = \theta^h - [\frac{\partial^2 Q^{\cdot}(\alpha,\beta;\tilde{x})}{\partial\theta\partial\theta^T}|_{\theta=\theta^h}]^{-1}\left[\frac{\partial Q^{\cdot}(\alpha,\beta;\tilde{x})}{\partial\theta}|_{\theta=\theta^h}\right]$$

such that

$$\frac{\partial Q^{\cdot}(\alpha,\beta;\tilde{x})}{\partial\theta} = \begin{pmatrix} \frac{\partial Q^\star(\alpha,\beta,\tilde{x})}{\partial\alpha} \\ \frac{\partial L^\star(\alpha,\beta,\tilde{x})}{\partial\beta} \end{pmatrix}$$

$$\frac{\partial^2 Q^*(\alpha,\beta;\tilde{x})}{\partial\theta\partial\theta^T} = \begin{pmatrix} \frac{\partial Q^\star(\alpha,\beta,\tilde{x})}{\partial\alpha^2} & \frac{\partial L^\star(\alpha,\beta,\tilde{x})}{\partial\alpha\,\partial\beta} \\ \frac{\partial L^\star(\alpha,\beta,\tilde{x})}{\partial\alpha\,\partial\beta} & \frac{\partial L^\star(\alpha,\beta,\tilde{x})}{\partial\beta^2} \end{pmatrix}$$

and

$$S_1 = \frac{\partial Q^{*2}}{\partial\alpha^2} = -\frac{n}{\alpha^2} - \sum_{i=1}^{n}(\frac{1}{(\int_0^\infty x^{-(\alpha+1)} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx)}(\int_0^\infty \ln(x)^2(\frac{\beta}{x})^\alpha + 2\ln(x)\ln(\frac{\beta}{x}) - \ln(\frac{\beta}{x})^2 + (\frac{\beta}{x})^\alpha \ln(\frac{\beta}{x})^2)(x^{-\alpha-1}(\frac{\beta}{x})^\alpha \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx) + \frac{(\int_0^\infty \ln(x) + (\frac{\beta}{x})^\alpha \ln(\frac{\beta}{x}))(x^{-\alpha-1}\exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx)^2}{(\int_0^\infty x^{-(\alpha+1)} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx)^2}$$

$$S_2 = \frac{\partial Q^{*2}}{\partial\beta^2} = -\frac{n\alpha}{\beta^2} - \sum_{i=1}^{n}(\frac{(\frac{\alpha^2}{\beta^2} - \frac{\alpha}{\beta^2} - \frac{\alpha^2}{\beta^2}(\frac{\beta}{x})^\alpha)(x^{-\alpha-1}\exp(-(\frac{\beta}{x})^\alpha))\mu_{\tilde{x}}(x)dx}{\int_0^\infty x^{-(\alpha+1)} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx} +$$

$$\frac{\int_0^\infty x^{-\alpha-1} (\frac{\beta}{x})^\alpha \frac{\alpha}{\beta} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx}{(\int_0^\infty x^{-(\alpha+1)} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx)^2} 1$$

$$S_3 = \frac{\partial Q^{*2}}{\partial\alpha\partial\beta} = -\frac{n}{\beta}$$

$$-\sum_{i=1}^{n}(\frac{-\ln(x)\frac{\alpha}{\beta} + \frac{\alpha}{\beta} + \frac{1}{\beta} - (\frac{\beta}{x})^\alpha \ln(x)\frac{\alpha}{\beta})(x^{-\alpha-1}\exp(-(\frac{\beta}{x})^\alpha))\mu_{\tilde{x}}(x)dx}{\left(\int_0^\infty x^{-(\alpha+l)} \exp\left(-\left(\frac{\beta}{x}\right)^\alpha\right)\mu_{\tilde{x}}(x)dx\right)\right)}$$

$$\int_0^\infty x^{-\alpha-1} \ln(x)\exp(-(\frac{\beta}{x})^\alpha) + x^{-\alpha-1}(\frac{\beta}{x})^\alpha \ln(\frac{\beta}{x})\exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx$$

$$+\frac{\int_0^\infty x^{-\alpha-1} (\frac{\beta}{x})^\alpha \frac{\alpha}{\beta} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\overline{x}}(x)dx}{(\int_0^\infty x^{-(\alpha+1)} \exp(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx)^2})$$

We replicate until $||\theta^{h+1} - \theta^h||$ approaches $\in$, where $\in > 0$ is a very small positive number. The Wald Method Using the following property, which is the inverse of Fisher's information matrix detriment, we must create the variance and covariance matrix for the parameters for which we need to estimate the confidence interval:

$$\left(-\frac{1}{I(\hat{\alpha},\hat{\beta})}\right)$$

Fisher information matrix can be founded from:

$$I(\hat{\alpha},\hat{\beta}) = -E\begin{pmatrix} S_1 & S_3 \\ S_3 & S_2 \end{pmatrix}$$

In other words, the matrix of variance and covariance for the parameters is

$$\Sigma = \frac{1}{E\begin{pmatrix} -S_1 & -S_3 \\ -S_3 & -S_2 \end{pmatrix}} = \begin{pmatrix} \hat{\sigma}^2(\hat{\alpha}) & \hat{\sigma}(\hat{\alpha},\hat{\beta}) \\ \hat{\sigma}(\hat{\alpha},\hat{\beta}) & \hat{\sigma}^2(\hat{\beta}) \end{pmatrix}$$

Consequently, the sample distribution and the convergence theory of MLE may be used to calculate the confidence intervals for the parameters that we want to estimate.

$$z = \frac{\hat{\alpha} - \alpha}{\sqrt{\hat{\alpha}^2(\hat{\alpha})}}$$

$$z = \frac{\hat{\beta} - \beta}{\sqrt{\hat{\beta}^2(\hat{\alpha})}}$$

Can be roughly calculated using a 95% confidence interval and a typical normal distribution. The confidence intervals for each parameter are as follows:

$$\hat{\alpha} - Z_{1-\frac{\Psi}{2}}\sqrt{\hat{\sigma}^2(\hat{\alpha})} < \alpha < \hat{\alpha} + Z_{1-\frac{\Psi}{2}}\sqrt{\hat{\sigma}^2(\hat{\alpha})}$$

$$\hat{\beta} - Z_{1-\frac{\psi}{2}}\sqrt{\hat{\sigma}^2(\hat{\beta})} < \beta < \hat{\beta} + Z_{1-\frac{\psi}{2}}\sqrt{\hat{\sigma}^2(\hat{\beta})}$$

Fuzzy Relative MLE Method

The fuzzy relative t function is defined by the following formula:

$$\begin{aligned} Q^\star &= \log\,(Q_0(\alpha,\beta;x \\ &= n\log\,(\alpha) + n\alpha\log\,(\beta) + \sum_{i=1}^{n} \log\,(\int_0^\infty x^{-(\alpha+1)}\ \exp\,(-(\frac{\beta}{x})^\alpha)\mu_{\widetilde{x}}(x)dx) \end{aligned}$$

$$\begin{aligned} &Q^*(\widehat{\alpha}_{fmle},\widehat{\beta}_{fmle},\widetilde{x}) \\ &\qquad = n\log\,(\widehat{\alpha}_{fmle}) + n\widehat{\alpha}_{\mathrm{fmle}}\log\,(\widehat{\beta}_{fmle}) \\ &\qquad + \sum_{i=1}^{n} \log\,(\int_0^\infty x^{-(\widehat{\alpha}_{fmle}+1)}\ \exp\,(-(\frac{\widehat{\beta}_{fmle}}{x})^{\widehat{\alpha}})\mu_{\widetilde{x}}(x)dx) \end{aligned}$$

$$R(\alpha,\beta;\tilde{x}) = \frac{Q^{\cdot}(\alpha,\beta;\tilde{x})}{Q\big(\hat{\alpha}_{fmle},\hat{\beta}_{fmle},\tilde{x}\big)}$$

$$= \frac{\log\,(\alpha) + n\alpha\log\,(\beta) + \sum_{i=1}^{n}\log\,(\int_0^\infty x^{-(\alpha+1)}\ \exp\,(-(\frac{\beta}{x})^\alpha)\mu_{\tilde{x}}(x)dx)}{\log\big(\hat{\alpha}_{fmle}\big) + n\hat{\alpha}_{\mathrm{fmle}}\log\big(\hat{\beta}_{fmle}\big) + \sum_{i=1}^{n}\log\,(\int_0^\infty x^{-(\hat{\alpha}_{fmle}+1)}\ \exp\,(-(\frac{\hat{\beta}_{fmle}}{x})^{\hat{\alpha}_{fmle}})\mu_{\tilde{x}}(}$$

Let $\hat{\alpha}(\beta)$ represent the MLE of α given β. The fuzzy relative MLE estimator of β can be established by maximising the likelihood function of the distribution, such that

$$\begin{aligned} Q_p(\beta) &= \max_\alpha Q^{\cdot}(\alpha,\beta;\tilde{x}) = Q^{\cdot}(\widehat{\alpha}(\beta),\beta;\tilde{x}) \\ &= \max_{\alpha}\ \log\,(\hat{\alpha}(\beta)) + n\hat{\alpha}(\beta)\log\,(\beta) \\ &\qquad + \sum_{i=1}^{n} \log\,(\int_0^\infty x^{-(\widehat{\alpha}(\beta)+1)}\ \exp\,(-(\frac{\beta}{x})^{\widehat{\alpha}(\beta)})\mu_{\widetilde{x}}(x)dx \end{aligned}$$

And we maximize $Q_p(\beta)$ by differentiating w.r.t β and equating it to zero as follows

$$R_P(\beta) = \frac{\partial Q_p(\beta)}{\partial\beta}$$

$$= \frac{n\hat{\alpha}(\hat{\beta})}{\hat{\beta}} + \sum_{i=1}^{n} \frac{x^{-(\hat{\alpha}(\hat{\beta})+1)}\hat{\alpha}(\beta)(\frac{\widehat{\beta}}{x})^{\hat{\alpha}(\hat{\beta})-1} \exp\,(-(\frac{\beta}{x})^{\hat{\alpha}(\hat{\beta})})\mu_{\tilde{x}}(x)}{\int_0^{\infty} x^{-(\hat{\alpha}(\hat{\beta})+1)} \exp\,(-(\frac{\widehat{\beta}}{x})^{\hat{\alpha}(\hat{\beta})})\mu_{\tilde{x}}(x)dx} = 0$$

In addition, we can find the relative MLE estimator of α as follows

$$\begin{aligned} Q_p(\alpha) &= \max_{\alpha} Q^{\cdot}(\alpha, \beta; \overline{x}) = Q^{\cdot}(\alpha, \widehat{\beta}(\alpha); \overline{x}) \\ &= \max\ \log(\alpha) + n\alpha \log(\hat{\beta}(\alpha)) \\ &\quad + \sum_{i=1}^{n} \log\,(\int_0^{\infty} x^{-(\alpha+1)} \exp\,(-(\frac{\hat{\beta}(\alpha)}{x})^{\alpha})\mu_{\tilde{x}}(x)dx \end{aligned}$$

Similarly, we maximize $Q_p(\alpha)$ by differentiating w.r.t α and equating it to zero as follows

$$\begin{aligned} R_P(\alpha) &= \frac{\partial L_p(\alpha)}{\partial \alpha} \\ &= \frac{1}{\widehat{\alpha}} + n \log(\widehat{\beta}(\alpha)) \\ &\quad + \sum_{i=1}^{n} \frac{x^{-(\alpha+1)} \exp\,(-(\frac{\widehat{\beta}(\alpha)}{x})^{\alpha}(\frac{\widehat{\beta}(\alpha)}{x})^{\alpha}) \log\,(\frac{\widehat{\beta}(\alpha)}{x})\mu_{\tilde{x}}(x) - (\alpha+2)x^{-(\alpha+2)} \exp\,(-(\frac{\widehat{\beta}(\alpha)}{x})^{\alpha})\mu_{\tilde{x}}(x)}{\int_0^{\infty} x^{-(\alpha+1)} \exp\,(-(\frac{\widehat{\beta}(\alpha)}{x})^{\alpha})\mu_{\tilde{x}}(x)dx} = 0 \end{aligned}$$

3. Applications with real Time Data

In this application the real time agriculture data from Kaggle.com [12], from that data we analyzed of Tamilnadu agriculture data set in 2018. We evaluated the data using inverse Weibull distribution with fuzzy parameters. Some data values are given in Table.1.The Maximum likelihood Estimation values for $\alpha = 1.1234$, $\beta = 1345.6789$. Hence, we finding Probability density function, Cumulative density function and Hazard rate function to the corresponding problem using inverse Weibull distribution with the Fuzzy parameter intervals $\alpha = \min 1.5$, $\alpha = \mathrm{Max}\, 2.5$, $\beta = \min 0.5$, $\alpha = \mathrm{Max}\, 1.5$.

Crop	Season	Area(ha)	(MT)	(kg/ha)	Value(₹)	(₹/kg)	Crop
Arecanut	Whole Year	6952	13543	986.2	1127614	2433.2	2.0112
Arhar/Tur	Kharif	41381	51632	986.2	6711998	14483.35	1.188571
Bajra	Kharif	31109	88494	986.2	5045880	10888.15	2.8308
Bajra	Rabi	15773	29516	986.2	2558381	5520.55	2.042692
Banana	Whole Year	87514	3483248	986.2	14194771	30629.9	37.39125
Black pepper	Whole	5571	1176	986.2	903616.2	1949.85	0.16

	Year						
Cardamom	Whole Year	3838	347	986.2	622523.6	1343.3	0.08
Cashewnut	Whole Year	85272	19701	986.2	13831118	29845.2	0.242
Castor seed	Kharif	4756	1526	986.2	771423.2	1664.6	0.333636
Coconut	Whole Year	439745	5.21E+09	986.2	71326639	153910.8	11263.83

Table.1. Agriculture data of Tamilnadu in the year of 2018

Mathematical Results of the Inverse Weibull Distribution

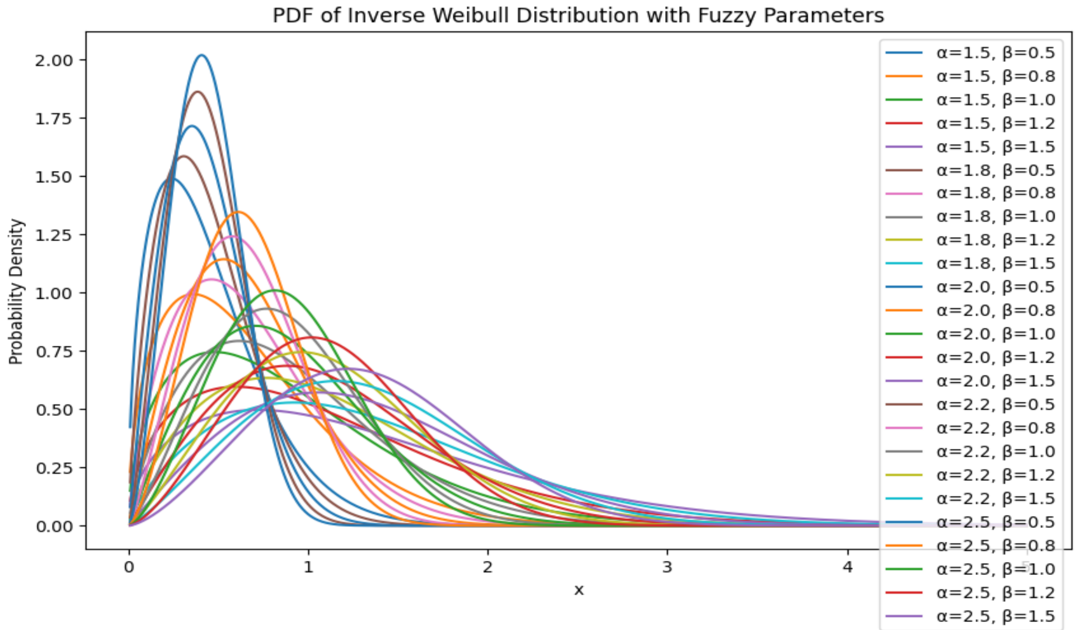

Figure 1: PDF of inverse Weibull distribution

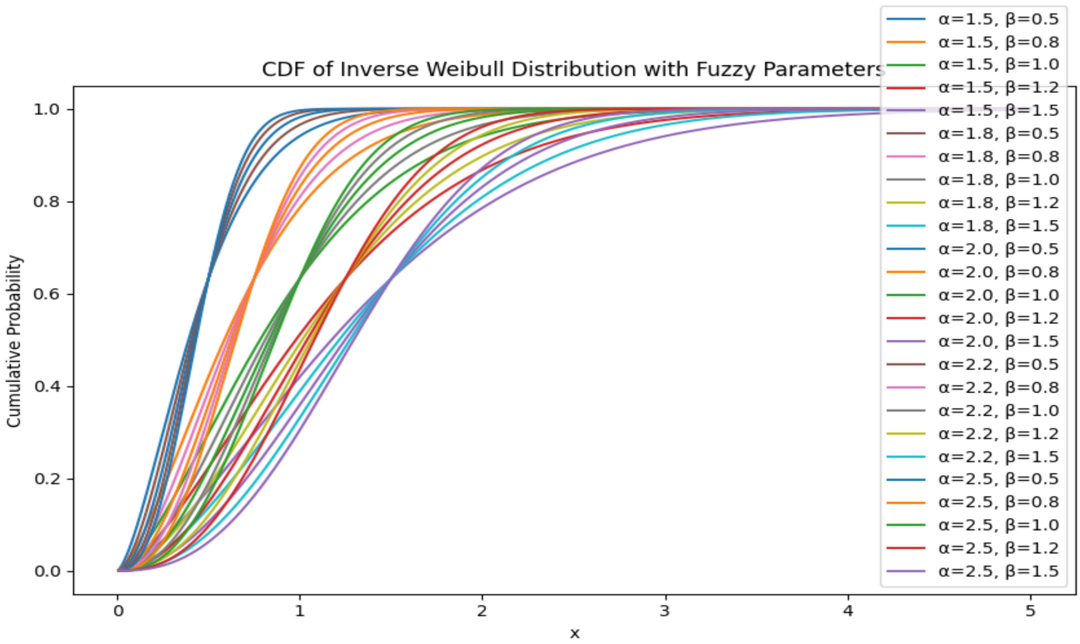

Figure 2: CDF of inverse Weibull distribution

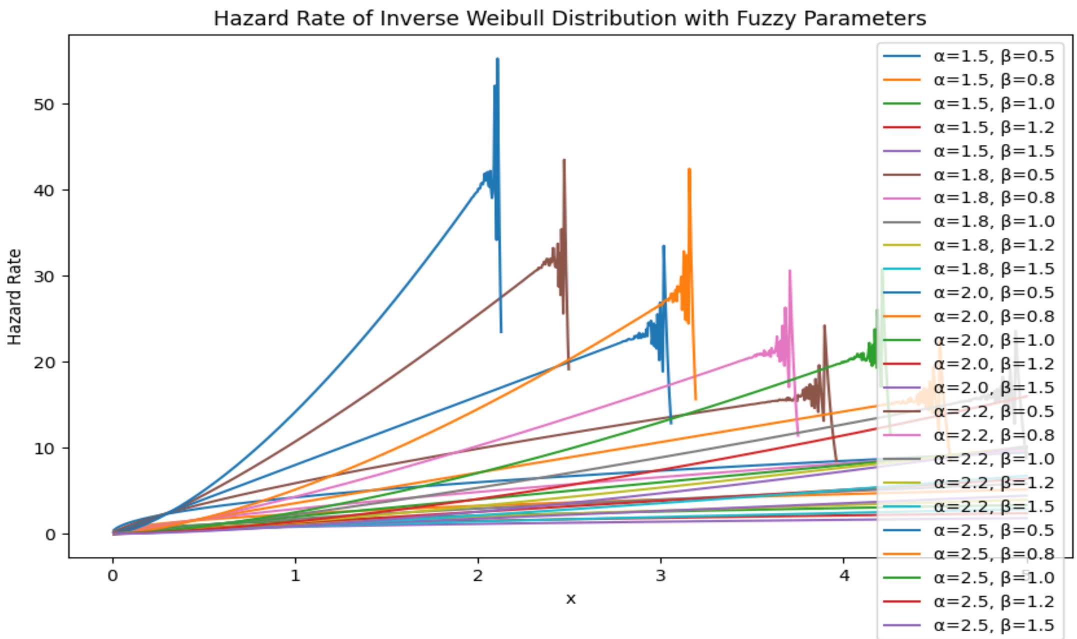

Figure 3: Hazard rate function of inverse Weibull distribution

4. Results and Discussion

The results of the investigation show that the suggested method does a good job of predicting the Inverse Weibull distribution using interval-value fuzzy parameters. With an emphasis on real-time scenarios, the parameters, probability density function, cumulative density function, and hazard rate function of the Inverse Weibull distribution are computed. The Inverse Weibull distribution is used to simulate data in many practical domains, such as biological research and the dynamic components of dependability. It may be used to mimic various failure characteristics, including motor-related ones. The results show that the suggested approach performs better in estimating the magnitude parameter and the accuracy function than the current traditional techniques. The suggested approach still has practical uses for assessing overall performance as well as the significance of the dependability function and scale value in reliability assessment. An essential part of the study was producing intervals of confidence for the reliability function and anticipated parameters using the parametric bootstrap technique. This approach offers a degree of uncertainty in the estimates and facilitates trustworthy inference. Analyzing the outcomes obtained with real-time data offers further proof of the validity and practicality of the suggested method. In addition to theoretical insights, the research offers a fact-based, real-world example. Kaggle.com provides support for the present research's findings [11]. In the inverse Weibull distribution to the agriculture problem of Tamilnadu, we found results for the Probability density function (Fig. 1), Cumulative density function (Fig. 2), and Hazard rate function (Fig. 3) using fuzzy parameters and interval values and supporting MATLAB tool. These results demonstrate how well the suggested technique performs in predicting the crop production for all the seasons, Whole Year, Kharif and Rabi, respectively, and provides reasonable predictions for the scale parameter.

5. Conclusion

In this study, the Inverse Weibull distribution model is applied to real-time data under fuzzy settings. The fraction of faulty items is represented by fuzzy parameters α and β. This value is produced using the three types distribution function of the inverse Weibull distribution with the interval parameter values, and the resulting curve is used to measure the performance of given data. Fuzzy operating characteristic curves are analyzed for various functions. That is the probability density function, cumulative density function, and hazard rate function for the inverse Weibull distribution. This study determined that the suggested plan can be used if the product's quality is ambiguous. The study found that the approach is quite effective in uncertain agriculture and settings.

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