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To date, numerous extensions of the exponentially weighted moving average, EWMA charts have been made. A new robust EWMA chart for the process mean is proposed. It enables easier detection of outliers and increase sensitivity to other forms of out-of-control situation when outliers are present.

Key words: Exponentially weighted moving average (EWMA), cumulative sum (CUSUM), Shewhart, process mean, sample mean, sample range, average run length (ARL)

Introduction

The EWMA chart is a good alternative to the Shewhart chart in the detection of small shifts. The EWMA chart constructed from the sample mean is first developed by Roberts (1959). Since then various extensions of the EWMA charts have been proposed. Sweet (1986) proposed two models to construct simultaneous control charts to monitor the mean and the variance of a process using the EWMA. Crowder (1987 & 1989) provided average run length (ARL) tables and graphs for the selection of the optimum values of the EWMA control chart parameters in the design of an EWMA chart. Ng & Case (1989) presented several EWMA control chart schemes based on individual measurement, sample mean, sample range and moving range statistics. Lucas & Saccucci (1990) showed that a fast initial response (FIR) feature is useful for the EWMA chart, especially for small values of smoothing constants. Rhoads, Montgomery & Mastrangelo (1996) proposed a scheme which is superior to that of Lucas & Saccucci (1990). MacGregor & Harris (1993) suggested an approach of using the EWMA based statistics in the monitoring of the process standard deviation. Gan (1990) proposed three modified EWMA charts for the Poisson data. A better procedure for using the EWMA chart for Poisson count is given by Borror, Champ & Rigdon (1998). Somerville, Montgomery & Runger (2002) developed a smoothing and filtering method using the EWMA and Poisson probabilities which separates the two distributions in a particle count data stream into a base process and an outlier process followed by applying statistical monitoring schemes to each of them. A Bernoulli EWMA is suggested to monitor the outlier process.

The EWMA control chart scheme for the sample mean proposed by Ng & Case (1989) is constructed by assuming that the data used in the computation of the limits are outlier free. This assumption may not be true in real situations since outliers often occur in the data used to compute the control limits. Outliers may consist of single unusual values which happen due to a sporadic special cause. Such outliers act only on occasional observations in a subgroup

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and not on subgroups as a whole. These outliers have to be detected, investigated and the special cause removed if possible. The presence of outliers will reduce the sensitivity of a control chart because the control limits are stretched so that the detection of the outliers themselves become more difficult. Furthermore, these stretched limits also make it more difficult for other types of out-of-control signals to be detected (Rocke, 1989, 1992). The purpose of this article is to propose a robust EWMA (EWMASMQ) chart for the process mean as an alternative which is superior to the standard EWMA (EWMASM) chart for the process mean. The EWMASMQ chart is constructed based on the limits that are set using an estimate of the process standard deviation using the average of the subgroup interquartile ranges (IQRs) rather than the average of the subgroup ranges in the case of the EWMASM. Thus, the EWMASMQ chart is less affected by outliers compared to the EWMASM chart. The next section gives an overview of the EWMASM chart.

The EWMASM Chart

The EWMASM chart is based on the following statistic (Ng & Case, 1989):

$$\hat{W}_t = \alpha \overline{X}_t + (1-\alpha)\hat{W}_{t-1} \tag{1}$$

where $\overline{X}_t$ is the mean of sample t and α is a weighting constant. In the estimation of the limits based on the process data, m subgroups of size n each are taken and then the average of the m sample means, $\overline{\overline{X}}$ is computed. $\overline{\overline{X}}$ is used as the starting point, i.e., $\hat{W}_0 = \overline{\overline{X}}$. The average of the m sample ranges is computed to be $\overline{R}$.

The upper and lower control limits of the EWMASM chart are

$$UCL = \overline{\overline{X}} + F_1\overline{R} \tag{2a}$$

and

$$LCL = \overline{\overline{X}} - F_1\overline{R} \tag{2b}$$

respectively, where

$$F_1 = \frac{3}{d_2\sqrt{n}}\sqrt{\frac{\alpha}{2-\alpha}} \tag{3}$$

In equation (3), d_2 is a standard constant whose value depends on the sample size n. The values of d_2 and F_1 (Ng & Case, 1989) for the various sample sizes n are given in Tables 1 and 2 respectively.

A Proposed EWMASMQ Chart For The Process Mean

Similar to the EWMASM chart, every observed sample mean is transformed into a corresponding EWMA before it is plotted on the EWMASMQ chart. Let $\overline{X}_t$ represents the mean of sample t. Every $\overline{X}_t$ will be transformed into a corresponding EWMA, $\hat{W}_t$, using the transformation

$$\hat{W}_t = \alpha \overline{X}_t + (1-\alpha)\hat{W}_{t-1}, \; t = 1, 2, \ldots . \tag{4}$$

When developing the EWMASMQ chart based on process data, m subgroups of size n each must be taken to compute the estimate of the proces mean. The average of the m sample means will be used as the starting point, i.e., $\hat{W}_0 = \overline{\overline{X}}$. For the EWMASMQ chart, the interquartile range, IQR is defined as $X_{(b)} - X_{(a)}$, where ( ) denotes the order statistics $a = [n/4] + 1$ and $b = n - a + 1$. Here, $[y]$ represents the greatest integer that is less than or equal to y. It is shown by Rocke (1992) that the mathematical expectation of IQR can be defined as

$$E(\text{IQR}) = d_2^Q \sigma_X \tag{5}$$

Table 1. Factors for the EWMASM and EWMASMQ Charts

n	a	b	d_2	d_2^Q
2	1	2	1.128	1.1284
3	1	3	1.693	1.6926
4	2	3	2.059	0.5940
5	2	4	2.326	0.9900
6	2	5	2.534	1.2835
7	2	6	2.704	1.5147
8	3	6	2.847	0.9456
9	3	7	2.970	1.1439
10	3	8	3.078	1.3121
11	3	9	3.173	1.4577
12	4	9	3.258	1.0737
13	4	10	3.336	1.2057
14	4	11	3.407	1.3235
15	4	12	3.472	1.4298
16	5	12	3.532	1.1400
17	5	13	3.588	1.2389
18	5	14	3.640	1.3269
19	5	15	3.689	1.4132
20	6	15	3.735	1.1806

Table 2. Factors of Control Limits for the EWMASM, F_1

n	α										
	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2	0.000	0.431	0.627	0.790	0.940	1.085	1.231	1.380	1.535	1.701	1.880
3	0.000	0.235	0.341	0.430	0.512	0.591	0.670	0.751	0.835	0.925	1.023
4	0.000	0.167	0.243	0.306	0.365	0.421	0.477	0.535	0.595	0.659	0.729
5	0.000	0.132	0.192	0.242	0.289	0.333	0.378	0.423	0.471	0.522	0.577
6	0.000	0.111	0.161	0.203	0.242	0.279	0.316	0.354	0.394	0.437	0.483
7	0.000	0.096	0.140	0.176	0.210	0.242	0.274	0.307	0.342	0.379	0.419
8	0.000	0.086	0.124	0.157	0.187	0.215	0.244	0.274	0.305	0.337	0.373
9	0.000	0.077	0.112	0.142	0.169	0.195	0.221	0.247	0.275	0.305	0.337
10	0.000	0.071	0.103	0.129	0.154	0.178	0.202	0.226	0.251	0.279	0.308
11	0.000	0.065	0.095	0.120	0.143	0.165	0.187	0.209	0.233	0.258	0.285
12	0.000	0.061	0.089	0.112	0.133	0.154	0.174	0.195	0.217	0.241	0.266
13	0.000	0.057	0.083	0.105	0.125	0.144	0.163	0.183	0.203	0.225	0.249
14	0.000	0.054	0.078	0.099	0.118	0.136	0.154	0.172	0.192	0.213	0.235
15	0.000	0.051	0.074	0.094	0.112	0.129	0.146	0.164	0.182	0.202	0.223
16	0.000	0.049	0.071	0.089	0.106	0.122	0.139	0.156	0.173	0.192	0.212
17	0.000	0.047	0.068	0.085	0.102	0.117	0.133	0.149	0.166	0.184	0.203
18	0.000	0.045	0.065	0.081	0.097	0.112	0.127	0.142	0.158	0.175	0.194
19	0.000	0.043	0.062	0.079	0.094	0.108	0.122	0.137	0.153	0.169	0.187
20	0.000	0.041	0.060	0.076	0.090	0.104	0.118	0.132	0.147	0.163	0.180

where d_2^Q is a constant whose value depends on the sample size n (see Table 1). The standard deviation, σ_X is estimated by

$$\hat{\sigma}_X = \frac{\overline{\text{IQR}}}{d_2^Q} \qquad (6)$$

where $\overline{\text{IQR}}$ is the average of the subgroup interquartile ranges.

Assuming that all the observed data are independent from one sample to another, then as t increases,

$$Var\left(\hat{W}_t\right) = Var\left(\overline{X}\right)\frac{\alpha}{2-\alpha} \qquad (7)$$

and

$$\sigma_{\hat{W}_t} = \frac{\sigma_X}{\sqrt{n}}\sqrt{\frac{\alpha}{2-\alpha}}. \qquad (8)$$

From equation (6), by using the average interquartile range to estimate the standard deviation of $\hat{W}_t$, $\hat{\sigma}_{\hat{W}_t}$, the following is obtained

$$\hat{\sigma}_{\hat{W}_t} = \frac{\overline{\text{IQR}}}{d_2^Q\sqrt{n}}\sqrt{\frac{\alpha}{2-\alpha}}. \qquad (9)$$

Thus, the limits of the EWMASMQ chart are

$$\text{Center Line} = \hat{W}_0 = \overline{\overline{X}} \qquad (10)$$

$$\text{Control Limits} = \hat{W}_0 \pm 3\hat{\sigma}_{\hat{W}_t} = \overline{\overline{X}} \pm 3\frac{\overline{\text{IQR}}}{d_2^Q\sqrt{n}}\sqrt{\frac{\alpha}{2-\alpha}} \qquad (11)$$

If

$$G_1 = \frac{3}{d_2^Q\sqrt{n}}\sqrt{\frac{\alpha}{2-\alpha}} \qquad (12)$$

then the control limits calculated based on equation (11) are

$$\text{UCL}_{\text{SMQ}} = \overline{\overline{X}} + G_1\overline{\text{IQR}} \qquad (13a)$$

and

$$\text{LCL}_{\text{SMQ}} = \overline{\overline{X}} - G_1\overline{\text{IQR}}. \qquad (13b)$$

Values of G_1 for different sample sizes, n, and smoothing constants, α, are listed in Table 3.

Table 3. Factors of Control Limits for the EWMASMQ, G_1

Sample Size, n	Value of α										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	0.0000	0.4313	0.6266	0.7897	0.9400	1.0854	1.2307	1.3795	1.5350	1.7005	1.8799
3	0.0000	0.2348	0.3411	0.4299	0.5117	0.5908	0.6699	0.7509	0.8355	0.9256	1.0233
4	0.0000	0.5793	0.8418	1.0608	1.2626	1.4580	1.6532	1.8530	2.0619	2.2842	2.5253
5	0.0000	0.3109	0.4517	0.5693	0.6776	0.7824	0.8872	0.9944	1.1065	1.2258	1.3552
6	0.0000	0.2189	0.3181	0.4009	0.4771	0.5509	0.6247	0.7002	0.7791	0.8631	0.9542
7	0.0000	0.1717	0.2495	0.3145	0.3743	0.4322	0.4901	0.5493	0.6112	0.6771	0.7486
8	0.0000	0.2573	0.3739	0.4712	0.5608	0.6476	0.7343	0.8231	0.9158	1.0146	1.1217
9	0.0000	0.2006	0.2914	0.3672	0.4371	0.5047	0.5723	0.6415	0.7138	0.7907	0.8742
10	0.0000	0.1659	0.2410	0.3037	0.3615	0.4174	0.4733	0.5306	0.5903	0.6540	0.7230
11	0.0000	0.1424	0.2068	0.2607	0.3103	0.3583	0.4062	0.4553	0.5067	0.5613	0.6205
12	0.0000	0.1850	0.2689	0.3388	0.4033	0.4657	0.5280	0.5919	0.6586	0.7296	0.8066
13	0.0000	0.1583	0.2300	0.2899	0.3450	0.3984	0.4518	0.5064	0.5635	0.6242	0.6901
14	0.0000	0.1390	0.2019	0.2545	0.3029	0.3498	0.3966	0.4445	0.4946	0.5480	0.6058
15	0.0000	0.1243	0.1806	0.2276	0.2709	0.3128	0.3547	0.3975	0.4423	0.4900	0.5418
16	0.0000	0.1509	0.2193	0.2764	0.3289	0.3798	0.4307	0.4828	0.5372	0.5951	0.6579
17	0.0000	0.1347	0.1958	0.2467	0.2937	0.3391	0.3845	0.4310	0.4795	0.5312	0.5873
18	0.0000	0.1223	0.1776	0.2239	0.2665	0.3077	0.3489	0.3910	0.4351	0.4820	0.5329
19	0.0000	0.1117	0.1623	0.2046	0.2435	0.2812	0.3188	0.3574	0.3976	0.4405	0.4870
20	0.0000	0.1304	0.1894	0.2387	0.2841	0.3281	0.3720	0.4169	0.4639	0.5140	0.5682

Comparison Of Performances

The performance of the EWMASMQ chart is compared to that of the EWMASM by performing a Monte-Carlo simulation. The following four different conditions are considered for the two control charts for the process mean:

(i) The In-control situation where the data are all standard normal random variables.

(ii) The Outliers situation where the data are a mixture of 95% standard normal and 5% data with five times the standard deviation. These outliers might represent an episodic phenomenon resulting from a sporadic special cause that control charts should detect.

(iii) The Special Cause situation where there is an additional $N(\delta,1)$ source of variability added to the subgroup means. Here, $\delta = \frac{\mu - \mu_0}{\sigma_0}$ where $\mu_0 = 0$ and $\sigma_0 = 1$ represent the nominal mean and standard deviation respectively while μ is the off-target mean. The values of $\delta \in \{0, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 4\}$ are considered. The control charts should detect this special cause.

(iv) The Outliers and Special Cause situation which consists of the data that are a mixture of 95% standard normal and 5% data with five times the standard deviation together with an additional component of variation, $N(\delta,1)$ added to the subgroup means.

Simulation studies based on the above four conditions are conducted using SAS version 8. Repeatedly, m = 10 and 20 subgroups of size n = 5 observations each are generated, control limits computed and the number of subgroups that fall outside the control limits are calculated. This procedure is repeated 10 000 times for a total of 10 000 $\times$ m subgroups. The proportions of out-of-control subgroups (based on 10 000 $\times$ m subgroups) are computed for the four different conditions and two different charting methods.

The results for the EWMASM chart are displayed in Tables 4 and 5 for m = 10 and 20 respectively. Similarly, Tables 6 and 7 give the results of the EWMASMQ chart for m = 10 and 20 respectively. An ideal procedure is the chart which gives a higher proportion for detecting out-of-control signals for the three conditions of Outliers, Special Cause and Outliers and Special Cause and a lower signal proportion for the In-control situation.

A comparison of the results in Tables 4 and 6 for m = 10 show that for fixed values of α and δ, the out-of-control proportions of the EWMASMQ chart are higher than the corresponding values of the EWMASM chart for the two out-of-control conditions of Outliers and Outliers and Special Cause. For example, when α = 0.5, the out-of-control proportion in Table 6 is 0.0242 while that in Table 4 is much lower at only 0.0047 for the Outliers condition. For the Outliers and Special Cause condition, the proportions of out-of-control in Table 4 when α = 0.5 are {0.0194, 0.0388, …, 0.9976} while the corresponding proportions in Table 6 are {0.0554, 0.0878, …, 0.9992} where the values of the former are all lower than that of the latter for $\delta \in \{0, 0.25, \ldots, 4\}$.

This shows that the EWMASMQ chart is more sensitive to out-of-control conditions when outliers are present in the data. The limits computed from the estimate of the interquartile ranges for the EWMASMQ chart are less influenced by outliers, compared to that of the EWMASM chart whose limits are computed based on the sample ranges. Thus, the EWMASMQ chart is more robust than the EWMASM and the former is a better alternative in the detection of a special cause when outliers are present. The EWMASMQ chart is also superior to the EWMASM when only outliers are present. The results in both Tables 4 and 6 indicate that for the Special Cause condition, the EWMASMQ chart is superior to the EWMASM for smaller values of δ and that both charts have comparable performances for larger values of δ.

Table 4. Proportions of Out-of-Control for the EWMASM Chart Under Four Different Conditions based on m = 10 and n = 5

		EWMASM α								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
In-Control		0.0000	0.0001	0.0003	0.0005	0.0009	0.0013	0.0017	0.0022	0.0025
Outliers		0.0000	0.0002	0.0009	0.0023	0.0047	0.0066	0.0093	0.0107	0.0126
	δ									
Special Cause	0	0.0030	0.0106	0.0168	0.0214	0.0256	0.0286	0.0311	0.0327	0.0338
	0.25	0.0377	0.0601	0.0638	0.0618	0.0600	0.0568	0.0550	0.0528	0.0504
	0.5	0.2395	0.2786	0.2502	0.2114	0.1816	0.1546	0.1353	0.1187	0.1056
	0.75	0.5162	0.5703	0.5337	0.4633	0.3946	0.3317	0.2829	0.2399	0.2045
	1	0.6813	0.7482	0.7484	0.7035	0.6341	0.5529	0.4772	0.4090	0.3483
	1.5	0.8246	0.8803	0.9051	0.9150	0.9097	0.8809	0.8296	0.7606	0.6802
	2	0.8858	0.9323	0.9559	0.9684	0.9750	0.9760	0.9684	0.9478	0.9077
	2.5	0.9181	0.9661	0.9839	0.9899	0.9929	0.9942	0.9948	0.9929	0.9847
	3	0.9469	0.9880	0.9953	0.9977	0.9985	0.9990	0.9992	0.9993	0.9986
	4	0.9904	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Outliers and Special Cause	0	0.0013	0.0056	0.0108	0.0154	0.0194	0.0234	0.0262	0.0287	0.0303
	0.25	0.0159	0.0296	0.0366	0.0381	0.0388	0.0399	0.0397	0.0401	0.0394
	0.5	0.1241	0.1568	0.1457	0.1255	0.1097	0.0961	0.0853	0.0773	0.0700
	0.75	0.3492	0.3908	0.3535	0.2971	0.2480	0.2071	0.1749	0.1490	0.1268
	1	0.5464	0.6025	0.5726	0.5070	0.4363	0.3643	0.3064	0.2574	0.2176
	1.5	0.7440	0.8079	0.8214	0.8027	0.7630	0.7000	0.6265	0.5484	0.4728
	2	0.8309	0.8847	0.9084	0.9169	0.9114	0.8881	0.8511	0.7953	0.7276
	2.5	0.8792	0.9270	0.9499	0.9609	0.9644	0.9605	0.9487	0.9243	0.8859
	3	0.9091	0.9553	0.9739	0.9819	0.9857	0.9857	0.9824	0.9728	0.9569
	4	0.9542	0.9872	0.9944	0.9967	0.9976	0.9981	0.9979	0.9964	0.9933

Table 5. Proportions of Out-of-Control for the EWMASM Chart Under Four Different Conditions based on m = 20 and n = 5

		EWMASM α								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
In-Control		0.0000	0.0003	0.0007	0.0012	0.0017	0.0020	0.0024	0.0026	0.0029
Outliers		0.0001	0.0017	0.0048	0.0080	0.0113	0.0140	0.0160	0.0180	0.0191
	δ									
Special Cause	0	0.0079	0.0170	0.0228	0.0258	0.0287	0.0306	0.0320	0.0330	0.0338
	0.25	0.1162	0.1045	0.0879	0.0753	0.0678	0.0612	0.0574	0.0538	0.0507
	0.5	0.5068	0.4269	0.3252	0.2510	0.2035	0.1677	0.1425	0.1222	0.1064
	0.75	0.7492	0.7439	0.6492	0.5316	0.4343	0.3545	0.2948	0.2463	0.2078
	1	0.8381	0.8695	0.8503	0.7784	0.6844	0.5848	0.4961	0.4182	0.3514
	1.5	0.9114	0.9392	0.9516	0.9547	0.9427	0.9071	0.8477	0.7712	0.6855
	2	0.9424	0.9658	0.9776	0.9836	0.9870	0.9861	0.9772	0.9533	0.9116
	2.5	0.9588	0.9831	0.9918	0.9951	0.9965	0.9973	0.9974	0.9951	0.9867
	3	0.9728	0.9943	0.9979	0.9991	0.9995	0.9996	0.9997	0.9996	0.9990
	4	0.9958	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Outliers and Special Cause	0	0.0039	0.0117	0.0183	0.0227	0.0268	0.0293	0.0318	0.0330	0.0338
	0.25	0.0545	0.0582	0.0524	0.0488	0.0465	0.0446	0.0442	0.0430	0.0420
	0.5	0.3232	0.2577	0.1893	0.1464	0.1188	0.0997	0.0867	0.0767	0.0684
	0.75	0.6173	0.5575	0.4401	0.3358	0.2653	0.2095	0.1731	0.1443	0.1225
	1	0.7609	0.7622	0.6824	0.5685	0.4679	0.3764	0.3086	0.2497	0.2067
	1.5	0.8699	0.9021	0.9029	0.8721	0.8161	0.7331	0.6411	0.5487	0.4626
	2	0.9154	0.9414	0.9537	0.9555	0.9462	0.9196	0.8770	0.8109	0.7322
	2.5	0.9392	0.9629	0.9749	0.9808	0.9815	0.9772	0.9635	0.9400	0.8995
	3	0.9534	0.9772	0.9878	0.9921	0.9936	0.9930	0.9893	0.9810	0.9664
	4	0.9763	0.9947	0.9979	0.9988	0.9991	0.9993	0.9990	0.9980	0.9954

Table 6. Proportions of Out-of-Control for the EWMASMQ Chart Under Four Different Conditions based on m = 10 and n = 5

		EWMASMQ α								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
In-Control		0.0000	0.0002	0.0006	0.0013	0.0023	0.0030	0.0038	0.0045	0.0052
Outliers		0.0010	0.0050	0.0119	0.0189	0.0242	0.0299	0.0336	0.0366	0.0387
	δ									
Special Cause	0	0.0059	0.0152	0.0228	0.0286	0.0329	0.0362	0.0388	0.0406	0.0417
	0.25	0.0465	0.0703	0.0745	0.0727	0.0697	0.0666	0.0639	0.0615	0.0591
	0.5	0.2462	0.2858	0.2588	0.2220	0.1912	0.1661	0.1459	0.1296	0.1162
	0.75	0.5140	0.5661	0.5314	0.4666	0.4001	0.3410	0.2915	0.2500	0.2161
	1	0.6794	0.7448	0.7439	0.6994	0.6302	0.5523	0.4802	0.4134	0.3562
	1.5	0.8228	0.8802	0.9040	0.9122	0.9038	0.8730	0.8208	0.7527	0.6738
	2	0.8859	0.9322	0.9546	0.9660	0.9726	0.9728	0.9639	0.9404	0.8989
	2.5	0.9205	0.9639	0.9816	0.9884	0.9918	0.9936	0.9940	0.9913	0.9817
	3	0.9468	0.9855	0.9945	0.9972	0.9983	0.9987	0.9990	0.9990	0.9980
	4	0.9869	0.9990	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Outliers and Special Cause	0	0.0087	0.0241	0.0377	0.0487	0.0554	0.0618	0.0652	0.0694	0.0711
	0.25	0.0454	0.0731	0.0852	0.0888	0.0878	0.0878	0.0868	0.0859	0.0852
	0.5	0.2216	0.2616	0.2424	0.2168	0.1924	0.1717	0.1563	0.1426	0.1317
	0.75	0.4726	0.5166	0.4805	0.4215	0.3636	0.3173	0.2744	0.2425	0.2128
	1	0.6425	0.7021	0.6873	0.6360	0.5669	0.4970	0.4334	0.3775	0.3288
	1.5	0.8011	0.8574	0.8773	0.8759	0.8537	0.8085	0.7480	0.6814	0.6076
	2	0.8699	0.9178	0.9399	0.9499	0.9512	0.9429	0.9213	0.8841	0.8322
	2.5	0.9084	0.9521	0.9711	0.9791	0.9826	0.9824	0.9777	0.9653	0.9437
	3	0.9357	0.9757	0.9876	0.9923	0.9942	0.9946	0.9933	0.9899	0.9822
	4	0.9769	0.9961	0.9984	0.9991	0.9992	0.9994	0.9994	0.9991	0.9982

It is shown by the results in Tables 4 and 6 that the EWMASM chart has lower false alarm (Type-I error) rates than the EWMASMQ chart. However, it should be noted that the superiority of the EWMASMQ chart in comparison to the EWMASM chart for the two out-of-control conditions of Outliers and Outliers and Special Cause far outweigh its deficiency in terms of a higher rate of false alarm. For example, if α = 0.3, the proportion of false alarm of the EWMASMQ chart is twice that of the EWMASM but the former is thirteen times more efficient in detecting outliers compared to the latter. Because a comparison between Tables 5 and 7 shows similar trend, it is suffice to discuss and draw conclusions based on the above comparisons.

Note that the evaluation of the performances of the control charts discussed in this section are made based on the proportions of subgroup points falling outside the control limits and are not based on the average run length (ARL) values because the charts' parameters are estimated from the subgroup data, i.e., the nominal values of these parameters are assumed to be unknown.

Table 7. Proportions of Out-of-Control for the EWMASMQ Chart Under Four Different Conditions based on m = 20 and n = 5

		EWMASMQ α								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
In-Control		0.0001	0.0006	0.0014	0.0020	0.0026	0.0030	0.0035	0.0038	0.0040
Outliers		0.0024	0.0115	0.0204	0.0268	0.0321	0.0350	0.0378	0.0402	0.0417
	δ									
Special Cause	0	0.0099	0.0197	0.0257	0.0295	0.0323	0.0343	0.0360	0.0369	0.0379
	0.25	0.1205	0.1093	0.0925	0.0809	0.0723	0.0661	0.0617	0.0581	0.0550
	0.5	0.5026	0.4270	0.3282	0.2570	0.2085	0.1733	0.1468	0.1273	0.1114
	0.75	0.7462	0.7395	0.6438	0.5312	0.4346	0.3581	0.2987	0.2511	0.2122
	1	0.8370	0.8680	0.8461	0.7756	0.6797	0.5825	0.4955	0.4195	0.3549
	1.5	0.9106	0.9389	0.9511	0.9541	0.9398	0.9025	0.8423	0.7658	0.6817
	2	0.9421	0.9657	0.9771	0.9832	0.9862	0.9851	0.9751	0.9500	0.9064
	2.5	0.9590	0.9822	0.9914	0.9948	0.9963	0.9970	0.9970	0.9943	0.9850
	3	0.9730	0.9935	0.9977	0.9989	0.9994	0.9996	0.9997	0.9995	0.9988
	4	0.9948	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Outliers and Special Cause	0	0.0181	0.0379	0.0503	0.0576	0.0626	0.0657	0.0673	0.0692	0.0698
	0.25	0.1183	0.1210	0.1101	0.1022	0.0973	0.0919	0.0895	0.0860	0.0845
	0.5	0.4536	0.3845	0.3052	0.2486	0.2090	0.1788	0.1589	0.1415	0.1295
	0.75	0.7082	0.6774	0.5755	0.4747	0.3945	0.3303	0.2797	0.2412	0.2100
	1	0.8139	0.8330	0.7857	0.7015	0.6075	0.5216	0.4444	0.3798	0.3252
	1.5	0.8981	0.9274	0.9352	0.9240	0.8918	0.8387	0.7714	0.6939	0.6137
	2	0.9345	0.9583	0.9695	0.9746	0.9717	0.9597	0.9365	0.8976	0.8420
	2.5	0.9533	0.9761	0.9860	0.9900	0.9914	0.9896	0.9833	0.9716	0.9492
	3	0.9672	0.9884	0.9917	0.9965	0.9973	0.9970	0.9956	0.9917	0.9844
	4	0.9894	0.9984	0.9993	0.9995	0.9997	0.9997	0.9996	0.9993	0.9983

An Example of Application

An example will be given to illustrate how the EWMASMQ chart is put to work in a real situation. The EWMASM chart is also constructed so that a comparison between the two approaches can be made. This example is based on the data from Wadsworth, Stephens and Godfrey (1986) and concerns the melt index of an extrusion grade polyethylene compound. As part of a study of the process, 20 subgroups of four each are taken. Table 8 gives the data of this process.

The limits of the EWMASM chart are computed using equations (2a) and (2b) while that of the EWMASMQ chart are calculated from equations (13a) and (13b). Figure 1 shows the EWMASM and EWMASMQ charts together with their respective control limits. The limits of the EWMASM chart are represented by UCL/LCL and those of the EWMASMQ chart by UCL_{SMQ} / LCL_{SMQ} .

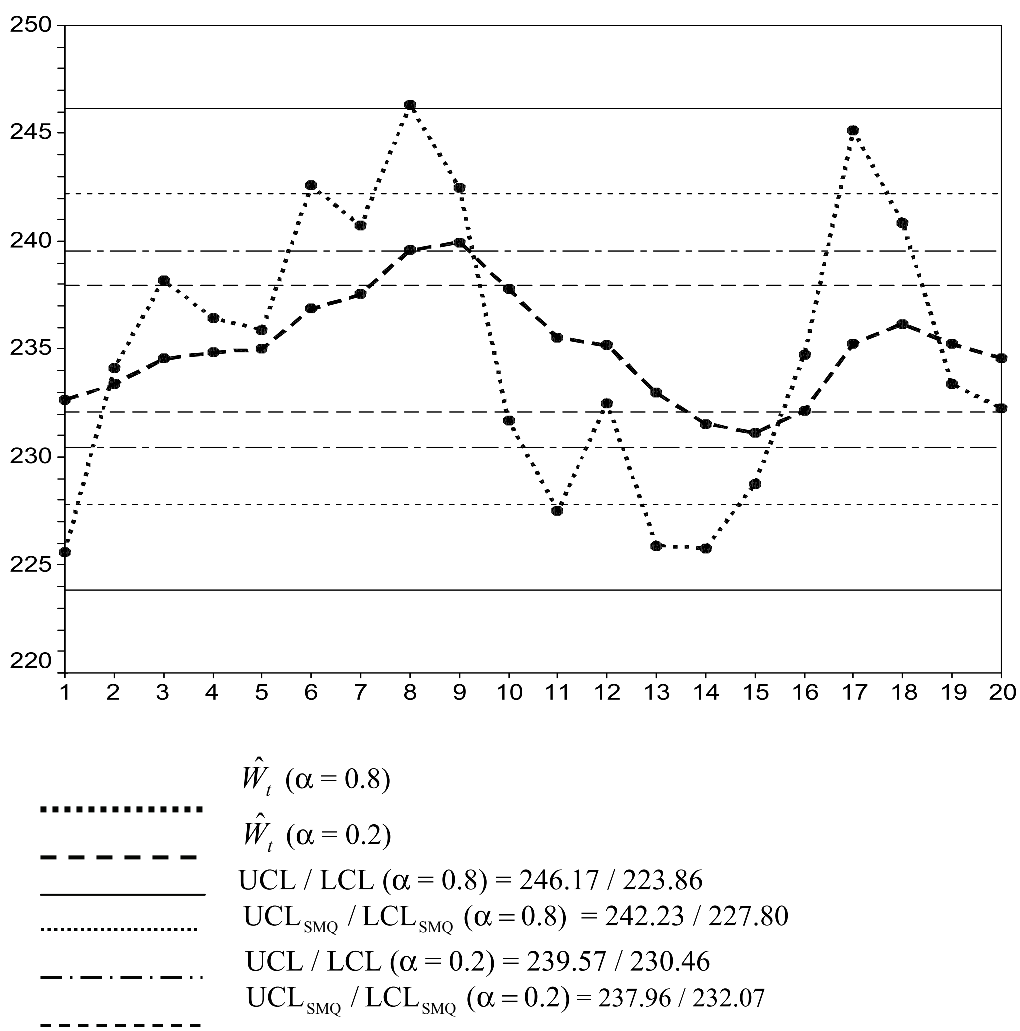

Figure 1. The EWMASM and EWMASMQ Charts for Melt Index Data

The EWMASM chart with α = 0.2 detects out-of-control points at subgroups 8 and 9 while the corresponding EWMASMQ chart signals at subgroups 8, 9, 14 and 15. The EWMASMQ chart detects two additional out-of-control points (i.e., subgroups 14 and 15) besides the two points at subgroups 8 and 9 which are also detected by the EWMASM chart. From Figure 1 for α = 0.2, a shift is observed that is gradually increasing between subgroups 1 and 9 followed by a shift which is gradually decreasing until subgroup 15. Here, both the EWMASM and EWMASMQ charts have successfully detected the upward shift but only the EWMASMQ chart managed to detect the downward shift. The presence of sample ranges with large values such as those in subgroups 3, 4, 6 and 8 cause the average sample range, $\overline{R}$ to be overestimated, hence widening the limits of the EWMASM chart so that the chart is less sensitive in detecting shifts in the mean. On the contrary, the EWMASMQ chart does not face this problem since its limits are computed based on the average sample interquartile range (IQR).

The EWMASM and EWMASMQ charts with α = 0.8 give more weight to the current sample average, $\overline{X}_t$ compared to the charts with α = 0.2. Thus, a weighting constant of α = 0.8 makes the two charts more sensitive to single subgroup averages with big or small values. Out-of-control points are detected at subgroups 1, 6, 8, 9, 11, 13, 14 and 17 by the EWMASMQ chart and at only subgroup 8 by the EWMASM chart. From the sample averages, $\overline{X}_t$ in Table 8, we notice that the values for subgroups 6, 8, 9 and 17 are somewhat bigger while those for subgroups 1, 11, 13 and 14 are somewhat smaller than the other sample averages. Investigations need to be carried out to search for assignable causes before these subgroups are classified as out-of-control points. It should be noted that the EWMASM chart with α = 0.8 fails to detect any subgroup average that plots below the LCL.

Table 8. Subgroups of Melt Index Measurements and the Computed Sample Means, Sample Ranges, Sample Interquartile Ranges and $\hat{W}_t$ Statistics

Sub. No., t	Observations				$\overline{X}_t$	R_t	IQR_t	$\hat{W}_t$	
	X_1	X_2	X_3	X_4				α=0.2	α=0.8
1	218	224	220	231	223.25	13	4	232.66	225.60
2	228	236	247	234	236.25	19	2	233.38	234.12
3	280	228	228	221	239.25	59	0	234.55	238.22
4	210	249	241	246	236	39	5	234.84	236.45
5	243	240	230	230	235.75	13	10	235.02	235.89
6	225	250	258	244	244.25	33	6	236.87	242.58
7	240	238	240	243	240.25	5	0	237.55	240.72
8	244	248	265	234	247.75	31	4	239.59	246.34
9	238	233	252	243	241.5	19	5	239.97	242.47
10	228	238	220	230	229	18	2	237.78	231.69
11	218	232	230	226	226.5	14	4	235.52	227.54
12	226	231	236	242	233.75	16	5	235.17	232.51
13	224	221	230	222	224.25	9	2	232.98	225.90
14	230	220	227	226	225.75	10	1	231.54	225.78
15	224	228	226	240	229.5	16	2	231.13	228.76
16	232	240	241	232	236.25	9	8	232.15	234.75
17	243	250	248	250	247.75	7	2	235.27	245.15
18	247	238	244	230	239.75	17	6	236.17	240.83
19	224	228	228	246	231.5	22	0	235.23	233.37
20	236	230	230	232	232	6	2	234.59	232.27
					$\overline{\overline{X}}$ = 235.0125	$\overline{R}$ = 18.75	$\overline{\text{IQR}}$ = 3.5		

Conclusion

This article discusses a new robust EWMA control chart for the sample mean which is referred to as the EWMASMQ chart. It is shown by simulation that the EWMASMQ chart is a superior alternative to the EWMASM chart when one is concern with the presence of outliers. Generally, the new EWMASMQ chart allows easier detection of outliers in the subgroups and is also more sensitive to other forms of out-of-control behavior when outliers are present. An example is given to show how the EWMASMQ chart works in a real situation. This example also illustrates the superiority of the EWMASMQ chart to the EWMASM chart, hence making the EWMASMQ as an attractive alternative to quality control practitioners.

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