Journal of Modern Applied Statistical Methods

Volume 3 | Issue 2 Article 23

11-1-2004

Statistics and Technology: Reflections on 35 Years of Change

James J. Higgins
Kansas State University, jhiggins@ksu.edu

Journal of Modern Applied Statistical Methods
November, 2004, Vol. 3, No. 2, 567-575

1538 – 9472/03/$95.00

Statistics and Technology: Reflections on 35 Years of Change

James J. Higgins
Department of Statistics
Kansas State University

From the days when statistical calculations were done on mechanical calculators to today, technology has transformed the discipline of statistics. More than just giving statisticians the power to crunch numbers, it has fundamentally changed the way we teach, do research, and consult. In this article, I give some examples of this from my 35 years as an academic statistician.

Introduction

When I began my undergraduate studies at the University of Illinois in 1961, the state of the art hand-held calculating device was the slide rule. I paid twenty-five dollars for mine, which was a lot of money in those days. The first statistical calculations I did were on a mechanical calculator, and the first book that I taught out of had a table of square roots in the appendix. I've seen mainframe computer centers and punch cards come and go. Now, powerful personal computers are commonplace, and a large fraction of the population has access to the internet.

All of this has fundamentally changed the discipline of statistics. It has changed what we teach and how we teach it. It has given statistical research a genuine experimental side to go along with theory, and it has changed the role of the statistical consultant. I've chosen examples from my experiences to illustrate the changes that have occurred and in some cases to suggest directions that the discipline might go in the next decade or so.

The Introductory Pre-Calculus Undergraduate Course Past and Present

The approaches of two very successful authors, William Mendenhall and David Moore, capture the essence of the changes that have occurred in the introductory undergraduate statistics course.

Mendenhall began publishing introductory statistics books in the 1960s at the time when there was rapid growth in the demand for introductory statistics as a general education course. He successfully took material that was previously only accessible to students with calculus and brought it to the pre-calculus audience. His organization, which has been replicated by many authors, includes elementary descriptive statistics, axiomatic probability and probability distributions, and a systematic treatment of inference (e.g., one-sample, two-sample, regression, analysis of variance).

Moore's books epitomize the changes in thinking that began to take place in the late 1970's and 1980's. His *Statistics: Concepts and Controversies* (Moore, 2001a) begins with sampling and experimental design. It then has a discussion of descriptive statistics that includes

James J. Higgins is a Professor of Statistics at Kansas State University. He has published a textbook on stochastic modeling and probability and a textbook on nonparametric statistics. He is an elected Fellow of the American Statistical Association. Contact him at jhiggins@ksu.edu.

contingency tables, correlation, and simple linear regression all done without reference to statistical inference. Only enough probability is covered to deal with inference, and then just the basics of inference are discussed.

The analysis of real data has become the standard for good exposition. Students can be expected to have, at a minimum, a two-variable, hand-held calculator so that the drudgery of computing means, standard deviations, and regression equations is eliminated. Most books display and interpret elements of computer output, and many instructors expect students to be proficient with some statistical software. The student can now concentrate on what it means rather than on how to compute the answer.

Beyond Data Analysis

Although I do a lot of data analysis in my introductory courses, data analysis is not the most important thing I do as a consulting statistician. Rather it is in the planning of studies that I think I have the greatest impact. In DeGroot (1987), C. R. Rao had this to say:

> "I believe that the two great methodologies in statistics are sample surveys, which is essentially collecting existing information, and design of experiments, where you generate observations to provide information on some given questions. Different types of data analysis are, of course, then applied depending upon what the statistician thinks is the right thing to do. They are not as fundamental as the data which are collected through principles of design and sample surveys."

Box (1990) was critical of the notion that statistics is a branch of mathematics. He wrote, "Statistics is, or should be, about scientific investigation and how to do it better…" His commentary is very thought provoking.

The difficulty in trying to teach design, sampling, or better scientific investigation in an introductory undergraduate course is that most undergraduates haven't had the opportunity to be involved in the process of scientific discovery. At most they may have done laboratory demonstrations that illustrate some scientific principle. The majority will not have dealt with a problem where they don't know the answer and have to take data to find it out. Thus they do not appreciate the most important reason to learn statistics, namely, scientific discovery.

Many instructors assign projects that illustrate discovery through data. Some projects are short so that they can be done in class, but they lack the complexity of real studies. Others are more extensive, but must of necessity take several weeks or even a semester to complete, see Hunter (1977). Here is where we could take the next step in the use of technology. I would like to see us merge video-game technology with our ability to simulate data from scientific studies to come up with interesting software that would invite student to conduct their own experiments in a computer lab.

Imagine, for instance, software that would simulate agricultural experiments. Students might have several varieties of corn that they could choose to plant, several options of fertilizer to use, a choice of whether to irrigate or not, several ways to control pests, different environments in which to plant the corn, different plots of ground upon which to do the experiment, and several responses to consider such as yield, plant damage, plant dry weight and the like. With computer graphics and animation showing a researcher planting the corn, applying the treatments, and harvesting afterward (all controlled by the student), the software would invite students to plan and carry out a scientific study in a way not unlike they might do in practice. It would be rather like using a flight simulator to teach the basics of flying an airplane. Students could be presented with many different scenarios that could be dealt with in a safe environment before they are turned loose to deal with the real world of scientific investigation.

Planet X

A few years ago our department was given the opportunity to design a studio classroom for one of our large introductory courses. The classroom that we came up with has 20 computers which are arranged on octagonal tables where students work in pairs. There are lots of opportunities for students to interact with each other. Unlike a lab, students are in the classroom for every class period. My colleague Deb Rumsey designed the classroom, and we set about developing a curriculum that would take advantage of it.

For years we had been using computers to simulate data for class examples, but we wanted to try something more elaborate. We decided to build a large database representing characteristics of individuals who belong to some population. We wanted to put together a story to give interest to the database, and we wanted students to have a lot of flexibility in terms of what data they could collect and what questions they could ask. Finally, we wanted to put some graphical and animation elements into the program to give it visual appeal.

We considered modeling a small city perhaps using census data to populate our database. I think this has merits, but it presents some pedagogical problems, too. Students may have preconceived notions that would taint their analysis of the data. For instance if a student asks questions of the data about race or gender, their biases might not only affect their interpretation of the data, but they could also lead to a class discussion that goes beyond statistics and into the realm of sociology where the instructor may not wish to go.

Such concerns led us to create Planet X, a place that is like Earth but with differences to be discovered. There are 4 ethnic groups on Planet X, 50 cities, 9 governmental regions, costal and inland cites, etc. The database has 500,000 inhabitants with 31 variables on each one representing various physical and social characteristics. Students can sample from the entire population or from various sub-populations. Computer animation shows a spaceship flying off to Planet X and going into orbit around the planet. Our students make contact with the inhabitants, gather data, and fly back to Earth where they do the analysis and write a report.

The philosophy behind Planet X is contrary to the conventional wisdom that it takes real data to engage students in statistics. I believe that data just need to be engaging, and whether the data are real or simulated is immaterial. Some students enjoy Planet X a lot. Others think it is hokey. Many students with a little guidance write reasonably good reports about what they've found out from their data analysis. The fact that they have something concrete to write about gives focus to their writing. Evaluations indicate that students develop a level of comfort with survey methodology that we do not necessarily find in our traditional classes.

The impediment in developing this is having someone with the time and technical expertise in graphics and animation programming to do the work. We were fortunate to have someone who knew enough about this to get something to work at K-State although it proved not to be transportable to other locations for various technical reasons. Ultimately it will take professional software developers to put together a sufficiently complex set of simulated scientific studies to make possible a true test of the usefulness of this type of technology in introductory statistics courses. I will simply offer the opinion that the potential there.

Statistics as an Undergraduate Discipline

Once, at a seminar by a statistician from the pharmaceutical industry, I asked the speaker whether his company hired undergraduate statistics majors to manage the large databases that his company maintains. He admitted that although they might do that, most of those they hired had little statistics background. His company simply hired those that had some computing. I thought what a lost opportunity not only for the company but also for statistics as an undergraduate discipline.

With a few notable exceptions, statistics lacks visibility as an undergraduate discipline in colleges and universities. See Minton (1983). Having taught in the Florida university system for 6 years, I noted with dismay that the new Florida Gulf Coast University, which was established in 1997, did not have a statistics program; let alone a statistics department. Even though we tout the importance of statistics in the information age, statistics wasn't even a blip on the radar screen of this modern university.

Part of this has to do with how statistics departments came into being. Almost all began at major universities with the primary mission to produce M.S. and Ph.D. statisticians. See Bancroft, et al. (1958) for an account of the state of the statistics profession in the 1950s. Entry to graduate school in statistics even today does not require an undergraduate degree in statistics.

Because statistics does not have a tradition as an undergraduate discipline in the same sense that mathematics does, there is not a clearly defined notion of what an undergraduate program in statistics about. This has troubled me for some time. What ideas and coursework are at the core of undergraduate statistics? Can these ideas be successfully taught in mathematics departments or departments of mathematical sciences where the majority of the undergraduate statistics programs now reside? What coursework would make a career path for the undergraduate statistics major? In the article "Nonmathematical Statistics: A New Direction for the Undergraduate Discipline", I attempted to answer these questions (Higgins, 1999).

Nonmathematical activities are very much a part of what a practicing statistician does and what customers of statistics need. They include things like managing large databases, planning studies in a team-oriented environment, ensuring protocol compliance, providing internet access to databases, and providing descriptive and graphical summaries of data (apart from the usual inferential statistics). I suggested eight courses that deal with these things that are not courses that would fit well within a traditional mathematics or mathematical sciences program. The titles are listed below. The article elaborates on the topics.

(1) The Scientific Process
(2) Planning and Managing Surveys
(3) Planning and Managing Scientific Studies
(4) Statistical Software for Data Management
(5) Statistical Graphics
(6) Computer Science in Statistics
(7) Communicating Statistical Ideas
(8) Management Principles for Statistics

These courses along with courses in inference could form the basis for a professional degree program in statistics. Students with this type of coursework could serve as "data specialists". It is not difficult to find job descriptions in industry, business, and government that require the skills of a data specialist. The very technology that enables these organizations to gather massive amounts of data also creates a potential bonanza of opportunities for the undergraduate statistics major with the right type of education.

There have been some good efforts to bring a common understanding of what the undergraduate statistics degree program is about. Articles by Bryce, et al. (2001) and Ritter, et. al. (2001) represents one of the latest attempts to do this. These articles made recommendations about the curriculum some of which incorporated ideas from my 1999 article. In the same issue, Moore (2001b) took on a different problem: how to grow undergraduate programs. His conclusion was that economic considerations compel statistics and mathematics to work together.

If Moore is correct, then undergraduate statistics cannot reach its full potential. The mathematical discipline by its very definition is not structured to support the kinds of nonmathematical courses that a professional undergraduate statistics program would need. The best that we could hope for in this case is that statistics would be a liberal arts degree option that could be fulfilled by students getting a degree in mathematics and taking a handful of courses in statistics.

Distance Education

Since the days of radio, colleges and universities have had some form of distance education. Kansas State University for years supported a radio station as part of its agricultural outreach. Modes of delivery have evolved from radio, to television, to video courses on demand, and finally to the internet.

In 1989, I was asked to have my introductory graduate methods course video taped to be used as a distance education course for a program for food inspectors. The course was taped in a special classroom that had a camera at the back of the room. I just did my thing teaching as I always would. The only concessions that I made for the camera were that I wrote with big chalk on the board so that my writing would show up on camera, and I wore long-sleeved shirts and ties. The production was very primitive, but it was also cost effective to produce which is an important consideration in putting together distance education material.

As it turned out, an educational television company that had national cable outlets obtained rights to the course for their distance education degree programs. Soon after that I began hearing from people from around the country who saw the course. My sister, who lives in Illinois and who did not know that I had done this, was clicking through the channels one evening when I came on the screen. Needless to say she was surprised. To my amazement those that saw the course seemed to like it. It found an audience that also surprised me: graduate students in nursing programs. Even though I talked about pigs, cows, wheat, and corn the course met a need for these students.

Later I redid the course for the internet. Here I made three choices that turned out to be right even though I had no previous experience with this form of teaching.

(1) I decided to make the presentation "linear" as I would in an ordinary classroom setting. I avoided the temptation to put in a lot of links and connections that would allow students to roam around and get away from the central flow of the material. I reasoned that if I were placed in the middle of a forest, I would not want to be given a lot of options for getting out, some of which might be dead ends. Rather I would like for someone to point to a single path as the way to go knowing at the end that I would be out of the forest.

(2) I divided the material that I would ordinarily teach in one class period into two parts, each with its concepts, reading assignments, and homework problems. Students have told me that they like this feature a lot.

(3) I presented the material in detailed outline form using PowerPoint slides rather than writing an online text. This allowed me to put in graphics and gizmos to give the pages some visual appeal while making the essential points as succinctly as I could. I require a textbook that students can refer to if they need additional explanation.

I have had over 1,500 students take this course. It is self-paced although I encourage students to finish within the semester that they sign up. We have 40 or more students a semester sign up for the course, and we offer it fall, spring, and summer. I use the Excel spreadsheet for computing because most students have access to it although I am well aware of its limitations. It is very satisfying knowing that this course is accepted by many universities and colleges around the country. I'm sure that a major part of the success of the course is the high level of motivation of the students who take it. I recently developed an undergraduate internet course for business majors. It is too early to tell how well my style will work with these students.

The use of the internet technology comes at a price. It took me over a year working part-time to develop each of my courses. Thus, internet courses are only cost effective if they can be rerun several times. I would not recommend anyone doing this without extra compensation or release time to do the work. Our department is reimbursed for my time by the Division of Continuing Education. Some of that comes to me indirectly as discretionary funds that I can use for travel, computer equipment, graders, and the like.

Mathematics, Computing, and Research

I took a pretty good dose of probability, analysis, and measure theory to go along with my statistics Ph.D. coursework. My dissertation was "Convergence Rates for Weighted Sums of Independent Random Variables" under Dave Hanson's direction. I chose the University of Missouri at Columbia in large part because I thought I would get a good background in mathematics to go along with statistics, and it has served me well even though now I consider myself to be an applied statistician. It was at the University of South Florida that I got my first significant exposure to applied research. Chris Tsokos, to whom I owe a great deal, directed me toward reliability theory which is an area that I have worked in since.

I've seen less emphasis on mathematics in statistics Ph.D. programs over the years. When I was in graduate school some version of measure theory was rather standard for Ph.D. students. Now I would say that it is far less common. I'm not sure whether this is good or not. Jacob Wolfowitz, who spent his last years at the University of South Florida, made it clear to me at one particular meeting of the curriculum committee that our students needed more mathematics not more applied statistics. I was never completely convinced by that, but who was I to argue.

What I think I can safely say is that computing has changed our expectations of what constitutes research in statistics. Tukey (1986) had this witty but profound insight about the role of computing:

> "In a world in which the price of calculation continues to decrease rapidly, but the price of theorem proving continues to hold steady or increase, elementary economics indicates that we ought to spend a large and larger fraction of our time on calculation."

In the same sense that physics has theoretical and experimental sides, statistics has these two sides too thanks to the capability to do computer simulations.

One of the courses that I took in my first year at Missouri was a course on computer simulation taught by Bill Bulgren. Although I have to confess that I was, and still am, a lousy programmer, I was really taken by the power of the Monte Carlo method to readily provide answers to difficult questions that could not be touched with standard analytical methods. The ideas that I learned in that course have influenced my research and teaching throughout my career.

A number of my papers have dealt with small sample properties of statistical methods, something that can be investigated with well-designed computer simulation studies. Work by Blair and Higgins (1980) shed light on some long-standing misconceptions about the power of nonparametric methods in the social and behavioral sciences. Specifically, an influential paper by Glass et al, (1972) concluded that nonparametric methods have low power and are not suitable for serious data analysis. Nothing could be further from the truth as asymptotic theory shows, but unfortunately even today these wrong ideas persist. Ironically the wrong ideas about rank tests arose from poorly designed simulation studies.

Advances in statistical methodology often involve the interplay of applications, experimental statistics, and theoretical statistics. The rank-transform methodology, which was first proposed by Iman, as a student of Conover, at Kansas State University, is such an example. At first it seemed to hold promise an easy way to do nonparametric statistics for the types of designed experiments that one typically encounters in practice. Simply replace observations by ranks and do the same linear models analysis on ranks that one would do on normally distributed data. See Conover and Iman (1981) for an overview. Unfortunately, the simulations that supported its use did not pick up problems in testing for interaction in factorial experiments. Simulations studies such as Sawilowsky, et al. (1989) and theoretical studies such as Thompson (1991) showed the deficiencies. Akritas and Arnold (1994) clarified the nonparametric hypotheses actually tested by the rank-transform methodology. The research has come full circle for our department as we just hired one of Akritas's students, Haiyan Wang, who is doing research along these lines.

Textbook Writing

Technology has had a significant effect on the content of my two textbooks, not to mention the fact that without a word processor I would never have had the patience to write the books.

My first book written jointly with Sallie Keller-McNulty was *Concepts in Probability and Stochastic Modeling* (Higgins & Keller-McNulty, 1995). Sallie, who was recently elected president of the American Statistical Association, was an M.S. student of mine at the University of South Florida and a colleague at Kansas State prior to becoming head of the statistics group at Los Alamos Laboratories. Our book came from a course that we developed for our computer science department. We decided to use modeling rather than inference as the theme around which to organize the material. In particular we included Markov chains and some elementary queuing theory in the course and did so early enough that it would not be treated as an after thought.

To make topics like this accessible to students who were not strong mathematically but who had programming skills, we made computer simulation of random events a key feature of the book. With this, one can ask students to investigate empirically some rather mathematically complicated random phenomena. For instance, it is a trivial matter to examine the zero crossings of a random walk by simulating 5000 or 10,000 tosses of a coin. Students find it surprising that so few crossings occur. One can approximate M/M/k queuing processes by simulating what we call Bernoulli queuing processes. Again the programming is nothing more than simulating tosses of multiple biased coins. Moreover with very little modification one can simulate non-homogeneous queuing processes and other rather complex systems. I now do the programming for the course with a spreadsheet where I not only can generate the data but graph it as well.

My other book *Introduction to Modern Nonparametric Statistics* (2004) was written for our nonparametric methods course. The audience is undergraduates and beginning graduate students in statistics and students from other areas, primarily biology, who need nonparametric methods for their research. Here again computing had a great deal to do with the approach that I took.

Many of the methods under the heading of nonparametric statistics are variations of permutation tests. The Wilcoxon rank-sum test with or without ties, the signed-rank test, the Kruskal-Wallis test, the Spearman test for correlation, the log-rank test for censored data, and various exact tests for contingency tables use some form of a permutation distribution of a statistic as the reference distribution for determining significance levels. The StatXact software, which came out in the late 1980s, was the first to exploit this in a comprehensive way.

My choice of topics for the book goes quite a bit beyond traditional rank tests, but I believe this is in keeping with a broader understanding of what now constitutes nonparametric statistics. Where possible I presented methods as special cases of permutation tests applied to scores. To deal with more complicated data structures, I included some bootstrap methods and a brief treatment of the rank-based, robust methods of Hettmansperger and McKean (1998).

Software is now catching up with the theory of nonparametric statistics although there is still a ways to go. In the early days, nonparametric methods were thought of as quick hand calculation methods suitable only for small data sets, but in fact many of the methods are computationally intensive. I believe that we are poised to see a rapid growth in the use of nonparametric methods now that exact methods and bootstrap methods are being included in several popular software packages. Scott Richter and I are working on a book that shows how to implement many of the popular nonparametric methods in SAS.

Consulting

I hold a joint appointment at Kansas State University with the College of Arts and Sciences and the College of Agriculture. For the agriculture part of my appointment I am one of six statistical consultants for Kansas State Research and Extension, formerly the Agricultural Experiment Station. Consulting is an integral part of what our department does, and even those who don't hold consulting appointments often are involved in consulting projects. It has been the source of research problems, classroom examples, and textbooks including the popular book *Analysis of Messy Data* by Milliken and Johnson (1984).

In the 1980's a large part of our consulting centered on statistical computing. We had a large computing lab, and most of those who needed statistical computing came to the lab to get their work done. Researchers now do their own computing on their desktop or laptop computers, and computer software supports more methods than ever before. This is both good and bad. It is good because statisticians can focus their efforts at the planning stages of a study as they should. It is bad because even good researchers may choose the wrong method for their analysis, and the statistician is not there to catch the error.

Because of the changing consulting role, the notion that a statistician is someone who provides statistical computing services at the behest of a client is not as prevalent as it once was. Most of the projects that I now deal with involve substantial issues of experimental design or sampling. It is not uncommon for me to receive credit for my contribution by being included as a co-author on scientific papers. This is a significant change in the way things were when I first began consulting.

I must comment on the controversial issue of how to evaluate the contribution of the academic consulting statistician. Is it service or is it research? In most cases the significant contribution is not in the methods that end up being used. These are often standard. Rather the contribution comes when the consulting statistician is able to recast the applied problem in such a way that it becomes apparent what methods should be applied. Even very good researchers in content areas have difficulty doing this. We should not discount the contribution of the statistician as mere service just because he or she has the education and experience to get it right. Many areas have a tradition of multiple-author papers and give due credit for them. In my opinion, we should do the same in statistics.

The Future

I don't suppose that statisticians as a group are any better equipped to discern the future than anyone else. If anything we are perhaps more cautious than most knowing the uncertainties inherent in extrapolating too far beyond the data. Thus, let me just offer an observation that many others have made. The ability of technology to produce huge amounts of high-dimensional data presents challenges for statisticians that cannot all be met with the methods that we now have. The need is apparent in such areas as engineering, genetics, space exploration, medicine, retailing, and homeland security. Even something as basic as creating data archives that can be accessed in a variety of usable forms presents significant technological and organizational challenges. In agricultural research, for instance, lack of data archiving results in a tremendous loss of information as data are discarded or lost after experiments are done and results are published. Whatever may emerge, methods for managing and analyzing large, high-dimensional databases will become increasingly important to society and one would hope to the discipline of statistics.

References

Akritas, J. G., Arnold, S. F. (1994). Fully nonparametric hypotheses for factorial designs I: Multivariate repeated measures designs. Journal of the American Statistical Association, *89*, 336-343.

Bancroft, T. A., Futransky, D., L., Harshbarger, B. (1958). Demand and supply of statisticians, now and in the future. *The American Statistician*, *12*, 20-24.

Blair, R. C., Higgins, J. J. (1980). A comparison of the power of Wilcoxon's rank-sum test to that of Student's t statistic under various nonnormal distributions. *Journal of Educational Statistics*, *5*, 309-335.

Box, G. (1990). Commentary. *Technometrics*, *33*, 251-252.

Bryce, G.R., Gould, R., Notz, W. I. (2001). Curriculum guidelines for bachelor of science degrees in statistical science. *The American Statistician*, *55*, 7-13.

Conover, W. J., Iman, R. L. (1981). Rank transformations as a bridge between parametric and nonparametric statistics. *The American Statistician*, *35*, 124-129.

DeGroot, Morris H. (1987). A conversation with C.R.Rao. *Statistical Science*, *2*, 53-67.

Glass, G. V., Peckham, P. D., Sanders, J.R. (1972). Consequences of failure to meet assumptions underlying the fixed effects analysis of variance and covariance. *Review of Educational Research*, *42*, 237-288.

Hettmansperger, T. P., McKean, J. W. (1998). *Robust Nonparametric Statistical Methods*. Arnold, London.

Higgins, J. J., Keller-McNulty, S. (1995). *Concepts in Probability and Stochastic Modeling*. Duxbury Press, Belmont, CA.

Higgins, J. J. (1999). Nonmathematical statistics: a new direction for the undergraduate discipline. The *American Statistician*, *53*, 1-6.

Higgins, J. J. (2004). *An Introduction to Modern Nonparametric Statistics*. Thompson-Books/Cole, Pacific Grove, CA.

Hunter, W. G. (1977). Some ideas about teaching design of experiments with 2^5 examples of experiments conducted by students. *The American Statistician*, *34*, 140-145.

Milliken, G. A., Johnson,D.E. (1984). *Analysis of Messy Data, Volume I: Designed Experiments.* Chapman & Hall, London.

Minton, P. D. (1983). The visibility of statistics as a discipline. *The American Statistician*, *72*, 284-289.

Moore, D. S. (2001a). *Statistics: Concepts and Controversies, 5th Edition*. W.H Freeman, New York.

Moore, D. S. (2001b). Undergraduate programs and the future of academic statistics. The *American Statistician*, *55*, 1-6.

Ritter, M. A., Starbuck, R. R., Hogg, R.V. (2001). Advice from prospective employers on training BS statisticians, 55, 14-18.

Sawilowsky, S. S., Blair, R. C., Higgins, J. J. (1989). An investigation of the type I error and power properties of the rank transform procedure in factorial ANOVA. *Journal of Educational Statistics*, *14*, 255-267.

Thomson, G. L. (1991). A unified approach to rank tests for multivariate and repeated measures designs. *Journal of the American Statistical Association*, *86*, 410-419.

Tukey, J. (1986). Sunset salvo. *The American Statistician*, *40*, 72-76.