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Large Sample and Bootstrap Intervals for the Gamma Scale Parameter Based on Grouped Data

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Interval estimation of the scale parameter of the gamma distribution using grouped data is considered in this article. Exact intervals do not exist and approximate intervals are needed Recently, Chen and Mi (2001) proposed alternative approximate intervals. In this article, some bootstrap and jackknife type intervals are proposed. The performance of these intervals is investigated and compared. The results show that some of the suggested intervals have a satisfactory statistical performance in situations where the sample size is small with heavy proportion of censoring.

Key words: bootstrap, gamma distribution, grouped data, interval estimation.

Introduction

In many practical studies, the collected data may not be complete observations, they may be in a form of counts of observations in certain intervals; such data is often called grouped data. Grouped data arise frequently in life testing experiments when inspecting the test units intermittently for failure, this procedure is frequently used because it requires less testing effort than continuous inspection. The data obtained from intermittent inspection consists only of the number of failures in each inspection interval. Other examples of natural occurrences of grouped data are given in Pettitt and Stephens (1977).

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In a recent article, Chen and Mi (2001) provided a general method for constructing intervals for the unknown parameters in the distribution using grouped data. Assume that the data are grouped in the classes $[0,t_1),[t_1,t_2),...,[t_{k-1},t_k),[t_k,\infty)$. The i-th interval is $[t_{i-1},t_i)$. Assume that $t_0=0$ and $t_{k+1}=\infty$. Let r_i be the number of failures in the i-th interval. Define the random variable $\zeta_n=\sum_{i=1}^{k} r_i t_i + r_{k+1} t_k$. It follows that $\frac{\zeta_n - ng(\lambda)}{\sqrt{n} s_n} \to N(0,1)$ in law as $n\to\infty$, where $g(\lambda)=\sum_{i=1}^{k} t_i p_i + t_k p_{k+1}$ and $s_n=\left(\sum_{i=1}^{k} t_i^2 \hat{p}_i + t_k^2 \hat{p}_{k+1}\right)-\left(\sum_{i=1}^{k} t_i \hat{p}_i + t_k \hat{p}_{k+1}\right)^2$. It follows that, asymptotically, $p\left(\frac{\zeta_n}{n}-z_{\alpha/2}\frac{s_n}{\sqrt{n}}<g(\lambda)<\frac{\zeta_n}{n}+z_{\alpha/2}\frac{s_n}{\sqrt{n}}\right)=1-\alpha$ When the function $g(\lambda)$ is monotone, an approximate $(1-\alpha)\%$ confidence interval for λ, call it the CM interval, can be obtained as

$$\left[g^{-1}\left(\frac{\zeta_n}{n}-z_{\alpha/2}\frac{s_n}{\sqrt{n}}\right), g^{-1}\left(\frac{\zeta_n}{n}+z_{\alpha/2}\frac{s_n}{\sqrt{n}}\right)\right].$$

However, the above interval possesses exact coverage probabilities and symmetry probabilities only for sufficiently large sample sizes. In this article, the properties of these intervals are investigated and some bootstrap based intervals that use the result of Chen and Mi (2001) are proposed. A similar problem has been investigated for the Burr type X distribution by Al-Nasser and Baklizi (2004).

Bootstrap Intervals

Let $x_1,\ldots,x_n$ be a random sample from the gamma distribution whose probability density function is given by

$$f(x,\lambda,c)=\frac{\lambda^c}{\Gamma(c)}x^{c-1}e^{-\lambda x},\ x>0.$$

Let r_i be the number of observations falling in the i-th interval $(t_{i-1},t_i), i=1,\ldots,k+1$. The joint probability function of $r_1,\ldots,r_{k+1}$ is multinomial with parameters n and $p_1,\ldots,p_{k+1}$. The following confidence intervals are based on the Bootstrap approach (Efron & Tibshirani, 1993). There are several Bootstrap based intervals discussed in the literature, the most common ones are the bootstrap –t interval, the percentile interval and the bias corrected and accelerated (BC_a) interval.

The Bootstrap – t Interval (BTS Intervals)

Let ζ_n be the random variable defined as $\zeta_n=\sum_{i=1}^{k} r_i t_i + r_{k+1}t_k$ calculated from the original data and let ζ_n^* be calculated from the bootstrap sample. Let z_α^* be the α quantile of the bootstrap distribution of $Z^*=\frac{(\zeta_n^*-\zeta_n)}{s_n^*}$, where s_n^* is estimated variance of ζ_n calculated from the bootstrap sample. The bootstrap-t interval for λ is given by $\left[g^{-1}\left(\zeta_n - z_{\alpha/2}^* s_n^*\right), g^{-1}\left(\zeta_n + z_{\alpha/2}^* s_n^*\right)\right]$ where z_α^* is determined by simulation

The Percentile Interval (PRC Interval)

Here, the bootstrap distribution of ζ_n^* are simulated by resampling repeatedly from the parametric model of the original data and calculating $\zeta_{n,i}^*, i=1,\ldots,B$ where B is the number of bootstrap samples. Let $\hat{G}$ be the cumulative distribution function of ζ_n^*, then the $1-\alpha$ interval is given by

$$\left[\hat{G}^{-1}\left(\frac{\alpha}{2}\right),\ \hat{G}^{-1}\left(1-\frac{\alpha}{2}\right)\right].$$

The Bias Corrected Interval (BC Interval)

The bias corrected interval (Efron, 1982) is calculated using the percentiles of the bootstrap distribution of ζ_n^*. The determination of the appropriate percentiles depends on a number ($\hat{z}_0$) called the bias correction. The $1-\alpha$ interval is given by

$$\left[\hat{G}^{-1}(\alpha_1),\ \hat{G}^{-1}(\alpha_2)\right]$$

where

$$\alpha_1=\Phi\left(2\hat{z}_0+z_{\alpha/2}\right),$$
$$\alpha_2=\Phi\left(2\hat{z}_0+z_{1-\alpha/2}\right),$$
$$\Phi(.)$$

is the standard normal cumulative distribution function, z_α is the α quantile of the standard normal distribution. The value of $\hat{z}_0$ are calculated as $\hat{z}_0=\Phi^{-1}\left(\frac{\#\{\zeta_n^*<\zeta_n\}}{B}\right)$.

The Bias Corrected and Accelerated Interval (BCa Interval)

The bias corrected and accelerated interval is calculated also using the percentiles of the bootstrap distribution of. The percentiles depend on two numbers $\hat{a}$ and $\hat{z}_0$ called the acceleration and the bias correction. The $1-\alpha$ interval is given by

$$\left(\hat{G}^{-1}(\alpha_1),\ \hat{G}^{-1}(\alpha_2)\right)$$

where

$$\alpha_1 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z_{\alpha/2}}{1-\hat{a}\left(\hat{z}_0 + z_{\alpha/2}\right)}\right),$$

$$\alpha_2 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z_{1-\alpha/2}}{1-\hat{a}\left(\hat{z}_0 + z_{1-\alpha/2}\right)}\right),$$

$$\Phi(.)$$

is the standard normal cummulative distribution function, z_α is the α quantile of the standard normal distribution. The values of $\hat{a}$ and $\hat{z}_0$ are calculated as follows;

$$\hat{a} = \frac{\sum_{i=1}^{n}\left(\zeta_n(.) - \zeta_n(i)\right)^3}{6\left\{\sum_{i=1}^{n}\left(\zeta_n(.) - \zeta_n(i)\right)^2\right\}^{3/2}}$$

where $\zeta_n(i)$ is calculated using the original data excluding the i-th observation and

$$\zeta_n(.) = \frac{\sum_{i=1}^{n}\zeta_n(i)}{n}.$$

The value of $\hat{z}_0$ is given, as before, by

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\#\left\{\zeta_n^* < \zeta_n\right\}}{B}\right).$$

Jacknife Intervals (JAC Intervals)

An interval based on the jacknife (Efron & Tibshirani, 1993) can be constructed as follows;

$$\zeta_n(.) \pm z_{\alpha/2}\hat{se}.,$$

where

$$\hat{se}^2 = \frac{n-1}{n}\sum_{i=1}^{n}\left(\zeta_n(.) - \zeta_n(i)\right)^2$$

is the jackknife estimate of the variance of ζ_n.

Intervals Based on the Bootstrap Standard Deviation (BSD Intervals)

An interval similar in form to the based on the jacknife can be constructed as follows;

$$\zeta_n \pm z_{\alpha/2}\tilde{se}.,$$

where

$$\tilde{se}^2 = \frac{1}{B-1}\sum_{i=1}^{B}\left(\zeta_{i,n}^* - \bar{\zeta}_n^*\right)^2,$$

$$\bar{\zeta}_n^* = \frac{1}{B}\sum_{i=1}^{B}\zeta_{i,n}^*$$

is the bootstrap estimate of the variance of ζ_n.

Small Sample Performance of the Intervals

For the confidence intervals with nominal confidence coefficient $(1-\alpha)$, the criterion of attainment of lower and upper error probabilities (Jennings, 1987) is used, which are both taken equal to $\frac{\alpha}{2}$. A simulation study is conducted to investigate the performance of the intervals. The sample sizes chosen are $n = 20, 30, 50, 100$. The number of groups $k+1$ is taken as 3, 5 and 9. The censoring proportion (cp) is taken as 0.6, 0.4, and 0.2. The confidence coefficient is taken as 95%, and the shape parameter r is taken as 0.4, 0.8, 1.2, 1.6, and 2. For each combination of the simulation indices 2000 samples were generated from the gamma distribution with $\lambda = 1$ The intervals are calculated, B = 2000 was used for bootstrap calculations. The following quantities are simulated for each interval using the results of the 2000 samples;

1. Lower error rates (L): The fraction of intervals that fall entirely above the true parameter.
2. Upper error rates (U): The fraction of intervals that fall entirely below the true parameter.
3. Total error rates (T): The fraction of intervals that did not contain the true parameter value.

The results are given in Tables 1-3.

Results

From the simulation results, it appears that for k = 2, small sample size (n = 20, 30) and heavy censoring (cp = 0.4,0.6), the CM intervals tend to be anti-conservative. This is also true for PRC, BC and BCa intervals. On the other hand, the BTS, BSD, and JAC intervals tend to attain the nominal sizes. As the censoring proportion is light to moderate, the PRC and the BTS intervals tend to be highly conservative while the BC and BCa intervals tend to be grossly anti-conservative. For larger sample sizes (n = 50, 100) all intervals attain their nominal sizes except for the BC and BCa intervals where they remain anticonservative. In situations where k = 2 and small sample size, all intervals are asymmetric. As k increases, the intervals tend generally to be more symmetric. The performance of the BC and BCa intervals improves considerably for larger values of k. Also their performance improves for higher values of r, that is, the more symmetric the parent gamma distribution, the more symmetric the BC and BCa intervals tend to be.

Conclusion

It appears that the intervals proposed by Chen and Mi (2001) have a good performance except for situations of small sample size and heavy censoring. In this case the BTS, JAC, and especially BSD intervals provide better alternatives.

Table 1. Results for K=2

		n		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
0.4	0.6	CM	0.045	0.038	0.083	0.033	0.039	0.071	0.017	0.025	0.042	0.023	0.027	0.049
		BTS	0.043	0.012	0.055	0.032	0.016	0.047	0.018	0.018	0.036	0.018	0.010	0.028
		PRC	0.013	0.012	0.025	0.010	0.016	0.026	0.007	0.018	0.025	0.009	0.026	0.035
		BC	0.027	0.104	0.130	0.022	0.079	0.100	0.017	0.079	0.096	0.017	0.059	0.076
		BCA	0.015	0.104	0.119	0.021	0.082	0.103	0.012	0.083	0.095	0.013	0.060	0.073
		JAC	0.045	0.038	0.083	0.033	0.039	0.071	0.017	0.025	0.042	0.023	0.027	0.049
		BSD	0.013	0.038	0.051	0.017	0.039	0.055	0.017	0.027	0.044	0.023	0.027	0.050
	0.4	CM	0.019	0.050	0.069	0.027	0.028	0.055	0.014	0.034	0.048	0.020	0.035	0.055
		BTS	0.019	0.017	0.036	0.026	0.009	0.035	0.018	0.014	0.032	0.030	0.014	0.043
		PRC	0.018	0.004	0.022	0.026	0.002	0.028	0.018	0.014	0.032	0.030	0.009	0.038
		BC	0.116	0.173	0.289	0.011	0.101	0.112	0.007	0.132	0.139	0.015	0.079	0.093
		BCA	0.042	0.160	0.202	0.009	0.100	0.109	0.007	0.124	0.131	0.014	0.079	0.092
		JAC	0.019	0.050	0.069	0.027	0.028	0.055	0.030	0.034	0.064	0.020	0.035	0.055
		BSD	0.019	0.050	0.069	0.027	0.028	0.055	0.017	0.034	0.051	0.020	0.035	0.055
	0.2	CM	0.007	0.067	0.074	0.023	0.085	0.108	0.011	0.055	0.066	0.015	0.034	0.048
		BTS	0.007	0.020	0.027	0.032	0.017	0.049	0.026	0.012	0.038	0.049	0.003	0.052
		PRC	0.037	0.000	0.037	0.032	0.005	0.037	0.026	0.004	0.030	0.049	0.002	0.051
		BC	0.006	0.224	0.230	0.005	0.238	0.242	0.005	0.234	0.239	0.005	0.173	0.178
		BCA	0.005	0.184	0.188	0.005	0.234	0.239	0.005	0.230	0.235	0.005	0.172	0.177
		JAC	0.007	0.067	0.074	0.023	0.085	0.108	0.011	0.055	0.066	0.015	0.034	0.048
		BSD	0.007	0.064	0.070	0.023	0.038	0.060	0.012	0.041	0.053	0.017	0.034	0.051
0.8	0.6	CM	0.033	0.037	0.070	0.013	0.037	0.050	0.030	0.022	0.051	0.035	0.016	0.051
		BTS	0.033	0.013	0.045	0.013	0.003	0.016	0.013	0.011	0.023	0.029	0.016	0.045
		PRC	0.000	0.042	0.042	0.003	0.020	0.023	0.006	0.024	0.030	0.009	0.029	0.038
		BC	0.005	0.178	0.182	0.013	0.080	0.093	0.028	0.047	0.075	0.027	0.035	0.061
		BCA	0.001	0.178	0.178	0.008	0.087	0.095	0.012	0.050	0.061	0.020	0.040	0.059
		JAC	0.033	0.037	0.070	0.013	0.037	0.050	0.030	0.022	0.051	0.035	0.029	0.063
		BSD	0.005	0.037	0.042	0.013	0.037	0.050	0.030	0.022	0.052	0.031	0.017	0.048
	0.4	CM	0.029	0.035	0.064	0.026	0.032	0.057	0.019	0.030	0.049	0.022	0.025	0.047
		BTS	0.025	0.024	0.049	0.024	0.022	0.045	0.021	0.015	0.036	0.030	0.014	0.044
		PRC	0.005	0.024	0.029	0.011	0.022	0.032	0.011	0.015	0.026	0.017	0.014	0.031
		BC	0.010	0.168	0.178	0.011	0.137	0.147	0.014	0.078	0.092	0.014	0.079	0.092
		BCA	0.005	0.168	0.173	0.005	0.137	0.141	0.013	0.078	0.091	0.011	0.079	0.089
		JAC	0.029	0.035	0.064	0.026	0.032	0.057	0.019	0.030	0.049	0.022	0.025	0.047
		BSD	0.029	0.035	0.064	0.026	0.032	0.057	0.019	0.030	0.049	0.022	0.026	0.048
	0.2	CM	0.013	0.056	0.069	0.025	0.046	0.071	0.021	0.041	0.062	0.022	0.030	0.051
		BTS	0.013	0.019	0.032	0.025	0.014	0.039	0.034	0.010	0.044	0.047	0.009	0.056
		PRC	0.013	0.007	0.020	0.025	0.006	0.031	0.034	0.003	0.036	0.047	0.004	0.051
		BC	0.002	0.214	0.216	0.010	0.183	0.193	0.008	0.105	0.113	0.009	0.098	0.106
		BCA	0.002	0.214	0.216	0.008	0.173	0.180	0.008	0.100	0.108	0.008	0.097	0.105
		JAC	0.013	0.056	0.069	0.025	0.046	0.071	0.021	0.041	0.062	0.022	0.030	0.051
		BSD	0.013	0.056	0.069	0.025	0.046	0.071	0.021	0.041	0.062	0.022	0.030	0.051
1.2	0.6	CM	0.010	0.038	0.048	0.015	0.031	0.046	0.017	0.024	0.041	0.017	0.024	0.041
		BTS	0.010	0.014	0.024	0.015	0.024	0.039	0.017	0.004	0.021	0.016	0.020	0.036
		PRC	0.000	0.014	0.014	0.002	0.027	0.029	0.006	0.018	0.024	0.003	0.039	0.041
		BC	0.010	0.089	0.099	0.017	0.063	0.080	0.018	0.050	0.068	0.023	0.024	0.047
		BCA	0.000	0.089	0.089	0.015	0.063	0.078	0.017	0.056	0.073	0.018	0.027	0.044
		JAC	0.010	0.038	0.048	0.051	0.031	0.081	0.017	0.024	0.041	0.017	0.024	0.041
		BSD	0.010	0.038	0.048	0.015	0.031	0.046	0.017	0.021	0.038	0.017	0.024	0.041
	0.4	CM	0.020	0.033	0.053	0.013	0.050	0.062	0.024	0.028	0.051	0.019	0.026	0.045
		BTS	0.020	0.032	0.052	0.032	0.017	0.049	0.024	0.014	0.037	0.022	0.021	0.042
		PRC	0.004	0.032	0.036	0.012	0.017	0.029	0.011	0.014	0.025	0.014	0.021	0.035
		BC	0.004	0.163	0.166	0.013	0.099	0.111	0.016	0.094	0.109	0.019	0.065	0.084
		BCA	0.003	0.163	0.166	0.011	0.100	0.111	0.012	0.094	0.105	0.016	0.066	0.082
		JAC	0.020	0.033	0.053	0.033	0.050	0.083	0.024	0.028	0.051	0.019	0.041	0.059
		BSD	0.020	0.033	0.053	0.013	0.050	0.062	0.024	0.034	0.058	0.019	0.034	0.053

Table 1. Continued

		n		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
	0.2	CM	0.016	0.050	0.066	0.018	0.059	0.077	0.024	0.051	0.075	0.020	0.026	0.045
		BTS	0.016	0.017	0.033	0.018	0.024	0.041	0.035	0.013	0.048	0.028	0.009	0.037
		PRC	0.016	0.006	0.022	0.017	0.011	0.028	0.035	0.008	0.043	0.028	0.009	0.037
		BC	0.003	0.167	0.169	0.008	0.188	0.195	0.012	0.145	0.157	0.015	0.113	0.127
		BCA	0.002	0.165	0.167	0.004	0.188	0.191	0.011	0.143	0.154	0.014	0.111	0.125
		JAC	0.016	0.050	0.066	0.018	0.059	0.077	0.024	0.051	0.075	0.020	0.026	0.045
		BSD	0.016	0.050	0.066	0.018	0.047	0.065	0.024	0.036	0.060	0.023	0.027	0.049
1.6	0.6	CM	0.034	0.046	0.080	0.023	0.021	0.044	0.017	0.018	0.035	0.025	0.024	0.049
		BTS	0.034	0.008	0.041	0.023	0.004	0.026	0.017	0.017	0.034	0.012	0.028	0.039
		PRC	0.000	0.044	0.044	0.000	0.021	0.021	0.001	0.042	0.043	0.005	0.050	0.054
		BC	0.016	0.066	0.081	0.019	0.052	0.071	0.029	0.054	0.083	0.030	0.038	0.067
		BCA	0.009	0.068	0.076	0.005	0.061	0.066	0.016	0.057	0.073	0.026	0.043	0.068
		JAC	0.034	0.046	0.080	0.023	0.021	0.044	0.017	0.018	0.035	0.025	0.024	0.049
		BSD	0.000	0.046	0.046	0.006	0.021	0.027	0.017	0.019	0.036	0.022	0.026	0.047
	0.4	CM	0.012	0.034	0.045	0.037	0.024	0.060	0.019	0.038	0.056	0.030	0.036	0.066
		BTS	0.048	0.010	0.058	0.036	0.009	0.045	0.019	0.017	0.036	0.025	0.020	0.045
		PRC	0.012	0.010	0.022	0.015	0.009	0.024	0.010	0.017	0.027	0.014	0.020	0.034
		BC	0.013	0.073	0.086	0.015	0.051	0.066	0.018	0.051	0.069	0.024	0.049	0.073
		BCA	0.010	0.073	0.083	0.015	0.051	0.066	0.016	0.052	0.068	0.019	0.050	0.068
		JAC	0.012	0.034	0.045	0.037	0.024	0.060	0.019	0.038	0.056	0.030	0.036	0.066
		BSD	0.012	0.034	0.045	0.015	0.024	0.038	0.019	0.031	0.049	0.018	0.034	0.052
	0.2	CM	0.015	0.063	0.078	0.018	0.043	0.061	0.023	0.048	0.071	0.017	0.038	0.055
		BTS	0.015	0.021	0.036	0.018	0.018	0.035	0.031	0.016	0.047	0.033	0.013	0.045
		PRC	0.014	0.008	0.022	0.018	0.007	0.024	0.031	0.008	0.039	0.033	0.007	0.040
		BC	0.005	0.225	0.230	0.010	0.130	0.140	0.009	0.108	0.117	0.015	0.068	0.083
		BCA	0.005	0.211	0.215	0.007	0.124	0.130	0.008	0.103	0.111	0.014	0.067	0.081
		JAC	0.015	0.063	0.078	0.018	0.043	0.061	0.023	0.048	0.071	0.017	0.038	0.055
		BSD	0.015	0.063	0.078	0.018	0.043	0.061	0.023	0.046	0.069	0.017	0.037	0.054
2.0	0.6	CM	0.043	0.019	0.062	0.007	0.021	0.028	0.015	0.023	0.038	0.023	0.021	0.044
		BTS	0.043	0.014	0.057	0.007	0.010	0.017	0.015	0.016	0.030	0.010	0.022	0.032
		PRC	0.000	0.018	0.018	0.000	0.036	0.036	0.000	0.042	0.042	0.001	0.041	0.042
		BC	0.000	0.038	0.038	0.007	0.055	0.061	0.015	0.045	0.059	0.035	0.023	0.057
		BCA	0.000	0.043	0.043	0.005	0.091	0.096	0.015	0.045	0.060	0.025	0.025	0.050
		JAC	0.043	0.019	0.062	0.007	0.021	0.028	0.015	0.023	0.038	0.023	0.021	0.044
		BSD	0.000	0.019	0.019	0.007	0.021	0.028	0.015	0.022	0.037	0.023	0.021	0.044
	0.4	CM	0.025	0.049	0.074	0.029	0.023	0.052	0.036	0.021	0.057	0.022	0.027	0.048
		BTS	0.025	0.013	0.038	0.029	0.012	0.041	0.018	0.009	0.027	0.013	0.019	0.031
		PRC	0.005	0.013	0.017	0.009	0.012	0.020	0.009	0.021	0.030	0.006	0.036	0.042
		BC	0.008	0.095	0.103	0.024	0.052	0.076	0.019	0.079	0.098	0.013	0.062	0.075
		BCA	0.005	0.097	0.102	0.014	0.052	0.066	0.015	0.083	0.097	0.012	0.067	0.079
		JAC	0.025	0.049	0.074	0.029	0.023	0.052	0.036	0.021	0.057	0.022	0.027	0.048
		BSD	0.025	0.049	0.074	0.027	0.023	0.050	0.032	0.022	0.053	0.015	0.027	0.041
	0.2	CM	0.033	0.035	0.068	0.024	0.047	0.070	0.014	0.048	0.062	0.018	0.039	0.057
		BTS	0.031	0.015	0.046	0.021	0.020	0.040	0.016	0.022	0.038	0.031	0.009	0.040
		PRC	0.009	0.015	0.023	0.021	0.008	0.028	0.016	0.007	0.023	0.031	0.009	0.040
		BC	0.005	0.197	0.201	0.009	0.157	0.165	0.011	0.099	0.109	0.014	0.071	0.084
		BCA	0.003	0.188	0.190	0.008	0.153	0.161	0.010	0.098	0.107	0.011	0.071	0.082
		JAC	0.033	0.081	0.114	0.024	0.047	0.070	0.014	0.048	0.062	0.018	0.039	0.057
		BSD	0.016	0.036	0.052	0.024	0.047	0.070	0.014	0.041	0.055	0.019	0.033	0.051

Table 2: Results for K=4

		n		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
0.4	0.6	CM	0.017	0.035	0.052	0.021	0.032	0.053	0.016	0.030	0.046	0.028	0.028	0.056
		BTS	0.013	0.012	0.025	0.014	0.015	0.028	0.009	0.021	0.029	0.011	0.025	0.036
		PRC	0.002	0.030	0.031	0.003	0.033	0.036	0.002	0.034	0.036	0.007	0.036	0.043
		BC	0.027	0.077	0.104	0.028	0.062	0.090	0.026	0.048	0.074	0.040	0.037	0.077
		BCA	0.020	0.084	0.104	0.022	0.063	0.085	0.021	0.052	0.073	0.037	0.038	0.074
		JAC	0.018	0.040	0.058	0.023	0.033	0.056	0.016	0.030	0.046	0.028	0.029	0.057
		BSD	0.014	0.039	0.053	0.018	0.035	0.052	0.014	0.030	0.044	0.026	0.029	0.055
	0.4	CM	0.015	0.048	0.063	0.022	0.046	0.068	0.019	0.035	0.054	0.019	0.040	0.059
		BTS	0.011	0.022	0.032	0.018	0.022	0.040	0.017	0.017	0.034	0.019	0.019	0.037
		PRC	0.007	0.015	0.022	0.015	0.019	0.034	0.013	0.017	0.030	0.017	0.021	0.037
		BC	0.027	0.113	0.140	0.022	0.097	0.119	0.018	0.081	0.099	0.016	0.073	0.088
		BCA	0.013	0.111	0.124	0.019	0.090	0.108	0.017	0.079	0.096	0.015	0.073	0.088
		JAC	0.015	0.055	0.070	0.022	0.049	0.071	0.020	0.036	0.055	0.019	0.040	0.059
		BSD	0.016	0.049	0.065	0.021	0.047	0.068	0.021	0.037	0.057	0.018	0.040	0.058
	0.2	CM	0.015	0.068	0.083	0.017	0.054	0.071	0.016	0.045	0.061	0.018	0.038	0.055
		BTS	0.014	0.034	0.048	0.020	0.019	0.039	0.024	0.011	0.034	0.036	0.010	0.046
		PRC	0.023	0.007	0.030	0.030	0.006	0.035	0.026	0.006	0.032	0.039	0.006	0.045
		BC	0.012	0.182	0.194	0.008	0.152	0.160	0.009	0.141	0.150	0.006	0.130	0.136
		BCA	0.010	0.170	0.180	0.008	0.142	0.149	0.009	0.134	0.142	0.006	0.126	0.132
		JAC	0.018	0.072	0.090	0.017	0.058	0.075	0.017	0.046	0.062	0.018	0.038	0.055
		BSD	0.017	0.053	0.069	0.017	0.041	0.058	0.017	0.037	0.054	0.018	0.035	0.052
0.8	0.6	CM	0.018	0.045	0.063	0.020	0.032	0.052	0.019	0.025	0.044	0.020	0.030	0.049
		BTS	0.008	0.018	0.025	0.013	0.016	0.029	0.009	0.019	0.027	0.012	0.037	0.048
		PRC	0.000	0.047	0.047	0.001	0.046	0.047	0.002	0.037	0.038	0.004	0.055	0.058
		BC	0.024	0.070	0.094	0.032	0.054	0.086	0.034	0.029	0.062	0.050	0.029	0.078
		BCA	0.015	0.076	0.091	0.025	0.061	0.086	0.030	0.033	0.062	0.044	0.033	0.077
		JAC	0.018	0.048	0.066	0.021	0.033	0.054	0.019	0.026	0.045	0.020	0.031	0.051
		BSD	0.008	0.047	0.055	0.015	0.035	0.050	0.014	0.026	0.040	0.018	0.030	0.048
	0.4	CM	0.018	0.050	0.067	0.017	0.043	0.060	0.015	0.038	0.053	0.024	0.028	0.052
		BTS	0.009	0.019	0.028	0.010	0.020	0.029	0.012	0.021	0.033	0.020	0.015	0.035
		PRC	0.003	0.018	0.021	0.005	0.024	0.029	0.006	0.028	0.034	0.015	0.018	0.033
		BC	0.022	0.087	0.109	0.026	0.090	0.115	0.022	0.064	0.086	0.029	0.043	0.072
		BCA	0.016	0.085	0.101	0.020	0.089	0.109	0.018	0.064	0.082	0.027	0.044	0.071
		JAC	0.021	0.055	0.075	0.023	0.045	0.068	0.016	0.038	0.054	0.024	0.030	0.054
		BSD	0.013	0.051	0.064	0.018	0.045	0.062	0.015	0.039	0.054	0.024	0.028	0.052
	0.2	CM	0.014	0.056	0.070	0.019	0.053	0.072	0.013	0.044	0.056	0.014	0.037	0.051
		BTS	0.008	0.019	0.027	0.017	0.020	0.036	0.015	0.012	0.027	0.023	0.010	0.033
		PRC	0.009	0.004	0.013	0.020	0.007	0.027	0.015	0.009	0.023	0.026	0.007	0.032
		BC	0.009	0.142	0.151	0.014	0.133	0.147	0.006	0.105	0.111	0.009	0.103	0.111
		BCA	0.006	0.135	0.141	0.013	0.128	0.140	0.005	0.101	0.106	0.009	0.100	0.108
		JAC	0.014	0.063	0.076	0.020	0.057	0.076	0.013	0.044	0.056	0.015	0.038	0.052
		BSD	0.013	0.044	0.056	0.019	0.045	0.063	0.012	0.037	0.049	0.015	0.035	0.050
1.2	0.6	CM	0.014	0.031	0.045	0.018	0.028	0.046	0.013	0.028	0.041	0.018	0.029	0.047
		BTS	0.007	0.014	0.021	0.006	0.016	0.022	0.005	0.027	0.032	0.008	0.044	0.052
		PRC	0.000	0.040	0.040	0.000	0.049	0.049	0.001	0.065	0.065	0.002	0.065	0.067
		BC	0.023	0.052	0.075	0.034	0.030	0.064	0.035	0.031	0.066	0.050	0.021	0.071
		BCA	0.011	0.058	0.069	0.028	0.039	0.067	0.030	0.039	0.069	0.049	0.026	0.074
		JAC	0.014	0.035	0.049	0.021	0.029	0.050	0.014	0.029	0.043	0.018	0.029	0.047
		BSD	0.004	0.034	0.038	0.007	0.028	0.035	0.009	0.027	0.036	0.014	0.029	0.042
	0.4	CM	0.010	0.047	0.057	0.015	0.034	0.049	0.019	0.039	0.058	0.020	0.030	0.050
		BTS	0.008	0.016	0.024	0.010	0.020	0.030	0.011	0.020	0.031	0.012	0.025	0.036
		PRC	0.004	0.022	0.025	0.003	0.024	0.026	0.006	0.029	0.035	0.006	0.030	0.036
		BC	0.013	0.074	0.087	0.024	0.054	0.078	0.029	0.055	0.084	0.035	0.038	0.073
		BCA	0.011	0.074	0.085	0.018	0.055	0.072	0.025	0.057	0.082	0.031	0.040	0.070
		JAC	0.011	0.054	0.065	0.015	0.035	0.050	0.021	0.040	0.061	0.020	0.030	0.050
		BSD	0.009	0.048	0.056	0.013	0.035	0.048	0.018	0.039	0.056	0.019	0.031	0.050

Table 2. Continued

		n		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
	0.2	CM	0.020	0.058	0.078	0.017	0.053	0.070	0.019	0.044	0.063	0.020	0.038	0.058
		BTS	0.015	0.019	0.033	0.016	0.021	0.036	0.019	0.020	0.038	0.025	0.018	0.043
		PRC	0.014	0.006	0.020	0.015	0.009	0.024	0.018	0.017	0.035	0.024	0.017	0.040
		BC	0.018	0.128	0.146	0.014	0.113	0.127	0.019	0.096	0.115	0.014	0.081	0.095
		BCA	0.014	0.122	0.136	0.013	0.110	0.123	0.019	0.092	0.111	0.014	0.079	0.092
		JAC	0.022	0.070	0.092	0.018	0.056	0.074	0.019	0.046	0.065	0.022	0.038	0.060
		BSD	0.019	0.052	0.071	0.016	0.049	0.065	0.019	0.042	0.061	0.022	0.037	0.059
1.6	0.6	CM	0.005	0.030	0.034	0.014	0.024	0.037	0.018	0.029	0.046	0.024	0.029	0.053
		BTS	0.002	0.012	0.014	0.009	0.014	0.023	0.006	0.035	0.040	0.007	0.053	0.060
		PRC	0.000	0.063	0.063	0.000	0.043	0.043	0.000	0.075	0.075	0.002	0.078	0.080
		BC	0.015	0.047	0.062	0.032	0.026	0.057	0.048	0.029	0.076	0.066	0.021	0.086
		BCA	0.009	0.057	0.065	0.025	0.029	0.053	0.041	0.034	0.074	0.061	0.024	0.085
		JAC	0.005	0.034	0.039	0.014	0.027	0.040	0.018	0.030	0.047	0.024	0.029	0.053
		BSD	0.002	0.030	0.032	0.007	0.025	0.031	0.009	0.027	0.036	0.018	0.026	0.044
	0.4	CM	0.013	0.040	0.052	0.019	0.042	0.060	0.019	0.026	0.045	0.020	0.031	0.050
		BTS	0.007	0.016	0.023	0.012	0.023	0.034	0.011	0.020	0.031	0.011	0.030	0.041
		PRC	0.000	0.035	0.035	0.001	0.048	0.049	0.002	0.028	0.030	0.007	0.039	0.046
		BC	0.013	0.071	0.083	0.021	0.073	0.094	0.030	0.040	0.069	0.032	0.036	0.067
		BCA	0.008	0.078	0.086	0.018	0.076	0.094	0.027	0.043	0.070	0.030	0.037	0.067
		JAC	0.013	0.043	0.056	0.020	0.045	0.064	0.020	0.028	0.048	0.020	0.031	0.051
		BSD	0.009	0.040	0.049	0.015	0.043	0.058	0.014	0.027	0.041	0.018	0.032	0.050
	0.2	CM	0.013	0.052	0.064	0.017	0.049	0.066	0.022	0.037	0.059	0.015	0.035	0.049
		BTS	0.011	0.017	0.028	0.011	0.015	0.026	0.018	0.015	0.033	0.014	0.015	0.029
		PRC	0.007	0.013	0.020	0.010	0.011	0.021	0.013	0.015	0.028	0.011	0.015	0.026
		BC	0.012	0.125	0.137	0.016	0.106	0.121	0.018	0.097	0.115	0.013	0.065	0.078
		BCA	0.011	0.121	0.132	0.013	0.103	0.116	0.018	0.096	0.114	0.012	0.064	0.076
		JAC	0.014	0.058	0.072	0.017	0.050	0.067	0.022	0.040	0.062	0.015	0.036	0.050
		BSD	0.013	0.043	0.055	0.015	0.043	0.058	0.022	0.035	0.057	0.015	0.032	0.047
2	0.6	CM	0.002	0.041	0.043	0.011	0.036	0.047	0.020	0.027	0.047	0.022	0.027	0.049
		BTS	0.000	0.019	0.019	0.004	0.031	0.035	0.008	0.046	0.054	0.007	0.070	0.076
		PRC	0.000	0.081	0.081	0.000	0.079	0.079	0.000	0.108	0.108	0.001	0.101	0.101
		BC	0.013	0.055	0.068	0.025	0.035	0.060	0.057	0.025	0.082	0.074	0.015	0.089
		BCA	0.002	0.070	0.072	0.020	0.042	0.062	0.049	0.033	0.082	0.069	0.019	0.087
		JAC	0.002	0.043	0.045	0.011	0.038	0.049	0.020	0.031	0.051	0.022	0.027	0.049
		BSD	0.000	0.038	0.038	0.002	0.035	0.036	0.009	0.025	0.034	0.013	0.024	0.036
	0.4	CM	0.014	0.040	0.054	0.011	0.039	0.049	0.011	0.042	0.053	0.020	0.034	0.054
		BTS	0.005	0.013	0.018	0.005	0.021	0.026	0.006	0.034	0.039	0.006	0.038	0.044
		PRC	0.000	0.026	0.026	0.001	0.037	0.038	0.003	0.051	0.053	0.003	0.063	0.066
		BC	0.017	0.057	0.074	0.020	0.054	0.074	0.024	0.049	0.073	0.039	0.034	0.073
		BCA	0.013	0.061	0.074	0.013	0.058	0.071	0.021	0.052	0.073	0.033	0.037	0.070
		JAC	0.014	0.042	0.056	0.011	0.044	0.055	0.013	0.044	0.056	0.020	0.036	0.056
		BSD	0.006	0.040	0.046	0.007	0.039	0.046	0.009	0.042	0.051	0.016	0.032	0.048
	0.2	CM	0.008	0.068	0.076	0.012	0.048	0.060	0.015	0.046	0.061	0.017	0.042	0.059
		BTS	0.008	0.025	0.033	0.008	0.017	0.024	0.012	0.021	0.033	0.015	0.021	0.036
		PRC	0.003	0.018	0.020	0.006	0.013	0.019	0.007	0.021	0.027	0.014	0.017	0.031
		BC	0.008	0.137	0.144	0.013	0.094	0.106	0.014	0.090	0.104	0.018	0.062	0.080
		BCA	0.006	0.134	0.140	0.011	0.092	0.103	0.012	0.088	0.100	0.017	0.062	0.079
		JAC	0.009	0.075	0.083	0.015	0.049	0.064	0.016	0.047	0.063	0.017	0.042	0.059
		BSD	0.008	0.059	0.066	0.012	0.043	0.055	0.015	0.041	0.056	0.016	0.039	0.055

Table 3: Results for K=8

		N		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
0.4	0.6	CM	0.022	0.048	0.069	0.019	0.039	0.057	0.021	0.033	0.053	0.021	0.021	0.042
		BTS	0.010	0.024	0.033	0.009	0.023	0.032	0.008	0.028	0.036	0.010	0.025	0.035
		PRC	0.001	0.046	0.047	0.001	0.047	0.048	0.003	0.042	0.044	0.005	0.040	0.045
		BC	0.042	0.073	0.114	0.045	0.054	0.099	0.049	0.036	0.085	0.052	0.022	0.074
		BCA	0.034	0.074	0.108	0.036	0.056	0.092	0.044	0.038	0.081	0.048	0.024	0.072
		JAC	0.022	0.051	0.073	0.022	0.040	0.062	0.021	0.033	0.054	0.022	0.022	0.043
		BSD	0.014	0.053	0.066	0.014	0.040	0.054	0.020	0.037	0.056	0.020	0.023	0.043
	0.4	CM	0.012	0.046	0.058	0.019	0.041	0.060	0.021	0.036	0.057	0.020	0.024	0.044
		BTS	0.010	0.024	0.033	0.011	0.022	0.032	0.012	0.020	0.032	0.012	0.015	0.027
		PRC	0.007	0.022	0.028	0.008	0.020	0.028	0.007	0.023	0.029	0.010	0.015	0.025
		BC	0.027	0.087	0.114	0.027	0.081	0.108	0.026	0.065	0.091	0.025	0.044	0.068
		BCA	0.016	0.080	0.096	0.023	0.077	0.099	0.024	0.065	0.088	0.024	0.044	0.068
		JAC	0.013	0.049	0.062	0.021	0.044	0.064	0.023	0.038	0.060	0.021	0.024	0.045
		BSD	0.012	0.047	0.059	0.020	0.044	0.064	0.022	0.039	0.061	0.021	0.027	0.048
	0.2	CM	0.011	0.075	0.086	0.014	0.059	0.072	0.011	0.045	0.056	0.016	0.040	0.055
		BTS	0.007	0.031	0.038	0.013	0.025	0.038	0.015	0.018	0.033	0.028	0.013	0.041
		PRC	0.012	0.005	0.016	0.018	0.007	0.025	0.024	0.005	0.028	0.036	0.008	0.043
		BC	0.011	0.156	0.166	0.011	0.149	0.159	0.009	0.124	0.132	0.010	0.111	0.121
		BCA	0.009	0.144	0.153	0.011	0.139	0.149	0.009	0.117	0.125	0.010	0.105	0.115
		JAC	0.013	0.082	0.095	0.015	0.062	0.077	0.012	0.046	0.058	0.016	0.040	0.055
		BSD	0.013	0.059	0.071	0.015	0.050	0.065	0.014	0.043	0.057	0.017	0.038	0.055
0.8	0.6	CM	0.016	0.040	0.056	0.021	0.033	0.054	0.023	0.032	0.055	0.020	0.039	0.059
		BTS	0.010	0.024	0.034	0.008	0.025	0.033	0.007	0.041	0.047	0.007	0.060	0.067
		PRC	0.000	0.063	0.063	0.001	0.053	0.054	0.001	0.073	0.074	0.001	0.076	0.077
		BC	0.038	0.053	0.090	0.042	0.034	0.076	0.059	0.032	0.090	0.067	0.031	0.097
		BCA	0.030	0.057	0.087	0.037	0.037	0.074	0.055	0.034	0.089	0.063	0.032	0.095
		JAC	0.017	0.043	0.060	0.023	0.038	0.060	0.024	0.034	0.058	0.020	0.041	0.061
		BSD	0.011	0.044	0.055	0.015	0.038	0.053	0.017	0.034	0.051	0.016	0.040	0.056
	0.4	CM	0.015	0.050	0.064	0.017	0.037	0.054	0.021	0.038	0.058	0.018	0.036	0.054
		BTS	0.005	0.027	0.032	0.005	0.022	0.027	0.012	0.028	0.040	0.009	0.032	0.041
		PRC	0.001	0.032	0.033	0.002	0.028	0.030	0.006	0.036	0.042	0.006	0.038	0.044
		BC	0.022	0.082	0.104	0.032	0.055	0.087	0.033	0.054	0.086	0.030	0.043	0.073
		BCA	0.019	0.080	0.099	0.027	0.054	0.081	0.031	0.054	0.085	0.028	0.044	0.071
		JAC	0.015	0.058	0.072	0.018	0.039	0.057	0.021	0.039	0.060	0.018	0.037	0.055
		BSD	0.011	0.055	0.066	0.016	0.040	0.055	0.021	0.041	0.062	0.018	0.038	0.056
	0.2	CM	0.012	0.062	0.074	0.013	0.052	0.065	0.013	0.038	0.050	0.017	0.034	0.050
		BTS	0.007	0.021	0.027	0.010	0.016	0.025	0.011	0.018	0.029	0.019	0.011	0.030
		PRC	0.007	0.007	0.014	0.011	0.007	0.018	0.012	0.015	0.026	0.022	0.011	0.033
		BC	0.014	0.121	0.135	0.014	0.112	0.126	0.012	0.103	0.115	0.014	0.082	0.096
		BCA	0.014	0.107	0.121	0.014	0.104	0.117	0.011	0.099	0.110	0.014	0.079	0.092
		JAC	0.015	0.065	0.080	0.013	0.055	0.068	0.013	0.038	0.051	0.017	0.034	0.050
		BSD	0.012	0.053	0.065	0.014	0.047	0.060	0.013	0.036	0.049	0.018	0.033	0.051
1.2	0.6	CM	0.009	0.038	0.047	0.013	0.034	0.047	0.013	0.031	0.044	0.024	0.028	0.052
		BTS	0.003	0.024	0.027	0.006	0.032	0.037	0.006	0.052	0.057	0.007	0.062	0.069
		PRC	0.000	0.072	0.072	0.000	0.091	0.091	0.000	0.097	0.097	0.002	0.094	0.096
		BC	0.023	0.042	0.065	0.035	0.031	0.066	0.059	0.020	0.079	0.082	0.012	0.094
		BCA	0.018	0.049	0.067	0.030	0.035	0.065	0.053	0.025	0.077	0.077	0.016	0.092
		JAC	0.009	0.042	0.051	0.014	0.037	0.051	0.013	0.034	0.047	0.024	0.028	0.052
		BSD	0.003	0.040	0.043	0.008	0.037	0.045	0.011	0.033	0.044	0.021	0.026	0.047

Table 3. Continued

		n		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
	0.4	CM	0.012	0.056	0.068	0.011	0.039	0.049	0.020	0.034	0.054	0.020	0.030	0.050
		BTS	0.005	0.029	0.033	0.005	0.027	0.032	0.008	0.029	0.037	0.010	0.035	0.044
		PRC	0.001	0.045	0.046	0.001	0.038	0.039	0.002	0.043	0.045	0.005	0.043	0.048
		BC	0.024	0.075	0.099	0.029	0.052	0.081	0.050	0.041	0.090	0.045	0.033	0.078
		BCA	0.018	0.073	0.090	0.026	0.052	0.078	0.045	0.042	0.086	0.042	0.033	0.075
		JAC	0.012	0.063	0.075	0.012	0.042	0.054	0.021	0.036	0.057	0.021	0.030	0.051
		BSD	0.009	0.059	0.067	0.009	0.042	0.050	0.018	0.036	0.054	0.019	0.030	0.049
	0.2	CM	0.011	0.059	0.070	0.017	0.043	0.059	0.016	0.046	0.062	0.015	0.030	0.045
		BTS	0.006	0.024	0.030	0.011	0.021	0.031	0.010	0.022	0.032	0.013	0.019	0.032
		PRC	0.004	0.014	0.018	0.011	0.015	0.026	0.011	0.019	0.030	0.015	0.018	0.033
		BC	0.016	0.115	0.131	0.021	0.075	0.095	0.017	0.082	0.099	0.017	0.059	0.076
		BCA	0.011	0.107	0.118	0.019	0.072	0.090	0.017	0.079	0.096	0.017	0.058	0.074
		JAC	0.014	0.062	0.076	0.017	0.044	0.061	0.016	0.048	0.064	0.016	0.031	0.046
		BSD	0.012	0.052	0.064	0.016	0.039	0.055	0.015	0.044	0.059	0.016	0.030	0.045
1.6	0.6	CM	0.005	0.035	0.040	0.011	0.031	0.042	0.011	0.033	0.044	0.020	0.031	0.051
		BTS	0.001	0.026	0.027	0.002	0.038	0.040	0.002	0.070	0.072	0.004	0.088	0.091
		PRC	0.000	0.080	0.080	0.000	0.112	0.112	0.000	0.136	0.136	0.000	0.121	0.121
		BC	0.029	0.033	0.062	0.036	0.026	0.062	0.051	0.019	0.070	0.095	0.009	0.104
		BCA	0.023	0.038	0.060	0.032	0.032	0.064	0.046	0.023	0.069	0.089	0.014	0.103
		JAC	0.007	0.039	0.046	0.011	0.033	0.044	0.011	0.034	0.045	0.021	0.031	0.052
		BSD	0.000	0.036	0.036	0.002	0.032	0.034	0.004	0.031	0.034	0.011	0.029	0.040
	0.4	CM	0.011	0.046	0.056	0.014	0.041	0.055	0.012	0.036	0.047	0.017	0.033	0.050
		BTS	0.006	0.029	0.035	0.004	0.029	0.033	0.005	0.038	0.043	0.006	0.047	0.053
		PRC	0.000	0.048	0.048	0.001	0.053	0.054	0.002	0.055	0.056	0.002	0.057	0.059
		BC	0.025	0.056	0.081	0.032	0.045	0.076	0.032	0.033	0.064	0.042	0.027	0.069
		BCA	0.018	0.056	0.074	0.028	0.046	0.074	0.028	0.035	0.063	0.040	0.028	0.068
		JAC	0.011	0.050	0.060	0.015	0.045	0.059	0.012	0.037	0.048	0.018	0.033	0.051
		BSD	0.009	0.049	0.058	0.009	0.041	0.050	0.009	0.035	0.044	0.014	0.033	0.047
	0.2	CM	0.013	0.055	0.068	0.010	0.047	0.057	0.017	0.044	0.061	0.014	0.035	0.048
		BTS	0.007	0.024	0.031	0.006	0.021	0.027	0.006	0.026	0.032	0.008	0.022	0.030
		PRC	0.005	0.018	0.022	0.004	0.016	0.020	0.005	0.022	0.027	0.008	0.021	0.029
		BC	0.020	0.091	0.111	0.014	0.081	0.095	0.021	0.068	0.089	0.019	0.055	0.074
		BCA	0.015	0.083	0.098	0.012	0.076	0.088	0.020	0.063	0.083	0.018	0.052	0.070
		JAC	0.014	0.062	0.076	0.011	0.049	0.060	0.017	0.045	0.062	0.014	0.035	0.049
		BSD	0.012	0.050	0.061	0.010	0.045	0.055	0.015	0.040	0.055	0.013	0.034	0.047
2	0.6	CM	0.002	0.037	0.039	0.010	0.032	0.041	0.013	0.029	0.042	0.019	0.029	0.048
		BTS	0.000	0.030	0.030	0.004	0.053	0.057	0.004	0.071	0.075	0.004	0.118	0.122
		PRC	0.000	0.112	0.112	0.000	0.108	0.108	0.000	0.135	0.135	0.000	0.156	0.156
		BC	0.019	0.036	0.054	0.046	0.017	0.063	0.066	0.016	0.082	0.102	0.013	0.115
		BCA	0.013	0.049	0.062	0.040	0.025	0.064	0.065	0.019	0.084	0.098	0.015	0.113
		JAC	0.002	0.046	0.048	0.011	0.037	0.048	0.014	0.030	0.044	0.019	0.030	0.048
		BSD	0.000	0.036	0.036	0.003	0.027	0.030	0.006	0.024	0.030	0.011	0.024	0.035
	0.4	CM	0.006	0.040	0.046	0.006	0.052	0.057	0.018	0.034	0.052	0.018	0.030	0.047
		BTS	0.001	0.027	0.028	0.002	0.045	0.047	0.008	0.045	0.053	0.007	0.049	0.056
		PRC	0.000	0.059	0.059	0.000	0.078	0.078	0.001	0.067	0.067	0.002	0.073	0.075
		BC	0.018	0.053	0.071	0.021	0.052	0.072	0.047	0.030	0.076	0.049	0.020	0.069
		BCA	0.015	0.056	0.070	0.017	0.056	0.073	0.040	0.034	0.074	0.044	0.022	0.066
		JAC	0.007	0.045	0.052	0.006	0.056	0.062	0.019	0.035	0.053	0.018	0.031	0.049
		BSD	0.003	0.040	0.043	0.004	0.051	0.055	0.013	0.034	0.047	0.015	0.028	0.043

Table 3. Continued

		n		20			30			50			100	
R	CP	M	L1	U1	T1	L2	U2	T2	L3	U3	T3	L4	U4	
	0.2	CM	0.014	0.065	0.078	0.010	0.048	0.057	0.014	0.048	0.062	0.018	0.037	0.055
		BTS	0.006	0.032	0.038	0.003	0.028	0.031	0.007	0.028	0.034	0.009	0.027	0.036
		PRC	0.001	0.029	0.030	0.001	0.029	0.030	0.004	0.027	0.031	0.007	0.031	0.038
		BC	0.018	0.100	0.117	0.015	0.086	0.101	0.022	0.071	0.093	0.023	0.049	0.071
		BCA	0.015	0.091	0.105	0.011	0.083	0.094	0.021	0.067	0.088	0.022	0.048	0.070
		JAC	0.015	0.069	0.084	0.010	0.051	0.061	0.014	0.050	0.064	0.018	0.038	0.056
		BSD	0.011	0.061	0.072	0.008	0.048	0.056	0.012	0.048	0.059	0.017	0.038	0.055

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