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# A Comparison of Risk Classification Methods for Claim Severity Data

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# A Comparison of Risk Classification Methods for Claim Severity Data

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The objective of this article is to compare several risk classification methods for claim severity data by using weighted equation which is written as a weighted difference between the observed and fitted values. The weighted equation will be applied to estimate claim severities which is equivalent to the total claim costs divided by the number of claims.

Key words: Risk classification, claim severity, claim cost.

## Introduction

The process of establishing premium rates for insuring uncertain events requires estimates which were made of two important elements; the probabilities or frequencies associated with the occurrence of such event, and the magnitude or severities of such event. The process of grouping risks of similar risk characteristics for the frequencies or severities is also known as risk classification. The risks may be categorized according to risk or rating factors. In motor insurance for instance, the driver's gender and claim experience, or the vehicle's make and capacity, may be considered as rating factors.

In the last forty years, researchers suggested various statistical procedures for risk classification. For example, Bailey and Simon (1960) suggested the minimum chi-squares, Bailey (1963) proposed the zero bias, Jung (1968) produced a heuristic method for minimum modified chi-squares, Ajne (1975) applied the method of moments also for minimum modified chi-squares, Chamberlain (1980) used the weighted least squares, Coutts (1984) produced the method of orthogonal weighted least squares with logit transformation, Harrington (1986) suggested the maximum likelihood procedure for models with functional form, and Brown (1988) proposed the bias and likelihood functions.

In the recent actuarial literature, research on risk classification methods is still continuing and developing. For example, Mildenhall (1999) studied the relationship between minimum bias and Generalized Linear Models (GLMs), Feldblum and Brosius (2003) provided minimum bias procedures for practicing actuary, Anderson *et al.* (2004) provided practical insights for GLMs analysis also for practicing actuary, Fu and Wu (2005) developed and generalized the minimum bias models, Ismail and Jemain (2005a) bridged the minimum bias and maximum likelihood methods for claim frequency data, and Ismail and Jemain (2005b) proposed the Negative Binomial and Generalized Poisson regressions as alternatives for the Poisson to handle over-dispersion.

In addition to statistical procedures, research on multiplicative and additive models has also been carried out. Bailey and Simon (1960) compared systematic bias and found that

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the multiplicative model overestimates the high risk classes, Jung (1968) and Ajne (1975) also found that the estimates for multiplicative model are positively biased, Bailey (1963) compared the models by producing two statistical criteria, i.e., minimum chi-squares and average absolute difference, Freifelder (1986) predicted the pattern of over and under estimation of the models if they were misspecified, Brown (1988) discussed the additive and multiplicative models which were derived from the maximum likelihood and minimum bias approaches, Jee (1989) compared the predictive accuracy of the models, Holler *et al.* (1999) compared their initial values sensitivity, and Mildenhall (1999) identified the GLMs with the additive and multiplicative models.

Based on the actuarial literature, studies for risk classification were centered on two main areas; risk classification methods, and multiplicative vs. additive models. The objective of this study is to compare several risk classification methods for multiplicative and additive models by using weighted equation which is written as a weighted difference between the observed and fitted values. In addition, the parameter solution for multiplicative and additive models will also be compared by using weighted solution. The weighted solution for multiplicative model is in the form of a weighted proportion of observed over fitted values, whereas for additive model, it is in the form of a weighted difference between observed and fitted values.

Although the weighted equation was previously suggested by Ismail and Abdul Aziz (2005a), the application was implemented on claim frequency data. Therefore, this study differs such that the weighted equation will be applied to estimate claim severity or average claim cost which is also equivalent to the total claim costs divided by the number of claims. Because the nature of claim frequency and severity data is different, the approach taken is also slightly modified.

It is well established that the claim cost distributions generally have positive support and are positively skewed. Because of these desired properties, the Gamma and Lognormal distributions have been widely used by the practitioners for modeling claim severities. As a comparison, several actuarial studies also reported severity results from Normal distribution. For example, Baxter *et al.* (1980) fit the U.K. own damage costs for privately owned and comprehensively insured vehicles to the weighted linear (additive) regression model by assuming that the variance is constant across the classes, McCullagh and Nelder (1989) reanalyzed the same data by fitting the costs to the Gamma by assuming that the coefficient of variation is constant across the classes and the mean is linear on reciprocal scale, Brockman and Wright (1992) fit the U.K. own damage costs for comprehensive policies also to the Gamma by using a log-linear (multiplicative) regression model, and Renshaw (1994) fit the U.K. motor insurance claim severity also to the Gamma log-linear regression model.

The fitting procedure for this study will be carried out by using two different approaches; classical and regression. The advantage of using the regression fitting procedure is that it can also be extended to other regression models, as long as the function of the fitted value is written in a specified linear form. In addition, the computation of the regression fitting procedure provides a faster convergence compared to the classical.

In this study, the risk classification methods will be compared on three types of severity data; Malaysian data, U.K. data (McCullagh & Nelder, 1989), and Canadian data (Bailey & Simon, 1960).

## Methodology

The related data sets for claim severity are $(c_i, y_i)$, $i = 1,2,...,n$, where $c_i$ and $y_i$ denotes the average claim cost adjusted for inflation and the claim count for the $i$th rating class. Therefore, the total claim cost is equal to the product of the claim count and the average claim cost, $y_i c_i$.

Let $\mathbf{x_i}$ be the vector of explanatory variables for the $i$th rating class, $\boldsymbol{\beta}$ the $p \times 1$ vector of regression parameters, and $\mathbf{f}$ the vector of fitted values.

If the model is assumed to be multiplicative, the fitted value is $f_i = \exp(\mathbf{x_i^T \beta})$, which can also be written as,

$$f_i = f_{i(-j)} \exp(\beta_j x_{ij}), \qquad (1)$$

where $f_{i(-j)}$ is the multiplicative fitted value without the $j$ th effect.

For an additive model, the fitted value is $f_i = \mathbf{x_i^T \beta}$, so that it can be written as,

$$f_i = f_{i(-j)} + \beta_j x_{ij}, \qquad (2)$$

where $f_{i(-j)}$ is the additive fitted value without the $j$ th effect.

Minimum Bias Models

The parameters for zero bias model are solved by equating (Bailey 1963),

$$\sum_i y_i c_i x_{ij} = \sum_i y_i f_i x_{ij}, \qquad j = 1,2,...p \qquad (3)$$

Therefore, Eq.(3) can also be written as a weighted difference between observed and fitted values,

$$\sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p, \qquad (4)$$

where the weight, $w_i$, is equal to $y_i x_{ij}$.

Substituting Eq.(1) for multiplicative model, Eq.(4) can be rewritten as,

$$\sum_i y_i c_i x_{ij} = \sum_i y_i f_{i(-j)} \exp(\beta_j x_{ij}) x_{ij}, \qquad j = 1,2,...,p \qquad (5)$$

The solution, $\exp(\beta_j)$, may be calculated from Eq.(5) because the value for $x_{ij}$ is either one or zero. The solution may be written as a weighted proportion,

$$\exp(\beta_j) = \sum_i v_i \frac{c_i}{f_{i(-j)}}, \qquad j = 1,2,..., p, \qquad (6)$$

where the weight, $v_i$, is equal to,

$$v_i = \frac{z_i}{\sum_i z_i}, \qquad (7)$$

and $z_i$ is $y_i f_{i(-j)} x_{ij}$.

If Eq.(2) is substituted for additive model, Eq.(4) can be rewritten as,

$$\sum_i y_i (c_i - f_{i(-j)}) x_{ij} = \sum_i y_i \beta_j x_{ij} x_{ij}, \; j = 1,2,..., p$$

Again, because the value for $x_{ij}$ is either one or zero, the solution, $\beta_j$, is obtainable and it is in the form of a weighted difference,

$$\beta_j = \sum_i v_i (c_i - f_{i(-j)}), \qquad , j = 1,2,..., p, \qquad (8)$$

where the weight, $v_i$, is also equal to Eq.(7). However, the equation for $z_i$ is $y_i x_{ij}$.

The parameters for minimum chi-squares model are solved by minimizing the chi-squares (Bailey & Simon 1960),

$$\chi^2 = \sum_i \frac{y_i (c_i - f_i)^2}{f_i},$$

or by equating,

$$\frac{\partial \chi^2}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p, \qquad (9)$$

where the weight, $w_i$, is

$$\frac{y_i(c_i + f_i)}{f_i^2}\frac{\partial f_i}{\partial \beta_j}.$$

The first derivative of the fitted value is equal to,

$$\frac{\partial f_i}{\partial \beta_j} = f_i x_{ij}, \qquad (10)$$

for multiplicative model and,

$$\frac{\partial f_i}{\partial \beta_j} = x_{ij}, \qquad (11)$$

for additive model.

Substituting Eq.(1) and Eq.(10) into Eq.(9) for multiplicative model, $\exp(\beta_j)$ is equal to Eq.(6), where the weight, $v_i$, is equal to Eq.(7). However, the value for $z_i$ is $y_i(c_i + f_i)x_{ij}$. If Eq.(2) and Eq.(11) are substituted into Eq.(9) for additive model, $\beta_j$ is equal to Eq.(8), where the weight, $v_i$, is also equal to Eq.(7). The value for $z_i$ is

$$\frac{y_i(c_i + f_i)}{f_i^2}x_{ij}.$$

Maximum Likelihood Models

Let $T_i = y_i C_i$ be the random variable for total claim costs. If $T_i$ is assumed to follow Normal distribution with mean $E(T_i) = y_i f_i$ and variance $Var(T_i) = \sigma^2$ (Brown 1988), the parameters are solved by using the likelihood equations,

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i(c_i - f_i) = 0, \qquad j = 1,2,\ldots,p, \qquad (12)$$

where $w_i$ is $y_i^2 \frac{\partial f_i}{\partial \beta_j}$. Substituting Eq.(1) and Eq.(10) into Eq.(12) for multiplicative model, $\exp(\beta_j)$ is equal to Eq.(6), where $v_i$ is equal to Eq.(7). However, the value for $z_i$ is $y_i^2 f_{i(-j)}^2 x_{ij}$. If Eq.(2) and Eq.(11) are substituted into Eq.(12) for additive model, $\beta_j$ is equal to Eq.(8), where $v_i$ is equal to Eq.(7). The value for $z_i$ is $y_i^2 x_{ij}$.

Let $T_i$ be Poisson distributed with mean $y_i f_i$. The likelihood equations can also be written as Eq.(12), but the value for $w_i$ is $\frac{y_i}{f_i}\frac{\partial f_i}{\partial \beta_j}$. Following the same procedure as the Normal distribution, the parameters for multiplicative model, $\exp(\beta_j)$, are equal to Eq.(6), where $v_i$ is equal to Eq.(7). However, the value for $z_i$ is $y_i f_{i(-j)} x_{ij}$. Therefore, the parameters for Poisson multiplicative are shown to be equivalent to the zero bias multiplicative. For additive model, $\beta_j$ is equal to Eq.(8), where $v_i$ is equal to Eq.(7). The value for $z_i$ is $\frac{y_i}{f_i}x_{ij}$.

If $T_i$ is exponentially distributed with mean $y_i f_i$, the likelihood equations can also be written as Eq.(12). However, the value for $w_i$ is $\frac{1}{f_i^2}\frac{\partial f_i}{\partial \beta_j}$. The parameters for multiplicative model, $\exp(\beta_j)$, are equal to Eq.(6), where $v_i$ is equal to Eq.(7). However, the value for $z_i$ is $x_{ij}$. For additive model, $\beta_j$ is equal to Eq.(8), where $v_i$ is equal to Eq.(7). The value for $z_i$ is $\frac{1}{f_i^2}x_{ij}$.

Let $T_i$ be Gamma distributed with mean $y_i f_i$ and variance $\sigma^2 y_i f_i^2$. The likelihood equations can also be written as Eq.(12), but the value for $w_i$ is $\frac{y_i}{f_i^2}\frac{\partial f_i}{\partial \beta_j}$. The parameters for multiplicative model, $\exp(\beta_j)$, are equal to

Eq.(6), where $v_i$ is equal to Eq.(7). However, the value for $z_i$ is $y_i x_{ij}$. For additive model, $\beta_j$ is equal to Eq.(8), where $v_i$ is equal to Eq.(7). The value for $z_i$ is $\frac{y_i}{f_i^2} x_{ij}$.

Other Models

The weighted equations shown by Eq.(4), Eq.(9) and Eq.(12) may also be extended to other error functions. For example, if the sum squares error is defined as (Brown 1988), $S = \sum_i y_i (c_i - f_i)^2$, the parameters are solved by using the least squares equations,

$$\frac{\partial S}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p, \tag{13}$$

where $w_i$ is $y_i \frac{\partial f_i}{\partial \beta_j}$. The parameters for multiplicative model, $\exp(\beta_j)$, are equal to Eq.(6), where $v_i$ is equal to Eq.(7). However, the value for $z_i$ is $y_i f_{i(-j)}^2 x_{ij}$. For additive model, $\beta_j$ is equal to Eq.(8), where $v_i$ is equal to Eq.(7). The value for $z_i$ is $y_i x_{ij}$. Therefore, the parameters for least squares additive are shown to be equivalent to the zero bias additive.

If the function of errors is a modified chi-squares which is defined as,

$$\chi^2_{\text{mod}} = \sum_i \frac{y_i}{c_i} (c_i - f_i)^2 ,$$

the weighted equation is equal to,

$$\frac{\partial \chi^2_{\text{mod}}}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p \tag{14}$$

where $w_i$ is

$$\frac{y_i}{c_i} \frac{\partial f_i}{\partial \beta_j} .$$

The parameters for multiplicative model, $\exp(\beta_j)$, are equal to Eq.(6) where $v_i$ is equal to Eq.(7). However, the value for $z_i$ is $\frac{y_i f_{i(-j)}^2}{c_i} x_{ij}$. For additive model, $\beta_j$ is equal to Eq.(8), where $v_i$ is equal to Eq.(7). The value for $z_i$ is $\frac{y_i}{c_i} x_{ij}$.

Table 1 summarizes the weighted equations and parameter solutions for all of the models discussed above. Based on the weighted equations and parameter solutions, the following conclusions can be made regarding the comparison of several risk classification methods which were discussed above:

- The parameter estimates for zero bias and Poisson multiplicative are equal. The parameter estimates for zero bias and least squares additive are also equal.
- The weighted equations and parameter solutions indicate that all models are similar. Each model is distinguished only by its weight.

Classical Fitting Procedure

In this study, the multiplicative and additive models will be fitted by using two different procedures; classical and regression. The classical fitting procedure was introduced by Bailey and Simon (1960). The procedure involves sequential iterations where each parameter, $\beta_j$, $j = 1,2,..., p$, is calculated individually in each sequence. In the first sequence, the value for $\beta_1^{(1)}$ is calculated by using the initial values, $\boldsymbol{\beta}^{(0)}$. The sequence is then repeated until the $p$th sequence, where $\beta_p^{(1)}$ is calculated and vector $\boldsymbol{\beta}^{(1)}$ is produced. The sequential iteration is then repeated until the values for $\boldsymbol{\beta}$ converged.

Table 1. Weighted equations and parameter solutions

| Models | $w_i$ for weighted equation, $\sum_i w_i(c_i - f_i) = 0$ | $z_i$ for multiplicative parameter solution, $\exp(\beta_j) = \sum_i v_i \frac{c_i}{f_{i(-j)}}$, where $v_i = \frac{z_i}{\sum_i z_i}$ | $z_i$ for additive parameter solution, $\beta_j = \sum_i v_i(c_i - f_{i(-j)})$, where $v_i = \frac{z_i}{\sum_i z_i}$ |
|---|---|---|---|
| Zero bias | $w_i = y_i x_{ij}$ | $z_i = y_i f_{i(-j)} x_{ij}$ | $z_i = y_i x_{ij}$ |
| Poisson | $w_i = \frac{y_i}{f_i} \frac{\partial f_i}{\partial \beta_j}$ | $z_i = y_i f_{i(-j)} x_{ij}$ | $z_i = \frac{y_i}{f_i} x_{ij}$ |
| Least squares | $w_i = y_i \frac{\partial f_i}{\partial \beta_j}$ | $z_i = y_i f_{i(-j)}^2 x_{ij}$ | $z_i = y_i x_{ij}$ |
| Minimum $\chi^2$ | $w_i = \frac{y_i(c_i + f_i)}{f_i^2} \frac{\partial f_i}{\partial \beta_j}$ | $z_i = y_i(c_i + f_i) x_{ij}$ | $z_i = \frac{y_i(c_i + f_i)}{f_i^2} x_{ij}$ |
| Normal | $w_i = y_i^2 \frac{\partial f_i}{\partial \beta_j}$ | $z_i = y_i^2 f_{i(-j)}^2 x_{ij}$ | $z_i = y_i^2 x_{ij}$ |
| Exponential | $w_i = \frac{1}{f_i^2} \frac{\partial f_i}{\partial \beta_j}$ | $z_i = x_{ij}$ | $z_i = \frac{1}{f_i^2} x_{ij}$ |
| Gamma | $w_i = \frac{y_i}{f_i^2} \frac{\partial f_i}{\partial \beta_j}$ | $z_i = y_i x_{ij}$ | $z_i = \frac{y_i}{f_i^2} x_{ij}$ |
| Minimum modified $\chi^2$ | $w_i = \frac{y_i}{c_i} \frac{\partial f_i}{\partial \beta_j}$ | $z_i = \frac{y_i f_{i(-j)}^2}{c_i} x_{ij}$ | $z_i = \frac{y_i}{c_i} x_{ij}$ |

As an example, the programming for zero bias multiplicative will be discussed here. The parameter solution is

$$\exp(\beta_j) = \sum_i v_i \frac{c_i}{f_{i(-j)}},$$

where

$$v_i = \frac{z_i}{\sum_i z_i}$$

and

$$z_i = y_i f_{i(-j)} x_{ij} .$$

Let $f_{i(-j)}$ be the $i$ th row of vector $\mathbf{f_{(-j)}}$. For multiplicative model, $\mathbf{f_{(-j)}} = \exp(\mathbf{X_{(-j)}}\boldsymbol{\beta}_{(-j)})$, where $\mathbf{X_{(-j)}}$ is the matrix of explanatory variables without the $j$ th column and $\boldsymbol{\beta}_{(-j)}$ the vector of regression parameters without the $j$ th row. Let $\mathbf{x_j}$ be the vector which is equivalent to the $j$ th column of matrix $\mathbf{X}$. Therefore, $x_{ij}$ is equal to the $i$ th row of vector $\mathbf{x_j}$. For each $j$, let $z_i = y_i f_{i(-j)} x_{ij}$. Therefore, the weight, $v_i$, is equal to $z_i$ divided by sum of $z_i$ for all $i$. Finally, the parameter solution, $\exp(\beta_j)$, is equal to the sum of $v_i \frac{c_i}{f_{i(-j)}}$ for all $i$.

An example of S-PLUS programming for zero bias multiplicative is given in Appendix A. Similar programming can also be used for all of the multiplicative and additive models which were discussed in this study. Each model should be differentiated only by three elements:

- The fitted values for multiplicative model are $\mathbf{f} = \exp(\mathbf{X}\boldsymbol{\beta})$ and $\mathbf{f_{(-j)}} = \exp(\mathbf{X_{(-j)}}\boldsymbol{\beta}_{(-j)})$. For additive model, the fitted values are $\mathbf{f} = \mathbf{X}\boldsymbol{\beta}$ and $\mathbf{f_{(-j)}} = \mathbf{X_{(-j)}}\boldsymbol{\beta}_{(-j)}$.
- The parameter solution is $\exp(\beta_j) = \sum_i v_i \frac{c_i}{f_{i(-j)}}$ for multiplicative model, and $\beta_j = \sum_i v_i (c_i - f_{i(-j)})$ for additive model.
- Each model has its own equation for $z_i$.

Regression Fitting Procedure

The regression fitting procedure involves standard iterations where all of the parameters, $\beta_j$, $j = 1,2,...,p$, are calculated simultaneously in each iteration. Because the parameters are solved simultaneously, the regression procedure provides a faster convergence compared to the classical procedure. In the first iteration, $\boldsymbol{\beta}^{(1)}$ are calculated by using initial values of $\boldsymbol{\beta}^{(0)}$. The iteration is then repeated until the values for $\boldsymbol{\beta}$ converged.

The parameters, $\beta_j$, $j = 1,2,...,p$, for regression fitting procedure are solved by minimizing $\sum_i w_i (c_i - f_i)^2$ or by equating,

$$\sum_i w_i (c_i - f_i) \frac{\partial f_i}{\partial \beta_j} = 0, \qquad , j = 1,2,...,p . \tag{15}$$

Therefore, the weighted equations for risk classification models shown by Eq.(4), Eq.(9), Eq.(12), Eq.(13) and Eq.(14) are also equivalent to Eq.(15). By using Taylor series approximation, the values for $\boldsymbol{\beta}$ in the $r$ th iteration is equal to,

$$\boldsymbol{\beta}^{(\mathbf{r})} = (\mathbf{Z}^{(\mathbf{r-1})\mathbf{T}}\mathbf{W}^{(\mathbf{r-1})}\mathbf{Z}^{(\mathbf{r-1})})^{-1}\mathbf{Z}^{(\mathbf{r-1})\mathbf{T}}\mathbf{W}^{(\mathbf{r-1})}(\mathbf{c} - \mathbf{s}^{(\mathbf{r-1})}) \tag{16}$$

where $\boldsymbol{\beta}^{(\mathbf{r})}$ and $\boldsymbol{\beta}^{(\mathbf{r-1})}$ are the values for $\boldsymbol{\beta}$ in the $r$ th and $r-1$ th iterations, $\mathbf{Z}^{(\mathbf{r-1})}$ the $n \times p$ matrix whose $ij$ th element is $\left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}^{(\mathbf{r-1})}}$, $\mathbf{W}^{(\mathbf{r-1})}$ the diagonal weight matrix evaluated at

$\boldsymbol{\beta}^{(\mathbf{r-1})}$, and $\mathbf{s}^{(\mathbf{r-1})}$ the vector whose $i$ th row is equal to

$$f_i(\boldsymbol{\beta}^{(\mathbf{r-1})}) - \sum_{j=1}^{p} \beta_j^{(r-1)} z_{ij}^{(r-1)} .$$

As an example, the programming for additive least squares will be discussed here. The weighted equation is

$$\sum_i y_i (c_i - f_i) \frac{\partial f_i}{\partial \beta_j} = 0, \; j = 1,2,..., p .$$

Therefore, the $i$ th diagonal element of the weight matrix is equal to $y_i$, which is free of $\boldsymbol{\beta}^{(\mathbf{r-1})}$. For an additive model, the $ij$ th element of matrix $\mathbf{Z}^{(\mathbf{r-1})}$ is equal to $\left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}^{(\mathbf{r-1})}} = x_{ij}$, which is also free of $\boldsymbol{\beta}^{(\mathbf{r-1})}$. Because $x_{ij}$ is the $ij$ th element of matrix $\mathbf{X}$ and the dimensions for $\mathbf{Z}^{(\mathbf{r-1})}$ and $\mathbf{X}$ are equal, $\mathbf{Z}^{(\mathbf{r-1})} = \mathbf{X}$ and $\mathbf{s}^{(\mathbf{r-1})} = \mathbf{f}(\boldsymbol{\beta}^{(\mathbf{r-1})}) - \mathbf{X}\boldsymbol{\beta}^{(\mathbf{r-1})} = 0$. Therefore, Eq.(16) for additive least squares is simplified into,

$$\boldsymbol{\beta}^{(\mathbf{r})} = \boldsymbol{\beta} = (\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{c}, \tag{17}$$

which is also equal to the Normal equation in the standard linear regression model. Eq.(17) also indicates that the parameters for additive least squares can be solved without any iteration.

For multiplicative model, the $ij$ th element of matrix $\mathbf{Z}^{(\mathbf{r-1})}$ is equal to, $\left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}^{(\mathbf{r-1})}} = f_i(\boldsymbol{\beta}^{(\mathbf{r-1})}) x_{ij}$. Therefore, the equation for $\mathbf{Z}^{(\mathbf{r-1})}$ may be written as,

$$\mathbf{Z}^{(\mathbf{r-1})} = \mathbf{F}^{(\mathbf{r-1})}\mathbf{X}, \tag{18}$$

where $\mathbf{F}^{(\mathbf{r-1})}$ is the diagonal matrix whose $i$ th diagonal elements is $f_i(\boldsymbol{\beta}^{(\mathbf{r-1})})$. The vector for $\mathbf{s}^{(\mathbf{r-1})}$ may be written as

$$\mathbf{s}^{(\mathbf{r-1})} = f(\boldsymbol{\beta}^{(\mathbf{r-1})}) - \mathbf{F}^{(\mathbf{r-1})}\mathbf{X}\boldsymbol{\beta}^{(\mathbf{r-1})} .$$

The advantage of using the regression fitting procedure is that besides multiplicative and additive models, the fitting can also be extended to other regression models as well. Therefore, the regression fitting procedure allows a variety of regression model to be created and applied, as long as the function of the fitted value is written as

$$f_i = \left( \sum_{j=1}^{p} \beta_j x_{ij} \right)^b, -1 \le b < 0, \;\; 0 < b \le 1 .$$

For example, if the fitted value is assumed to follow an inverse function, i.e., $b = -1$, the $ij$ th element of matrix $\mathbf{Z}^{(\mathbf{r-1})}$ is equal to $\left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}^{(\mathbf{r-1})}} = -\{f_i(\boldsymbol{\beta}^{(\mathbf{r-1})})\}^2 x_{ij}$. Therefore, the equation for $\mathbf{Z}^{(\mathbf{r-1})}$ may also be written as Eq.(18). However, the $i$ th diagonal element of matrix $\mathbf{F}^{(\mathbf{r-1})}$ is equal to $-\{f_i(\boldsymbol{\beta}^{(\mathbf{r-1})})\}^2$.

An example of S-PLUS programming for least squares multiplicative is given in Appendix B. Similar programming can also be used for either of the multiplicative, additive or inverse models. Each programming should be differentiated only by three elements:

- The vector for the fitted values is equal to $\mathbf{f} = \exp(\mathbf{X}\boldsymbol{\beta})$ for multiplicative model, $\mathbf{f} = \mathbf{X}\boldsymbol{\beta}$ for additive model, and $\mathbf{f} = (\mathbf{X}\boldsymbol{\beta})^{-1}$ for inverse model.

- $\mathbf{Z}^{(\mathbf{r-1})} = \mathbf{X}$ for additive model, and $\mathbf{Z}^{(\mathbf{r-1})} = \mathbf{F}^{(\mathbf{r-1})}\mathbf{X}$ for multiplicative and inverse models. However, the $i$ th diagonal element of matrix $\mathbf{F}^{(\mathbf{r-1})}$ is equal to $f_i(\boldsymbol{\beta}^{(\mathbf{r-1})})$ for multiplicative model, and $-\{f_i(\boldsymbol{\beta}^{(\mathbf{r-1})})\}^{-2}$ for inverse model.

- Each model has its own weight matrix.

## Results

### Malaysian Data

The risk classification methods will be compared on the Malaysian private car Third Party Property Damage (TPPD) average claim costs data. Specifically, the TPPD claim covers the legal liability for third party property loss or damage caused by or arising out of the use of an insured motor vehicle. The data, which was obtained from an insurance company in Malaysia and was supplied by the General Insurance Association of Malaysia (PIAM), was based on 170,000 private car policies in a three-year period of 1998-2000. The data consists of claim counts and average claim costs which were already paid as well as outstanding. The average claim costs, which were already adjusted for inflation, were given in Ringgit Malaysia (RM). The risks for the claims were associated with five rating factors; coverage type, vehicle make, vehicle use and driver's gender, vehicle year, and location. Altogether, there were $2 \times 2 \times 3 \times 4 \times 5 = 240$ cross-classified rating classes of claim severities to be estimated. The complete data is available by contacting the author.

The claim severities were fitted to all of the multiplicative and additive models which were discussed in this study. However, the fitting involves only 108 data points because 132 of the rating classes have zero claim count. In addition, the models will be evaluated by using two different tests; chi-squares and average absolute difference. The average absolute difference is equal to (Bailey and Simon 1960)

$$\frac{\sum_i y_i |c_i - f_i|}{\sum_i y_i c_i}.$$

Table 2 and Table 3 give the parameter estimates, chi-squares and average absolute difference for multiplicative and additive models of the Malaysian data.

Table 2: Multiplicative models for Malaysian data

| Parameters | Zero bias /Poisson | Least squares | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 7,467.43 | 7,459.97 | 7,460.65 | 7,493.35 | 7,229.48 | 7,480.11 | 7,486.69 |
| $\exp(\beta_2)$ Non-comp | 1.15 | 1.15 | 1.17 | 1.13 | 1.16 | 1.15 | 1.10 |
| $\exp(\beta_3)$ Foreign | 1.08 | 1.07 | 1.08 | 1.07 | 1.20 | 1.08 | 1.08 |
| $\exp(\beta_4)$ Female | 0.90 | 0.90 | 0.90 | 0.93 | 0.80 | 0.89 | 0.88 |
| $\exp(\beta_5)$ Business | 0.20 | 0.20 | 0.20 | 0.20 | 0.21 | 0.20 | 0.20 |
| $\exp(\beta_6)$ 2-3 years | 0.78 | 0.78 | 0.78 | 0.79 | 0.74 | 0.78 | 0.78 |
| $\exp(\beta_7)$ 4-5 years | 0.69 | 0.70 | 0.70 | 0.69 | 0.66 | 0.69 | 0.69 |
| $\exp(\beta_8)$ 6+ years | 0.73 | 0.73 | 0.73 | 0.72 | 0.72 | 0.73 | 0.73 |
| $\exp(\beta_9)$ North | 0.94 | 0.94 | 0.94 | 0.93 | 0.92 | 0.94 | 0.93 |
| $\exp(\beta_{10})$ East | 0.86 | 0.85 | 0.87 | 0.84 | 0.88 | 0.87 | 0.83 |
| $\exp(\beta_{11})$ South | 0.93 | 0.93 | 0.94 | 0.94 | 1.04 | 0.93 | 0.93 |
| $\exp(\beta_{12})$ East M'sia | 0.94 | 0.95 | 0.97 | 0.94 | 1.06 | 0.94 | 0.89 |
| $\chi^2$ ($10^{-5}$) | 3.76 | 3.77 | 3.73 | 3.89 | 6.69 | 3.77 | 4.08 |
| Absolute difference | 0.07 | 0.06 | 0.07 | 0.07 | 0.11 | 0.07 | 0.07 |

Table 3: Additive models for Malaysian data

| Parameters ($10^{-2}$) | Zero bias /Least squares | Poisson | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 74.08 | 74.07 | 74.07 | 74.74 | 72.82 | 74.09 | 74.10 |
| $\exp(\beta_2)$ Non-comp | 8.06 | 8.03 | 9.31 | 7.23 | 5.37 | 8.01 | 5.26 |
| $\exp(\beta_3)$ Foreign | 4.36 | 4.55 | 4.67 | 4.10 | 10.16 | 4.72 | 4.34 |
| $\exp(\beta_4)$ Female | -6.18 | -6.51 | -6.26 | -4.77 | -12.32 | -6.77 | -7.16 |
| $\exp(\beta_5)$ Business | -40.79 | -40.83 | -40.75 | -40.13 | -39.69 | -40.88 | -41.09 |
| $\exp(\beta_6)$ 2-3 years | -15.51 | -15.60 | -15.56 | -15.42 | -18.34 | -15.70 | -15.78 |
| $\exp(\beta_7)$ 4-5 years | -21.90 | -22.03 | -21.90 | -22.64 | -23.23 | -22.19 | -22.34 |
| $\exp(\beta_8)$ 6+ years | -19.53 | -19.54 | -19.72 | -20.38 | -19.15 | -19.56 | -19.15 |
| $\exp(\beta_9)$ North | -3.76 | -3.70 | -3.60 | -4.23 | -3.99 | -3.64 | -3.86 |
| $\exp(\beta_{10})$ East | -8.53 | -7.77 | -6.88 | -8.78 | -6.44 | -7.18 | -9.69 |
| $\exp(\beta_{11})$ South | -3.92 | -3.83 | -3.72 | -3.87 | 2.42 | -3.75 | -4.03 |
| $\exp(\beta_{12})$ East M'sia | -3.36 | -3.53 | -1.97 | -3.45 | 2.27 | -3.72 | -6.46 |
| $\chi^2$ ($10^{-5}$) | 3.71 | 3.70 | 3.66 | 3.81 | 6.21 | 3.70 | 4.01 |
| Absolute difference | 0.06 | 0.06 | 0.06 | 0.07 | 0.11 | 0.06 | 0.07 |

The classical and regression fitting procedures give equal values for parameter estimates. However, the regression procedure provides a faster convergence.

The multiplicative and additive models give similar parameter estimates. The smallest chi-squares is given by the minimum chi-squares model. Except for the exponential model, all models provide similar values for absolute difference.

U.K. Data

The U.K. data provides information on the Own Damage claim counts and average claim costs for privately owned and comprehensively insured vehicles (McCullagh & Nelder 1989). The average claim costs, which were already adjusted for inflation, were given in Pound Sterling. The risks for the claims were associated with three rating factors; policyholder's age, car group and vehicle age. Altogether, there were $8 \times 4 \times 4 = 128$ cross-classified rating classes of claim severities to be estimated.

The claim severities were fitted to all of the multiplicative and additive models which were discussed in this study. In addition, the severities were also fitted to the inverse models because McCullagh and Nelder (1989) also fit the severities to the Gamma regression model by assuming that the regression effects were linear on the reciprocal scale. The fitting involves only 123 data points because five of the rating classes have zero claim count.

Table 4, Table 5 and Table 6 give the parameter estimates, chi-squares and average absolute difference for multiplicative, additive and inverse models of the U.K. data.

Table 4: Multiplicative models for U.K. data

| Parameter | Zero bias /Poisson | Least squares | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 297.57 | 309.81 | 313.59 | 279.34 | 302.38 | 286.75 | 257.91 |
| $\exp(\beta_2)$ 21-24 yrs | 0.98 | 0.94 | 0.95 | 1.05 | 0.90 | 1.00 | 1.08 |
| $\exp(\beta_3)$ 25-29 yrs | 0.91 | 0.88 | 0.87 | 0.97 | 1.01 | 0.94 | 1.04 |
| $\exp(\beta_4)$ 30-34 yrs | 0.88 | 0.86 | 0.84 | 0.96 | 0.75 | 0.89 | 1.01 |
| $\exp(\beta_5)$ 35-39 yrs | 0.70 | 0.67 | 0.67 | 0.75 | 0.72 | 0.73 | 0.79 |
| $\exp(\beta_6)$ 40-49 yrs | 0.77 | 0.75 | 0.73 | 0.81 | 0.76 | 0.79 | 0.89 |
| $\exp(\beta_7)$ 50-59 yrs | 0.78 | 0.76 | 0.75 | 0.83 | 0.79 | 0.80 | 0.89 |
| $\exp(\beta_8)$ 60+ yrs | 0.78 | 0.77 | 0.74 | 0.82 | 0.75 | 0.80 | 0.90 |
| $\exp(\beta_9)$ B | 0.99 | 0.98 | 0.99 | 0.96 | 1.06 | 1.00 | 0.99 |
| $\exp(\beta_{10})$ C | 1.16 | 1.15 | 1.16 | 1.14 | 1.17 | 1.17 | 1.16 |
| $\exp(\beta_{11})$ D | 1.48 | 1.48 | 1.50 | 1.53 | 1.60 | 1.49 | 1.45 |
| $\exp(\beta_{12})$ 4-7 yrs | 0.91 | 0.90 | 0.91 | 0.95 | 0.89 | 0.92 | 0.91 |
| $\exp(\beta_{13})$ 8-9 yrs | 0.70 | 0.69 | 0.70 | 0.74 | 0.66 | 0.71 | 0.69 |
| $\exp(\beta_{14})$ 10+ yrs | 0.49 | 0.48 | 0.51 | 0.50 | 0.48 | 0.50 | 0.46 |
| $\chi^2$ ($10^{-4}$) | 3.10 | 3.13 | 3.07 | 3.27 | 4.50 | 3.12 | 3.40 |
| Absolute difference | 0.08 | 0.08 | 0.08 | 0.08 | 0.11 | 0.08 | 0.08 |

Table 5: Additive models for U.K. data

| Parameter | Zero bias /Least squares | Poisson | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 298.67 | 288.34 | 303.94 | 273.49 | 291.89 | 278.98 | 241.88 |
| $\exp(\beta_2)$ 21-24 yrs | -5.60 | 0.31 | -7.53 | 17.58 | -10.84 | 4.96 | 34.01 |
| $\exp(\beta_3)$ 25-29 yrs | -24.64 | -16.95 | -30.52 | -2.01 | 15.31 | -9.91 | 26.37 |
| $\exp(\beta_4)$ 30-34 yrs | -33.22 | -29.34 | -43.39 | -7.76 | -47.35 | -26.59 | 14.17 |
| $\exp(\beta_5)$ 35-39 yrs | -87.89 | -75.74 | -89.26 | -64.78 | -44.23 | -64.82 | -33.45 |
| $\exp(\beta_6)$ 40-49 yrs | -66.99 | -60.27 | -75.55 | -50.51 | -45.84 | -54.15 | -13.68 |
| $\exp(\beta_7)$ 50-59 yrs | -63.35 | -55.64 | -70.12 | -45.49 | -36.19 | -48.60 | -10.87 |
| $\exp(\beta_8)$ 60+ yrs | -63.15 | -56.91 | -72.15 | -47.39 | -44.32 | -51.10 | -10.32 |
| $\exp(\beta_9)$ B | -2.46 | -0.21 | -0.50 | -7.03 | 8.19 | 2.04 | -0.30 |
| $\exp(\beta_{10})$ C | 34.18 | 35.45 | 35.05 | 33.89 | 25.86 | 36.41 | 35.84 |
| $\exp(\beta_{11})$ D | 108.66 | 108.76 | 113.74 | 123.07 | 97.83 | 108.90 | 96.09 |
| $\exp(\beta_{12})$ 4-7 yrs | -24.21 | -21.54 | -21.98 | -10.57 | -30.60 | -19.62 | -20.39 |
| $\exp(\beta_{13})$ 8-9 yrs | -76.75 | -72.26 | -71.63 | -59.08 | -96.51 | -69.12 | -74.38 |
| $\exp(\beta_{14})$ 10+ yrs | -126.63 | -121.21 | -118.78 | -111.15 | -147.85 | -117.94 | -128.54 |
| $\chi^2$ ($10^{-4}$) | 3.41 | 3.35 | 3.32 | 3.55 | 4.88 | 3.40 | 3.77 |
| Absolute difference | 0.09 | 0.09 | 0.09 | 0.09 | 0.11 | 0.09 | 0.09 |

Table 6: Inverse models for U.K. data

| Parameter ($10^4$) | Poisson | Least squares | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 32.79 | 31.30 | 31.23 | 35.10 | 37.44 | 34.11 | 37.44 |
| $\exp(\beta_2)$ 21-24 yrs | 2.41 | 4.16 | 3.12 | -0.28 | -0.74 | 1.01 | -0.74 |
| $\exp(\beta_3)$ 25-29 yrs | 4.75 | 6.26 | 6.11 | 2.25 | 0.46 | 3.50 | 0.46 |
| $\exp(\beta_4)$ 30-34 yrs | 5.30 | 6.39 | 6.81 | 2.50 | 0.83 | 4.62 | 0.83 |
| $\exp(\beta_5)$ 35-39 yrs | 14.97 | 16.61 | 16.11 | 12.41 | 11.36 | 13.70 | 11.36 |
| $\exp(\beta_6)$ 40-49 yrs | 10.28 | 11.12 | 11.73 | 8.41 | 5.97 | 9.69 | 5.97 |
| $\exp(\beta_7)$ 50-59 yrs | 9.96 | 10.98 | 11.30 | 7.78 | 5.89 | 9.16 | 5.89 |
| $\exp(\beta_8)$ 60+ yrs | 9.75 | 10.58 | 11.26 | 7.88 | 5.32 | 9.20 | 5.32 |
| $\exp(\beta_9)$ B | 0.70 | 0.93 | 0.68 | 2.06 | 0.65 | 0.38 | 0.65 |
| $\exp(\beta_{10})$ C | -5.68 | -5.29 | -5.60 | -5.11 | -5.95 | -6.14 | -5.95 |
| $\exp(\beta_{11})$ D | -13.77 | -13.55 | -13.90 | -14.27 | -13.60 | -14.21 | -13.60 |
| $\exp(\beta_{12})$ 4-7 yrs | 3.95 | 4.21 | 3.99 | 2.65 | 3.88 | 3.66 | 3.88 |
| $\exp(\beta_{13})$ 8-9 yrs | 16.83 | 17.14 | 16.33 | 15.45 | 17.95 | 16.51 | 17.95 |
| $\exp(\beta_{14})$ 10+ yrs | 41.74 | 41.97 | 38.52 | 43.50 | 47.09 | 41.54 | 47.09 |
| $\chi^2$ ($10^{-4}$) | 3.10 | 3.13 | 3.07 | 3.27 | 3.37 | 3.12 | 3.37 |
| Absolute difference | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 |

As expected, the parameter estimates for classical and regression fitting procedures are equal and the regression fitting procedure provides a faster convergence.

The parameter estimates for multiplicative, additive and inverse models are similar. In particular, the parameter estimates for Gamma inverse model are equal to the parameter estimates produced by McCullagh and Nelder (1989). The smallest chi-squares is also given by the minimum chi-squares model. Except for the exponential model, all models provide equal values for absolute difference.

Canadian Data

The Canadian data, which was obtained from Bailey and Simon (1960), provides information on liability claim counts and average claim costs for private passenger automobile insurance. The data involves two rating factors; merit and class. Altogether, there were $4 \times 5 = 20$ cross-classified rating classes of claim severities to be estimated.

The claim severities were fitted to all of the multiplicative and additive models which were discussed in this study. Table 7 and Table 8 give the parameter estimates, chi-squares and average absolute difference for multiplicative and additive models of the Canadian data.

As expected, the multiplicative and additive models give similar parameter estimates. The smallest chi-square is also given by the minimum chi-squares model. Except for the exponential model, all models provide equal values for absolute difference.

Table 7: Multiplicative models for Canadian data

| Parameter | Zero bias /Poisson | Least squares | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 292.00 | 292.10 | 291.97 | 291.08 | 294.57 | 291.92 | 292.07 |
| $\exp(\beta_2)$ Merit X | 0.99 | 0.99 | 0.99 | 1.00 | 0.97 | 0.99 | 0.98 |
| $\exp(\beta_3)$ Merit Y | 0.99 | 0.99 | 0.99 | 0.99 | 1.00 | 0.99 | 0.99 |
| $\exp(\beta_4)$ Merit B | 1.06 | 1.05 | 1.06 | 1.07 | 1.05 | 1.06 | 1.06 |
| $\exp(\beta_5)$ Class 2 | 1.09 | 1.08 | 1.09 | 1.09 | 1.12 | 1.09 | 1.08 |
| $\exp(\beta_6)$ Class 3 | 1.02 | 1.02 | 1.02 | 1.03 | 0.98 | 1.02 | 1.02 |
| $\exp(\beta_7)$ Class 4 | 1.17 | 1.17 | 1.17 | 1.18 | 1.16 | 1.17 | 1.17 |
| $\exp(\beta_8)$ Class 5 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 |
| $\chi^2$ ($10^{-4}$) | 4.95 | 4.96 | 4.95 | 5.45 | 8.03 | 4.95 | 4.99 |
| Absolute difference | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 |

Table 8: Additive models for Canadian data

| Parameter | Zero bias /Least squares | Poisson | Minimum $\chi^2$ | Normal | Exponential | Gamma | Minimum modified $\chi^2$ |
|---|---|---|---|---|---|---|---|
| $\exp(\beta_1)$ Intercept | 291.95 | 291.87 | 291.83 | 291.06 | 294.77 | 291.80 | 291.94 |
| $\exp(\beta_2)$ Merit X | -4.24 | -4.05 | -3.38 | 0.59 | -10.11 | -3.92 | -5.37 |
| $\exp(\beta_3)$ Merit Y | -3.45 | -3.58 | -3.51 | -3.95 | 1.00 | -3.68 | -3.71 |
| $\exp(\beta_4)$ Merit B | 17.11 | 17.53 | 17.58 | 20.28 | 15.49 | 17.92 | 17.44 |
| $\exp(\beta_5)$ Class 2 | 25.16 | 25.35 | 25.75 | 25.13 | 35.64 | 25.54 | 24.63 |
| $\exp(\beta_6)$ Class 3 | 4.71 | 4.68 | 4.80 | 8.26 | -6.92 | 4.65 | 4.43 |
| $\exp(\beta_7)$ Class 4 | 51.08 | 51.18 | 51.28 | 53.30 | 47.12 | 51.30 | 51.01 |
| $\exp(\beta_8)$ Class 5 | -22.92 | -22.99 | -22.79 | -23.62 | -25.33 | -23.05 | -23.38 |
| $\chi^2$ ($10^{-4}$) | 4.68 | 4.67 | 4.67 | 5.10 | 8.20 | 4.67 | 4.71 |
| Absolute difference | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 |

## Conclusion

This study compared several risk classification methods for multiplicative and additive models by using weighted equation which is written as a weighted difference between the observed and fitted values. In addition, the parameter solutions for multiplicative and additive models were also compared by using weighted solution. The weighted solution for multiplicative model is in the form of a weighted proportion of observed over fitted values, whereas the weighted solution for additive model is in the form of a weighted difference between observed and fitted values.

In this study, the weighted equation was applied to estimate claim severity or average claim cost which is also equivalent to the total claim costs divided by the number of claims. The risk classification methods were compared on three types of severity data; Malaysian private motor third party property damage data, U.K. private vehicles own damage data from McCullagh and Nelder (1989), and data from Bailey and Simon (1960) on Canadian private automobile liability.

The fitting procedure were carried out by using two different approaches; classical and regression. The advantage of using the regression fitting procedure is that besides multiplicative and additive models, the fitting can also be extended to other regression models, as long as the function of the fitted value is written in a specified linear form. The inverse models were also fitted to the U.K. data because McCullagh and Nelder (1989) also fit the same data to the Gamma regression model by assuming that the regression effects were linear on the reciprocal scale.

As expected, the multiplicative and additive models give similar parameter estimates. The smallest chi-squares for multiplicative, additive and inverse models is given by the minimum chi-squares model. Except for the exponential model, all models provide similar values for absolute difference.

## References


Ajne, B. (1975). A Note on the Multiplicative Ratemaking Model. *ASTIN Bulletin*, *8*(2), 144-153.

Anderson, D., Feldblum, S., Modlin, C., Schirmacher, D., Schirmacher, E. & Thandi, N. (2004). A practitioner's guide to generalized linear models. *Casualty Actuarial Society Discussion Paper Program*, 1-115.

Bailey, R. A. & Simon, L. J. (1960). Two studies in automobile insurance ratemaking, *ASTIN Bulletin*, *4*(1), 192-217.

Bailey, R. A. (1963). Insurance rates with minimum bias. *Proceedings of the Casualty Actuarial Society, 50*(93), 4-14.

Baxter, L. A., Coutts, S. M., & Ross, G. A. F. (1980). Applications of linear models in motor insurance. *Proceedings of the 21st International Congress of Actuaries*, Zurich. 11-29.

Brockmann, M. J. & Wright, T. S. (1992). Statistical motor rating: Making effective use of your data. *Journal of the Institute of Actuaries*, *119*(3), 457-543.

Brown, R. L. (1988). Minimum bias with generalized linear models. *Proceedings of the Casualty Actuarial Society*, *75*(143), 187-217.

Chamberlain, C. (1980). Relativity pricing through analysis of variance. *Casualty Actuarial Society Discussion Paper Program*, 4-24.

Coutts, S. M. (1984). Motor insurance rating: An actuarial approach. *Journal of the Institute of Actuaries, 111,* 87-148.

Feldblum, S. & Brosius, J. E. (2003). The minimum bias procedure: A practitioner's guide. *Proceedings of the Casualty Actuarial Society*, *90*(172), 196-273.

Freifelder, L. (1986). Estimation of classification factor relativities: A modeling approach. *Journal of Risk and Insurance*, *53,* 135-143.

Fu, L. & Wu, C. P. (2005). Generalized minimum bias models. *Casualty Actuarial Society Forum*, *Winter,* 72-121.

Harrington, S. E. (1986). Estimation and testing for functional form in pure premium regression models. *ASTIN Bulletin*, *16,* 31-43.

Holler, K. D., Sommer, D. & Trahair, G. (1999). Something old, something new in classification ratemaking with a novel use of GLMs for credit insurance. *Casualty Actuarial Society Forum*, *Winter,* 31-84.

Jee, B. (1989). A comparative analysis of alternative pure premium models in the automobile risk classification system. *Journal of Risk and Insurance*, *56,* 434-459.

Jung, J. (1968). On automobile insurance ratemaking. *ASTIN Bulletin*. 5*(1)*, 41-48.

McCullagh, P. & Nelder, J. A. (1989). *Generalized linear models* (2nd Ed.). London: Chapman and Hall.

Mildenhall, S. J. (1999). A systematic relationship between minimum bias and generalized linear models. *Proceedings of the Casualty Actuarial Society*, *86*(164), 93-487.

Ismail, N. & Jemain, A. A. (2005a). Bridging minimum bias and maximum likelihood methods through weighted equation. *Casualty Actuarial Society Forum, Spring,* 367-394.

Ismail, N. & Jemain, A. A. (2005b). Handling overdispersion with negative binomial and generalized poisson regression models. *Proposal for the Casualty Actuarial Society Forum 2007.* Sub-committee members: Lewandowski, J.J., Holler, K.D. & White, J.

Renshaw, A. E. (1994). Modelling the claims process in the presence of covariates. *ASTIN Bulletin*, *24*(2), 265-285.

## APPENDIX A

S-PLUS programming for classical fitting procedure (Zero Bias Multiplicative)

```
ZeroBias.multi <- function(data)
{
# To identify matrix X, vector cost, and vector count from the data
   X <- as.matrix(data[,-(1:2)])
   cost <- as.vector(data[,1])
   count <- as.vector(data[,2])
# To set initial values for vector beta
   new.expbeta <- rep(c(1), dim(X)[2])
# To start the iteration
   for (i in 1:50)
   {
# To start the sequence
      for (j in 1:dim(X)[2])
      {
         expbeta <- new.expbeta
         fitted <- as.vector(exp(X%*%log(expbeta)))
         fitted.noj <- as.vector(exp(X[,-j]%*%log(expbeta[-j])))
         z <- as.vector(count*fitted.noj*X[,j])
         v <- as.vector(z/sum(z))
         new.expbeta[j] <- as.vector(sum(v*(cost/fitted.noj)))
      }
   }
# To calculate fitted values, chi-squares, and absolute difference
   fitted <- as.vector(exp(X%*%log(expbeta)))
   chi.square <- sum((count*(cost-fitted)^2)/fitted)
   abs.difference <- sum(count*abs(cost-fitted))/sum(count*cost)
# To list programming output
   list (expbeta=expbeta, chi.square=chi.square,
   abs.difference=abs.difference)
}
```

## APPENDIX B

S-PLUS programming for regression fitting procedure (Least Squares Multiplicative)

```
LeastSquares.Reg <- function(data)
{
# To identify matrix X, vector cost, and vector count
   X <- as.matrix(data[,-(1:2)])
   cost <- as.vector(data[,1])
   count <- as.vector(data[,2])
# To set initial values for vector beta
   new.beta <- c(5, rep(c(1), dim(X)[2]-1))
# To start the iteration
   for (i in 1:20)
   {
      beta <- new.beta
      fitted <- as.vector(exp(X%*%beta))
      Z <- diag(fitted)%*%X
      W <- diag(count)
      r.s <- cost-fitted+as.vector(Z%*%beta)
      new.beta <- as.vector(solve(t(Z)%*%W%*%Z)%*%t(Z)%*%W%*%r.s)
   }
# To calculate fitted values, chi-squares, and absolute difference
   fitted <- as.vector(exp(X%*%new.beta))
   chi.square <- sum((count*(cost-fitted)^2)/fitted)
   abs.difference <- sum(count*abs(cost-fitted))/sum(count*cost)
# To list programming output
   list  (expbeta=exp(new.beta), chi.square=chi.square,
   abs.difference=abs.difference)
}
```