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Monitoring structural change is performed not by hypothesis testing but by model selection using a modified Bayesian information criterion. It is found that concerning detection accuracy and detection speed, the proposed method shows better performance than the hypothesis-testing method. Two advantages of the proposed method are also discussed.

Key words: Modified Bayesian information criterion, model selection, monitoring, structural change

Introduction

Deciding whether a time series has a structural change is tremendously important for forecasters and policymakers. If the data generating process (DGP) changes in ways not anticipated, then forecasts lose accuracy. In the real world, not only historical analysis but also real-time analysis should be performed, because new data arrive steadily and the data structure changes gradually. Given a previously estimated model, the arrival of new data presents the challenge of whether yesterday's model can explain today's data. This is why real-time detection of structural change is an essential task. Such forward-looking methods are closely related to the sequential test in the statistics literature but receive little attention in econometrics except for Chu, Stinchcombe, and White (1996) and Leisch, Hornik, and Kuan (2000).

Chu et al. (1996) has proposed two tests for monitoring potential structural changes: the fluctuation and CUSUM monitoring tests. In their fluctuation test, when new observations are obtained, estimates are computed sequentially from all available data (historical and newly obtained sample) and compared to the estimate based only on the historical sample. The null hypothesis of no change is rejected if the difference between these two estimates becomes too large. One drawback of their test is however that it is less sensitive to a change occurring late in the monitoring period.

Leisch et al. (2000) proposed the generalized fluctuation test which includes the fluctuation test of Chu et al. (1996) as a special case and shown that their tests have roughly equal sensitivity to a change occurring early or late in the monitoring period. Two drawbacks of their test are however that there is no objective criterion in selecting the window sizes, and that it has low power in the case of small samples.

In this article, a model-selection-based monitoring of structural change is presented. The existence of structural change is examined, not by hypothesis testing but by model selection using a modified Bayesian information criterion proposed by Liu, Wu, and Zidek (1997). Liu et al. (1997) presented segmented linear regression model and proposed model-selection method in determining the number and location of changepoints. Their criterion has been applied to examine what happened in historical data sets while it has not been applied to examine what happens in real time.

Therefore this criterion is applied to monitor structural change. In this method, whether the observed time series contains a structural change is determined as a result of model selection from a battery of alternative models with and without structural change.

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Another contribution of this article is the introduction of minimum length of each segment (L). Liu et al. (1997) pay little attention to this topic and make the minimum length equivalent to the number of explanatory variables. This possibly leads to over-fit problem in samples. In order to overcome this problem, $L = 10$ is set arbitrarily and practically in simulations and obtain better performance than the Liu et al. method.

The rest of the article is organized as follows. The hypothesis- testing method and the model-selection method are reviewed briefly. Next, simulation results are shown to illustrate the efficacy of the proposed method. Finally, conclusions and discussions are presented.

Methodology

Leisch et al. (2000) hypothesis-testing method considered the following regression model

$$y_i = x_i'\beta_i + \varepsilon_i,\ i = 1,...,T, T+1,..., \tag{1}$$

where x_i is the $n \times 1$ vector of explanatory variables, and ε_i is a i.i.d. disturbance term. Suppose an economist is currently at time T and has observed historical data $(y_i, x_i')', i = 1,...,T$. He takes as given that the parameter vector β_i was constant and unknown historically. Consider testing the null hypothesis that β_i remains constant against the alternative that β_i changes at some unknown point in the future.

Leisch et al. (2000) first considered tests based on recursive estimates and show that Chu et al. (1996) fluctuation test is a special case of this class of tests. They write the Chu et al. fluctuation test as

$$\max - RE_T(\tau) = \max_{k=T+1,...,[T\tau]} \frac{k}{\hat{\sigma}_T \sqrt{T}} \left\| Q_T^{1/2}(\hat{\beta}_k - \hat{\beta}_T) \right\| \tag{2}$$

where $\hat{\beta}_k = \left(\sum_{i=1}^{k} x_i x_i' \right)^{-1} \sum_{i=1}^{k} x_i y_i$,

$Q_T = \frac{1}{T}\sum_{i=1}^{T} x_i x_i'$, $\hat{\sigma}_T^2 = \frac{1}{T}\sum_{t=1}^{T} (y_t - x_t'\hat{\beta}_T)^2$, and $\|\bullet\|$ denotes the maximum norm. The period from time $T+1$ through $[T\tau]$, $\tau > 1$, is the expected monitoring period. For a suitable boundary function q,

$$\lim_{T\to\infty} P\left\{ \begin{array}{l} \frac{k}{\hat{\sigma}_T \sqrt{T}} \left\| Q_T^{1/2}(\hat{\beta}_k - \hat{\beta}_T) \right\| < q(k/T), \\ for\ all\ T+1 \le k \le [T\tau] \end{array} \right\}$$

$$= P\left\{ \left\| W^0(t) \right\| < q(t),\ for\ all\ 1 \le t \le \tau \right\}, \tag{3}$$

where W^0 is the generalized Brownian bridge on $[0, \infty]$, as shown by Chu et al. (1996), and

$$q(t) = \sqrt{t(t-1)[a^2 + \log(\frac{t}{t-1})]}, \tag{4}$$

where $t = k/T$. The limiting distribution of $\max - RE_T(\tau)$ is thus determined by the boundary crossing probability of W^0 on $[1, \tau]$. Choosing $a^2 = 7.78$ and $a^2 = 6.25$ gives 95% and 90% monitoring boundaries, respectively.

Leisch et al. (2000) next considered tests based on moving estimates. Define the moving OLS estimates computed from windows of a constant size $[Th]$, where $0 < h \le 1$ and $[Th] > n$, as

$$\tilde{\beta}_T(k, [Th]) = \left(\sum_{i=k-[Th]+1}^{k} x_i x_i' \right)^{-1} \sum_{i=k-[Th]+1}^{k} x_i y_i,$$
$$k = [Th], [Th]+1, ... \tag{5}$$

They propose tests on the maximum and range of the fluctuation of moving estimates:

$$\max-ME_{T,h}(\tau) = \max_{k=T+1,\ldots,[T\tau]} \frac{[Th]}{\hat{\sigma}_T\sqrt{T}} \left\| Q_T^{1/2}(\tilde{\beta}_T(k,[Th]) - \hat{\beta}_T) \right\|, \tag{6}$$

$$range-ME_{T,h}(\tau) = \max_{i=1,\ldots,n} \frac{[Th]}{\hat{\sigma}_T\sqrt{T}} \left(\max_{k=T+1,\ldots,[T\tau]} [Q_T^{1/2}(\tilde{\beta}_T(k,[Th]) - \hat{\beta}_T)]_i - \min_{k=T+1,\ldots,[T\tau]} [Q_T^{1/2}(\tilde{\beta}_T(k,[Th]) - \hat{\beta}_T)]_i \right). \tag{7}$$

The following asymptotic results are obtained:

$$\lim_{T\to\infty} P\left\{ \frac{[\mathrm{Th}]}{\hat{\sigma}_{\mathrm{T}}\sqrt{T}} \left\| Q_T^{1/2}(\tilde{\beta}_T(k,[Th]) - \hat{\beta}_T) \right\| < z(h)\sqrt{2\log_+(k/T)}, \text{ for all } T+1 \le k < [T\tau] \right\} = [F_1(z(h),\tau)]^n ; \tag{8}$$

$$\lim_{T\to\infty} P\left\{ \max_{i=1,\ldots,n} \frac{[Th]}{\hat{\sigma}_T\sqrt{T}} \left(\max_{k=T+1,\ldots,J} [Q_T^{1/2}(\tilde{\beta}_T(k,[Th]) - \hat{\beta}_T)]_i - \min_{k=T+1,\ldots,J} [Q_T^{1/2}(\tilde{\beta}_T(k,[Th]) - \hat{\beta}_T)]_i \right) < z(h)\sqrt{2\log_+(J/T)}, \text{ for all } T+1 \le J < [T\tau] \right\} = [F_2(z(h),\tau)]^n ; \tag{9}$$

where $\log_+ t = 1$ if $t \le e$, $\log_+ t = \log t$ if $t > e$. In contrast with the boundary-crossing probability of (4), the $F_i(i=1,2)$ do not have analytic forms. Nevertheless, the critical values $z(h)$ can be obtained via simulations, and some typical values are shown in Leisch et al. (2000).

Liu et al. model-selection method

Liu et al. (1997) considered the following segmented linear regression model

$$y_t = x_t\beta_i + \varepsilon_t,\ t = T_{i-1}+1,\ldots,T_i,\ i = 1,\ldots,m+1, \tag{10}$$

where $T_0 = 0$ and $T_{m+1} = T$.

The indices $(T_1,\ldots,T_m)$, or the changekpoints, are explicitly treated as unknown. In addition, the following conditions are newly imposed:

$$T_i - T_{i-1} \ge L \ge n \text{ for all } i\,(i = 1,\ldots,m+1). \tag{11}$$

Changepoints too close to each other or to the beginning or end of the sample cannot be considered, as there are not enough observations to identify the subsample parameters.

In subsequent simulations, comparisons are made between $L=1$ and $L=10$ in the case of $n=1$, and it is concluded that the latter shows better performance than the former.

The purpose of this method is to estimate the unknown parameter vector β_i together with the changepoints when T observations on y_t are available. Their estimation method is based on the least-squares principle. The estimates of the regressive parameters and the changepoints are all obtained by minimizing the sum of squared residuals

$$S_T(T_1,\ldots,T_m) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} (y_t - x_t\beta_i)^2 . \tag{12}$$

Liu et al. (1997) estimated m, the number of changepoints, and $T_1,\ldots,T_m$, by minimizing the modified Schwarz's criterion (Schwarz 1978)

$$\mathrm{LWZ} = T\ln S_T(\hat{T}_1,\ldots,\hat{T}_m)/(T-q) + qc_0(\ln(T))^{2+\delta_0} \tag{13}$$

where $q = n(m+1)+m$, and c_0 and δ_0 are some constants such as $c_0 > 0$ and $\delta_0 > 0$. Liu et al. (1997) recommended using $\delta_0 = 0.1$ and $c_0 = 0.299$; here small simulations are implemented to examine how the structural change detection is affected by changing these two parameter values in the next Section. This criterion is an extended version of Yao (1988) such as

$$\text{YAO} = T \ln S_T(\hat{T}_1,...,\hat{T}_m)/T + q\ln(T) . \tag{14}$$

So, LWZ and YAO differ in the severity of their penalty for overspecification. In general, in model selection, a relatively large penalty term would be preferable for easily identified models. A large penalty will greatly reduce the probability of overestimation while not unduly risking underestimation. Because the optimal penalty is model dependent, however, no globally optimal pair of (c_0, δ_0) can be recommended.

In subsequent simulations, some alternative pairs of (c_0, δ_0) are considered and compared in selecting structural change models. In the model-selection method using the LWZ criterion in the case of possibly one structural change, for example, the following procedure is carried out. First, the OLS estimation for no structural change model ($m = 0$ in equ. 10) is performed, and the LWZ value is stored. Next the OLS estimations for one structural change models obtained by changing the changepoint on the condition of (11) are carried out, and the LWZ values are stored. Finally, the best model is selected using the minimum LWZ procedure from alternative models with and without structural change.

Results

Historical analysis of the structural change using the Liu et al. criterion.

Liu et al. recommended setting the parameters in their information criterion as $\delta_0 = 0.1$ and $c_0 = 0.299$, but they have not shown the efficacy of these parameter values via simulations in which several alternative pairs of (c_0, δ_0) are considered. Such simulations are implemented. The following two DGPs are considered:

DGP 1: $y_t = 2 + e_t,\ t = 1,...,T,$

DGP 2: $y_t = 2 + e_t$ if $t \le T/2$, $y_t = 2.8 + e_t$ if $t > T/2$,

where e_t is generated from i.i.d. $N(0,1)$. Considered are historical samples of sizes $T = 50$, 100, 200, 400, $L = 1$, 10, $c_0 = 0.01$, 0.05, 0.1, 0.3, 0.5, and $\delta_0 = 0.01$, 0.05, 0.1, 0.2. The number of replications is 1,000.

Table 1 shows frequency counts of selecting structural change models using the Liu et al. information criterion. First consider comparing the performances between two pairs of $(c_0 = 0.1, \delta_0 = 0.05)$ and $(c_0 = 0.299, \delta_0 = 0.1)$.

The former significantly outperforms the latter, particularly in the structural-change cases of $T = 50$ and 100. The pair of $(c_0 = 0.299, \delta_0 = 0.1)$ imposes too heavy penalty to select structural change models correctly. Next consider comparing between $L = 1$ and $L = 10$. The latter outperforms the former, particularly in small samples of $T = 50$ and 100. In the case of $L = 1$, it happens to occur that a structural change is incorrectly detected in the beginning or end of the sample.

Monitoring structural change via the Leisch et al. simulations

In Leisch et al. (2000), the DGP for examining empirical size is the same as DGP 1. They show the performances of $\max - RE$, $\max - ME$, and $range - ME$ tests and consider moving window sizes $h = 0.25$, 0.5, 1, and $\tau = 10$ for the expected monitoring period $[T\tau]$.

However, the DGP for examining empirical power is not the same as DGP2. The mean changes from 2.0 to 2.8 at $1.1T$ or $3T$. Similarly to Leisch et al. (2000), only the results for the 10% significance level are reported. All experiments were repeated 1,000 times.

Table 1

Frequency counts of selecting structural change models

L	T	c_0	δ_0 for DGP1				δ_0 for DGP2			
			0.01	0.05	0.1	0.2	0.01	0.05	0.1	0.2
1	50	0.01	0.590	0.585	0.583	0.576	0.968	0.968	0.968	0.968
1	50	0.05	0.379	0.360	0.347	0.305	0.919	0.913	0.908	0.893
1	50	0.1	0.192	0.172	0.155	0.128	0.826	0.817	0.798	0.746
1	50	0.3	0.024	0.018	0.012	0.008	0.330	0.301	0.266	0.200
1	50	0.5	0.001	0.000	0.000	0.000	0.080	0.068	0.055	0.028
1	100	0.01	0.653	0.647	0.639	0.623	0.999	0.999	0.999	0.999
1	100	0.05	0.360	0.348	0.327	0.269	0.994	0.994	0.992	0.987
1	100	0.1	0.147	0.137	0.112	0.070	0.959	0.951	0.946	0.925
1	100	0.3	0.003	0.002	0.001	0.001	0.591	0.530	0.456	0.349
1	100	0.5	0.001	0.000	0.000	0.000	0.183	0.142	0.112	0.056
1	200	0.01	0.697	0.692	0.677	0.656	1.000	1.000	1.000	1.000
1	200	0.05	0.297	0.275	0.244	0.186	1.000	1.000	1.000	1.000
1	200	0.1	0.077	0.064	0.047	0.026	0.999	0.999	0.999	0.997
1	200	0.3	0.000	0.000	0.000	0.000	0.906	0.881	0.843	0.707
1	200	0.5	0.000	0.000	0.000	0.000	0.514	0.430	0.331	0.202
1	400	0.01	0.710	0.703	0.684	0.660	1.000	1.000	1.000	1.000
1	400	0.05	0.271	0.240	0.202	0.136	1.000	1.000	1.000	1.000
1	400	0.1	0.047	0.034	0.026	0.014	1.000	1.000	1.000	1.000
1	400	0.3	0.000	0.000	0.000	0.000	0.999	0.999	0.998	0.992
1	400	0.5	0.000	0.000	0.000	0.000	0.952	0.924	0.868	0.706
10	50	0.01	0.368	0.366	0.362	0.352	0.933	0.932	0.931	0.930
10	50	0.05	0.212	0.206	0.200	0.176	0.869	0.863	0.851	0.840
10	50	0.1	0.111	0.101	0.091	0.075	0.763	0.745	0.726	0.694
10	50	0.3	0.009	0.008	0.008	0.003	0.344	0.315	0.279	0.212
10	50	0.5	0.001	0.001	0.001	0.001	0.101	0.080	0.059	0.028
10	100	0.01	0.461	0.457	0.449	0.434	0.993	0.993	0.992	0.992
10	100	0.05	0.230	0.217	0.200	0.164	0.980	0.977	0.974	0.969
10	100	0.1	0.091	0.078	0.068	0.051	0.948	0.943	0.930	0.907
10	100	0.3	0.001	0.000	0.000	0.000	0.552	0.504	0.446	0.342
10	100	0.5	0.000	0.000	0.000	0.000	0.157	0.132	0.094	0.039
10	200	0.01	0.550	0.539	0.531	0.512	1.000	1.000	1.000	1.000
10	200	0.05	0.226	0.210	0.191	0.147	1.000	1.000	0.999	0.999
10	200	0.1	0.071	0.058	0.046	0.028	0.999	0.999	0.999	0.997
10	200	0.3	0.001	0.001	0.000	0.000	0.900	0.875	0.840	0.722
10	200	0.5	0.000	0.000	0.000	0.000	0.514	0.423	0.320	0.176
10	400	0.01	0.596	0.587	0.580	0.552	1.000	1.000	1.000	1.000
10	400	0.05	0.176	0.155	0.124	0.094	1.000	1.000	1.000	1.000
10	400	0.1	0.033	0.026	0.019	0.006	1.000	1.000	1.000	1.000
10	400	0.3	0.000	0.000	0.000	0.000	0.999	0.999	0.997	0.981
10	400	0.5	0.000	0.000	0.000	0.000	0.942	0.918	0.869	0.696

One fundamental difference between the Leisch et al. method and the proposed method is whether the changepoint is estimated. In the Leisch et al. method, the changepoint estimation cannot be performed. In order to do so, another step is needed. As in Chu et al. (1996), for example, it is possible to define the changepoint by the point at which the maximum of the LR statistics is obtained for the period from the starting point to the first hitting point. In contrast, the proposed method presents not only the first hitting point but also the changepoint simultaneously. This is because in the proposed method, from a battery of alternative models obtained by changing the changepoint on the condition of (11), including no structural change model, the best model is selected in each monitoring point. Therefore, the proposed method is very computer intensive.

Table 2 shows frequency counts of selecting structural change models. In the LWZ criterion used are a pair of $(c_0 = 0.1, \delta_0 = 0.05)$, considering the results of the preceding simulation results. In the cases of no structural change, the YAO criterion ($L = 1$ and $L = 10$) and the LWZ criterion ($L = 1$) show poor performance. In contrast, the performance of the LWZ criterion ($L = 10$) is comparable to other hypothesis-testing methods. In addition, it is shown that the more samples are obtained, the better performances are realized, because larger penalty ($\ln(T)^{2.05}$) is imposed in the LWZ criterion than in the YAO criterion ($\ln(T)$).

In the cases of structural change, the proposed method using the LWZ criterion ($L = 10$) outperforms other hypothesis-testing methods, particularly in the late change case. The max – *ME*, and *range* – *ME* tests with small window sizes of $h = 1/4$ and $h = 1/2$ shows poor performances in small samples.

More interesting features are shown in Table 3. Concerning the mean of detection delay, the proposed method using the LWZ criterion ($L = 10$) significantly outperforms other hypothesis-testing methods. One fundamental drawback of the Leisch et al. method is that it remains unknown how small h should be. The smaller h is used, the quicker detection is obtained, but the lower power is also realized.

Conclusion

In this article, a model-selection-based monitoring of structural change was presented. The existence of structural change was examined not by hypothesis testing but by model selection using a modified Bayesian information criterion proposed by Liu, Wu, and Zidek (1997). It was found that concerning detection accuracy and detection speed, the proposed method shows better performance than the hypothesis-testing method of Leisch, Hornik, and Kuan (2000).

This model-selection-based method has two advantages in comparison to the hypothesis-testing method. First, by the introduction of a modified Bayesian information criterion, the subjective judgment required in the hypothesis-testing procedure for determining the levels of significance is completely eliminated, and a semiautomatic execution becomes possible. Second, the model-selection-based method frees time series analysts from complex works of hypothesis testing. In order to provide better data description, different alternative models should usually be considered by changing the number of structural changes. In the conventional framework of hypothesis testing, however, different alternative models lead to different test statistics (Bai & Perron, 1998). In the model-selection method, any model change can be made very simply and the performance of the new model is evaluated consistently using the information criterion.

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Table 2. Frequency counts of selecting structural change models

T	CP	YAO		LWZ		max-RE	max-ME			range-ME		
		L=1	L=10	L=1	L=10		h=1/4	h=1/2	h=1	h=1/4	h=1/2	h=1
25		0.838	0.401	0.424	0.146	0.088	0.091	0.104	0.121	0.058	0.065	0.081
50		0.822	0.472	0.245	0.104	0.078	0.090	0.108	0.105	0.051	0.064	0.049
100		0.852	0.538	0.138	0.067	0.073	0.109	0.109	0.109	0.065	0.055	0.061
200		0.840	0.582	0.054	0.031	0.084	0.090	0.105	0.108	0.054	0.053	0.045
300		0.868	0.590	0.020	0.012	0.087	0.090	0.098	0.103	0.060	0.055	0.042
25	28	0.986	0.961	0.941	0.890	0.931	0.685	0.832	0.925	0.108	0.277	0.660
50	55	1.000	0.999	0.994	0.989	0.996	0.948	0.992	1.000	0.206	0.640	0.950
100	110	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.607	0.966	0.999
200	220	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.955	0.999	1.000
300	330	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000
25	75	0.999	0.996	0.994	0.994	0.691	0.445	0.660	0.823	0.034	0.100	0.417
50	150	1.000	1.000	1.000	1.000	0.953	0.828	0.966	0.992	0.072	0.386	0.858
100	300	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	0.320	0.847	0.999
200	600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.824	0.998	1.000
300	900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.979	1.000	1.000

Note: CP denotes change point.

Table 3. The mean and standard deviation of detection delay

T	Change point	YAO		LWZ		max-RE
		L=1	L=10	L=1	L=10	
25	28	16(23)	25(26)	17(24)	25(25)	28(36)
50	55	15(18)	21(21)	20(26)	27(30)	25(27)
100	110	15(11)	19(11)	22(18)	25(19)	24(16)
200	220	16(11)	19(10)	24(16)	26(15)	27(14)
300	330	16(11)	19(10)	27(15)	27(14)	30(15)
25	75	15(13)	19(13)	21(20)	25(20)	69(45)
50	150	15(11)	19(11)	23(17)	25(17)	104(69)
100	300	16(10)	19(9)	26(14)	27(13)	127(75)
200	600	17(10)	19(10)	30(15)	31(14)	147(73)
300	900	18(10)	20(9)	34(15)	34(15)	165(80)

T	Change point	max-ME			range-ME		
		h=1/4	h=1/2	h=1	h=1/4	h=1/2	h=1
25	28	22(22)	23(25)	24(19)	32(10)	30(14)	33(21)
50	55	30(32)	26(26)	32(19)	49(15)	44(26)	46(29)
100	110	25(16)	30(11)	44(13)	73(43)	60(47)	58(14)
200	220	30(10)	42(13)	62(16)	75(63)	66(20)	82(16)
300	330	37(11)	53(15)	76(19)	71(50)	80(15)	102(18)
25	75	26(28)	27(29)	29(28)	37(8)	35(13)	40(22)
50	150	39(47)	34(38)	37(29)	48(16)	55(32)	61(45)
100	300	30(28)	33(19)	47(18)	74(43)	77(67)	69(33)
200	600	31(12)	45(15)	64(25)	98(114)	74(24)	91(17)
300	900	38(12)	55(18)	78(30)	84(86)	87(17)	111(19)

Note: The number in each parenthesis indicates standard deviation.

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