Jouínal of Modeín Applied Statistical Methods

Volume 24 | Issue 1 Article 8

Modulo Arithmetic Triangle of Numbers

M. Devika[1], Dr. R. Sivaraman[2]

[1]Ph.D. Research Scholar, Post Graduate and Research Department of Mathematics
Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai, India
Email: mdevika1969@gmail.com
[2]Associate Professor, Post Graduate and Research Department of Mathematics
Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai, India
Email: rsivaraman1729@yahoo.co.in

Recommended Citation

M. Devika[1], Dr. R. Sivaraman[2] (2025). Modulo Arithmetic Triangle of Numbers. Jouínal of ModeínApplied Statistical Methods, 24(1), https://doi.oíg/10.56801/Jmasm.V24.i1.8

Journal of Modern Applied Statistical Methods

2025, Vol. 24, No. 1, ISSN 1538 – 9472

Doi: 10.56801/Jmasm.V24.i1.8

Modulo Arithmetic Triangle of Numbers

M. Devika[1], Dr. R. Sivaraman[2]

[1]Ph.D. Research Scholar, Post Graduate and Research Department of Mathematics
Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai, India
Email: mdevika1969@gmail.com
[2]Associate Professor, Post Graduate and Research Department of Mathematics
Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai, India
Email: rsivaraman1729@yahoo.co.in

Abstract

Numerous numerical patterns and triangles have surfaced, and each of these triangles has unique qualities related to the numbers that make them up. In this work, we have examined a number triangle made up of whole numbers ordered according to modulo arithmetic and we have looked at a few intriguing characteristics of its elements. We have shown many theorems that aid in our comprehension of the composition and patterns of numbers using this number triangle.

Keywords: Number Triangle, Modulo Arithmetic, Perfect Square, Properties of Number Triangle

1. Introduction

Numerous intriguing number triangles have surfaced, and each of these triangles has examined significant characteristics related to the numbers inside the triangle. In this article, we will use modulo arithmetic to build a basic number triangle in which the whole numbers are organized in each row in a certain sequence. This article will explain the fascinating mathematical characteristics of this number triangle. In this work, we have proved eight theorems.

2. Construction of Number Triangle

Take a look at the number triangle made of whole numbers that follows.

Figure 1 shows that row p includes p numbers if we take p to be the row number, where p is a natural integer. This triangle is analogous to the most famous Pascal's Triangle whose entries are connected to binomial coefficients.

Further we notice from Figure 1, that the number located in p^{th} row and q^{th} position (when read from left to right) where $p \geq 1$, $q \geq 1$ is given by

$$a(p,q) \equiv (p^2 + q^2 + 1)\, mod(p+q) \qquad (2.1)$$

1

0 1

3 4 1

3 3 5 1

3 2 3 6 1

3 1 3 3 7 1

3 0 9 0 3 8 1

3 9 8 9 12 3 9 1

3 9 7 7 9 13 3 10 1

3 9 6 5 6 9 14 3 11 1

3 9 5 3 3 5 9 15 3 12 1

3 9 4 1 0 1 4 9 16 3 13 1

3 9 3 16 15 16 19 3 9 17 3 14 1

• • • • • • • • • • • • • •

Figure 1: Modulo Arithmetic Triangle of Numbers

We now offer some findings on the number triangle shown in Figure 1 using equation (2.1).

Properties of Number Triangle

3. Theorem 1

For $p \geq 1$,

$$a(p,p) \equiv 1\, mod\, 2p \qquad (3.1)$$

Proof: Using (2.1) we have

$$a(p,p) \equiv (p^2 + p^2 + 1)\, mod(p+p)$$

$$\equiv (2p^2 + 1)\, mod(2p)$$

$$\text{Thus, } a(p,p) \equiv 1 \bmod 2p$$

■

4. Theorem 2

For $p \geq 3$,

$$a(p, p-1) \equiv (p+1) \bmod (2p-1) \qquad (4.1)$$

Proof: Using (2.1) we have

$$a(p,p-1) \equiv (p^2 + (p-1)^2 + 1) \bmod (2p-1)$$
$$\equiv (2p^2 - 2p + 2) \bmod (2p-1)$$
$$\equiv [2p(p-1) - p + 1 + p + 1] \bmod (2p-1)$$
$$\equiv [2p(p-1) - 1(p-1) + p + 1] \bmod (2p-1)$$
$$\equiv [(2p-1)(p-1) + (p+1)] \bmod (2p-1)$$
$$\text{Thus, } a(p,p-1) \equiv (p+1) \bmod (2p-1).$$

■

5.Theorem 3

For p≥ 3,

$$a(p,p-2) \equiv 3 \bmod (2p-2) \qquad (5.1)$$

Proof: Using (2.1) we have

$$a(p,p-2) \equiv (p^2 + (p-2)^2 + 1) \bmod (2p-2)$$
$$\equiv (2p^2 - 4p + 4 + 1) \bmod (2p-2)$$
$$\equiv [2p(p-2) - 2p + 4 + 2p + 1] \bmod (2p-2)$$
$$\equiv [(2p-2)(p-2) + (2p+1)] \bmod (2p-2)$$
$$\equiv (2p+1) \bmod (2p-2)$$
$$\equiv (2p - 2 + 3) \bmod (2p-2)$$
$$\text{Thus, } a(p,p-2) \equiv 3 \bmod (2p-2).$$

■

6. Theorem 4

For $p \geq 3$,

$$a(p,1) \equiv 3 \bmod (p+1) \qquad (6.1)$$

Proof: Using (2.1) we have

$$a(p,1) \equiv (p^2 + 1^2 + 1) \bmod (p+1)$$
$$\equiv (p^2 + 1^2 + 2p - 2p + 1) \bmod (p+1)$$

$$\equiv [(p+1)^2 - 2p - 2 + 2 + 1)]\ mod(p+1)$$

$$\equiv (-2(p \mp 1) + 3)\ mod(p+1)$$

Thus, $a(p,1) \equiv 3\ mod\ (p+1)$

■

7. Theorem 5

For $p \geq 7$,

$$a(p, p-4) \equiv 9\ mod\ (2p-4) \qquad (7.1)$$

Proof: Using (2.1) we have

$$a(p, p-4) \equiv (p^2 + (p-4)^2 + 1)\ mod(2p-4)$$

$$\equiv (2p^2 - 8p + 16 + 1)\ mod(2p-4)$$

$$\equiv [2p(p-4) - 4p + 16 + 4p + 1]\ mod(2p-4)$$

$$\equiv [(2p-4)(p-4) + (4p+1)]\ mod(2p-4)$$

$$\equiv (4p - 8 + 9)\ mod\ (2p-4)$$

$$\equiv (2(2p-4) + 9)\ mod\ (2p-4)$$

$$\equiv 9\ mod\ (2p-4)$$

Thus, $a(p, p-4) \equiv 9\ mod\ (2p-4)$

■

8. Theorem 6

For $p \geq 8$,

$$a(p,2) \equiv 9\ mod\ (p+2) \quad (8.1)$$

Proof: Using (2.1) we have

$$a(p,2) \equiv (p^2 + 2^2 + 1)\ mod(p+2)$$

$$\equiv (p^2 + 4 + 4p - 4p + 1)\ mod(p+2)$$

$$\equiv ((p+2)^2 + 1 - 4p)\ mod(p+2)$$

$$\equiv (1 - 4p)\ mod\ (p+2)$$

$$\equiv (9 - 4p - 8)\ mod\ (p+2)$$

$$\equiv (9 - 4(p+2)) mod\ (p+2)$$

$$\equiv 9\ mod\ (p+2)$$

$$\text{Thus, } a(p,2) \equiv 9 \; mod \; (p+2)$$

■

9. Theorem 7

$$a(p,q) \equiv 0 \; mod(p+q) \; if \; and \; only \; if \; 2pq \equiv 1 \; mod(p+q) \quad (9.1)$$

Proof: Using (2.1) we have

$$a(p,q) \equiv \; (p^2+q^2+1) \; mod(p+q)$$

$$a(p,q) \equiv 0 \; mod(p+q) \Leftrightarrow (p^2+q^2+1) \equiv 0 \; mod(p+q)$$

$$\Leftrightarrow (p+q)^2+1 \equiv 2pq \; mod(p+q)$$

$$\Leftrightarrow 1 \equiv 2pq \; mod(p+q)$$

$$\Leftrightarrow 2pq \equiv 1 \; mod(p+q)$$

$$\text{Thus, } a(p,q) \equiv 0 \; mod(p+q) \Leftrightarrow 2pq \equiv 1 \; mod(p+q)$$

■

10. Theorem 8

$a(p,q)$ is a perfect square of the form m^2 if and only if $2pq+(m^2-1) \equiv 0 \; mod(p+q)$ for some integer m (10.1)

Proof: Using (2.1) we have

$$a(p,q) \equiv \; (p^2+q^2+1) \; mod(p+q)$$

$a(p,q)$ is a perfect square $\Leftrightarrow (p^2+q+1) \equiv m^2 \; mod(p+q)$

$$\Leftrightarrow (p^2+q^2+1)+2pq \equiv m^2+2pq \; mod(p+q)$$

$$\Leftrightarrow (p+q)^2+1 \; \equiv m^2+2pq \; mod(p+q)$$

$$\Leftrightarrow 2pq+(m^2-1) \equiv 0 \; mod(p+q)$$

$a(p.q)$ is a perfect square $\Leftrightarrow 2pq+(m^2-1) \equiv 0 \; mod(p+q)$

■

11. Conclusion

Using a basic number triangle (as shown in Figure 1), which is made up of whole numbers created in each row by modulo arithmetic operations, such that the p^{th} row has p whole numbers. In this work, we have proven eight fascinating theorems using the built number triangle's elements. These findings will aid in our comprehension of the composition and characteristics of the triangle under consideration. Similarly by considering suitable function similar to that of in (2.1), we can construct various other number triangles and investigate their properties. One can also consider mod 2, mod 3, and so on for the entries of the number triangle in Figure 1, and look out for some patterns obtained through such entries.

REFERENCES

[1] R. Sivaraman, J. Suganthi, P.N. Vijayakumar, R. Sengothai, Generalized Pascal's Triangle and its Properties, NeuroQuantology, Vol. 20, No. 5, 2022, 729 – 732.

[2] R. Sivaraman, On Some Properties of Leibniz's Triangle, Mathematics and Statistics, Vol. 9, No. 3, (2021), pp. 209 – 217.

[3] R. Sivaraman, Generalized Pascal's Triangle and Metallic Ratios, International Journal of Research – Granthaalayah, Volume 9, Issue 7, July 2021, pp. 179 – 184.

[4] R. Sivaraman, Number Triangles and Metallic Ratios, International Journal of Engineering and Computer Science, Volume 10, Issue 8, August 2021, pp. 25365 – 25369.

[5] R. Sivaraman, Fraction Tree, Fibonacci Sequence and Continued Fractions, International Conference on Recent Trends in Computing (ICRTCE – 2021), Journal of Physics: Conference Series, IOP Publishing, 1979 (2021) 012039, 1 – 10.

[6] R. Sivaraman, Polygonal Properties of Number Triangle, German International Journal of Modern Science, 17, September 2021, pp. 10 – 14.

[7] R. Sivaraman, Number Tree and its Applications, AUT AUT Research Journal, Volume XI, Issue VII, July 2020, pp. 79 – 85.

[8] M. Niqui, Exact arithmetic on the Stern - Brocot tree, Journal of Discrete Algorithms 5 (2) (2007), pp. 356-379.

[9] H. Lennerstad, L. Lundberg, Generalizations of the floor and ceiling functions using the Stern-Brocot tree, Research Report 2006:2, Blekinge Institute of Technology, Sweden, 2006.

[10] B.P. Bates, M.W. Bunder, K.P. Tognetti, Linkages between the Gauss map and the Stern_Brocot tree, Acta Mathematica Academiae Paedagogicae Nyíregyháziensis 22 (3) (2006) 217-235.

[11] A. Dinesh Kumar, R. Sivaraman, On Some Properties of Fabulous Fraction Tree, Mathematics and Statistics, Vol. 10, No. 3, (2022), pp. 477 – 485.

[12] R. Sivaraman, Generalized Lucas, Fibonacci Sequences and Matrices, Purakala UGC CARE Group I Journal, Volume 31, Issue 18, April 2020, pp. 509 – 515.

[13] A. Dinesh Kumar, R. Sivaraman, Analysis of Limiting Ratios of Special Sequences, Mathematics and Statistics, Vol. 10, No. 4, (2022), pp. 825 – 832,

[14] R. Sivaraman, Summing Through Triangle, International Journal of Mechanical and Production Engineering Research and Development, Volume 10, Issue 3, June 2020, pp. 3073 – 3080.

[15] Dinesh Kumar A, Sivaraman R. Ramanujan Summation for Pascal's Triangle. Contemp. Math. 2024 Feb. 29;5(1):817-25.