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Joseph Liouville's 'Mathematical Works Of Évariste Galois'

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Translations, Ephemerals, & Biographies
Joseph Liouville's 'Mathematical Works Of Évariste Galois'

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Liouville's 1846 introduction to the mathematical works of Galois is translated from French to flowing (American) English. It gave an overview of the tragic circumstances of the undergraduate mathematician whose originality led to major advances in abstract Algebra.

Key words: Galois, Liouville, *Journal de Mathématiques Pures et Appliquées*

Introduction

The mathematics of Évariste Galois (1811 – 1832) flourished throughout the 20th century. It centered on groups of permutations, currently known in group theory as Galois connections or Galois representations. Its application has greatly expanded, emerging from the bowels of abstract Algebra, proliferating even as a key component of Wiles' (1995) solution to one of the most famous problems in mathematics, the 17th century Fermat's Last Theorem.

In the 1820s, Galois became interested in mathematics at the Lycée of Louis-le-Grand. Although he excelled, he was denied admission to the more prestigious École Polytechnique because his other coursework was wanting. Subsequently, he returned to the Lycée and became a student of Louis Paul Émile Richard (1795 – 1849). Richard (who never published) was also the mentor of other illustrious students: Joseph Le Verrier (1811 – 1877), Joseph Alfred Serret (1819 – 1885), and Charles Hermite (1822 – 1901).

Galois' tragic demise abruptly truncated a promising academic career to about a half dozen presentations and publications. (Approximately a dozen personal letters exist, www.galois-group.net/gtp/index//EN/.) Either inadvertently or deliberately, development of his scholarly work was hindered by Sylvestre François Lacroix (1765 – 1843), Jean Baptiste Joseph Fourier (1768 – 1830), and Augustin Louis Cauchy (1789 – 1857), but was facilitated by Marie-Sophie Germain (1776 – 1831), Siméon Denis Poisson (1781 – 1840), and Jacques Charles François Sturm (1803 – 1855).

Galois' work may have been forever lost were it not for the dedication of Joseph Liouville (1809 – 1882), one of the most influential mathematicians of the middle of the 19th century. In 1836, he became the Founding Editor of *Journal de Mathématiques Pures et Appliquées* (extant today as an Elsevier Science journal), known fondly as *Liouville's Journal*.

Liouville's (1846) introduction to Galois' work was written in French. We translate it in flowing (American) English. It is followed by a scan of the first page (Figure 1) as it originally appeared in *Liouville's Journal*.

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References

- Liouville, J. (1846). Oeuvres mathématiques d'Évariste Galois. *Journal de Mathématiques Pures et Appliquées*, 11, 381-384.
- Wiles, A. (1995). Modular elliptic curves and Fermat's last theorem. *Annals of Mathematics*, 141(3), 443-551.
- www.galois-group.net/gtp/index//EN/

Notice: Mathematical Works of Évariste Galois
Toul, October 30, 1846
J. Liouville

The ingenious and profound geometrician, whose works are presented here, died being scarcely twenty years old. Even so, he managed to squander fruitlessly the greater part of the last two years of such a short a life, in political activism, nightclubs, or imprisoned under lock and key at Sainte-Pélagie. He was born October 26, 1811. In the month of May, 1832, he succumbed to a fatal duel, undoubtedly the culmination of some frivolous quarrel. It removed him from the mathematical sciences, of which he would have cultivated with considerable brilliance!

The primary work of Évariste Galois has as a goal the conditions of resolving equations by using radicals. The author posed the basis of a general theory which he applied in detail to the equations of which the degree is a prime number. At the early age of sixteen, and on the benches of the Louis-le-Grand College, Galois enrolled in this difficult subject matter, where his contented studies were encouraged by the excellent Professor and gentleman, Mr. Richard [1]. He presented several memoirs to the Academy containing the results of his work. Apart from a few fragments and some notes, however, nothing remains extant with the exception of the article he submitted to them on January 17, 1831.

The reviewers [2], in their referees' report, reproached the young analyst for his obscure draft. Their reproach, which had already been addressed (Galois himself tells us) in his preceding communications, was well-founded, as he must have admitted. Perhaps, his exaggerated desire for conciseness was the cause of this defect, and is something which one must endeavor to refrain from when dealing with the abstract and mysterious matters of pure Algebra. Clarity is, indeed, all the more necessary when one has intention of leading the reader away from the beaten roads into the desert.

"When dealing with transcendental questions," said Descartes, "strive to be transcendently clear." Galois had frequently neglected this precept; and we understand that famous geometricians had deemed it suitable to

attempt to try to lead this genius novice, inexperienced albeit on the right path, by the severity of their wise council. The author that they censured was in front of them, passionate, active, and he would benefit from their advice.

But at present all that has changed. Alas, Galois is no more! Let us cease carrying on with useless criticisms; let us leave its defects, and instead see its qualities.

While bowing to the wishes of Évariste's friends, I engaged myself, under the watchful eyes of his brother [3], so to speak, to the attentive study of all the publications and manuscripts he left. I believed it my duty and primary goal to research and arrange them, in order to bring forth to the best of my abilities that which was novel in his works.

My zeal was soon rewarded. I experienced great pleasure the moment when, after having filled in the minor gaps, I recognized both the scope and precision of the method that Galois proved. In particular, this is his beautiful theorem: In order for an irreducible equation of a primary degree to be solvable by radicals, it is necessary and sufficient that all the roots are rational functions of any two among them. This method, truly worthy of the attention of the geometricians, would be enough by itself to ensure our compatriot being ranked with the small number of the scientists who have deserved the title of 'Inventor'.

We will initially reproduce the material Galois published from 1828 – 1830 which appeared in the *Annales* of Mr. Gergonne and in the *Bulletin des Sciences* of Mr. Ferussac. Then, his unedited writings follow. Finally, we propose some commentary to complete certain passages and clarify a few delicate points.

On the eve of his death, and in anticipation of the disastrous fate which awaited him, Galois quickly traced the summary of the broad ideas which occupied him, and addressed it, in the form of a letter, to his best friend, Mr. Auguste Chevalier. This last writing, a sort of scientific testament, which we will place as a preface to his posthumous works, is impossible to read without emotion while contemplating the circumstances under which it was composed.

This letter appeared in 1832, in the *Revue Encyclopédique* (September issue, page 568). An obituary notice on Mr. Galois appeared

in the same issue (page 744). We did not believe it necessary to include it in our collection to follow. It contains interesting details, but, for the most part, does not pertain to science. Moreover, certain extreme assertions and opinions concerning persons and things would only motivate others to rise to contradict him. It is true that even in the eyes of one who was furthest removed from his opinions, the author of this notice excused him in advance due to professed tender friendship in unity with Galois. As for those who neither knew nor had seen this unfortunate young man, we will contain ourselves in our role of geometrician (and the observations which we will allow ourselves in publishing his works under the inspiration of his family) to only portray his mathematics.

Appendix

The ordinary requirements of our publication and the extent of Galois' works prevent us from presenting it in a single issue. We shall present his material in two parts, in this volume and in the next.

[1] Mr. Le Verrier, Mr. Hermite, and other distinguished scholars were in Mr. Richard's class. These excellent students glorify their teacher.

[2] Misters Lacroix and Poisson were signatories to the reviewer's report. We can see their conclusions in the report were expressed somewhat dryly by the way in which Mr. Lacroix expressed himself in the sixth edition of his *Complements of the Elements of Algebra*, page 345: "In 1831, a young Frenchman, Évariste Galois, dead the following year, had announced, in a memoir presented to the Academy of Sciences, that in order for an irreducible equation of the first degree to be solvable by radicals, it is necessary and sufficient that any two of the roots be known and the others could be deduced rationally; but this memoir appeared almost unintelligible to the reviewers responsible for examining it."

[3] Mr. Alfred Galois.

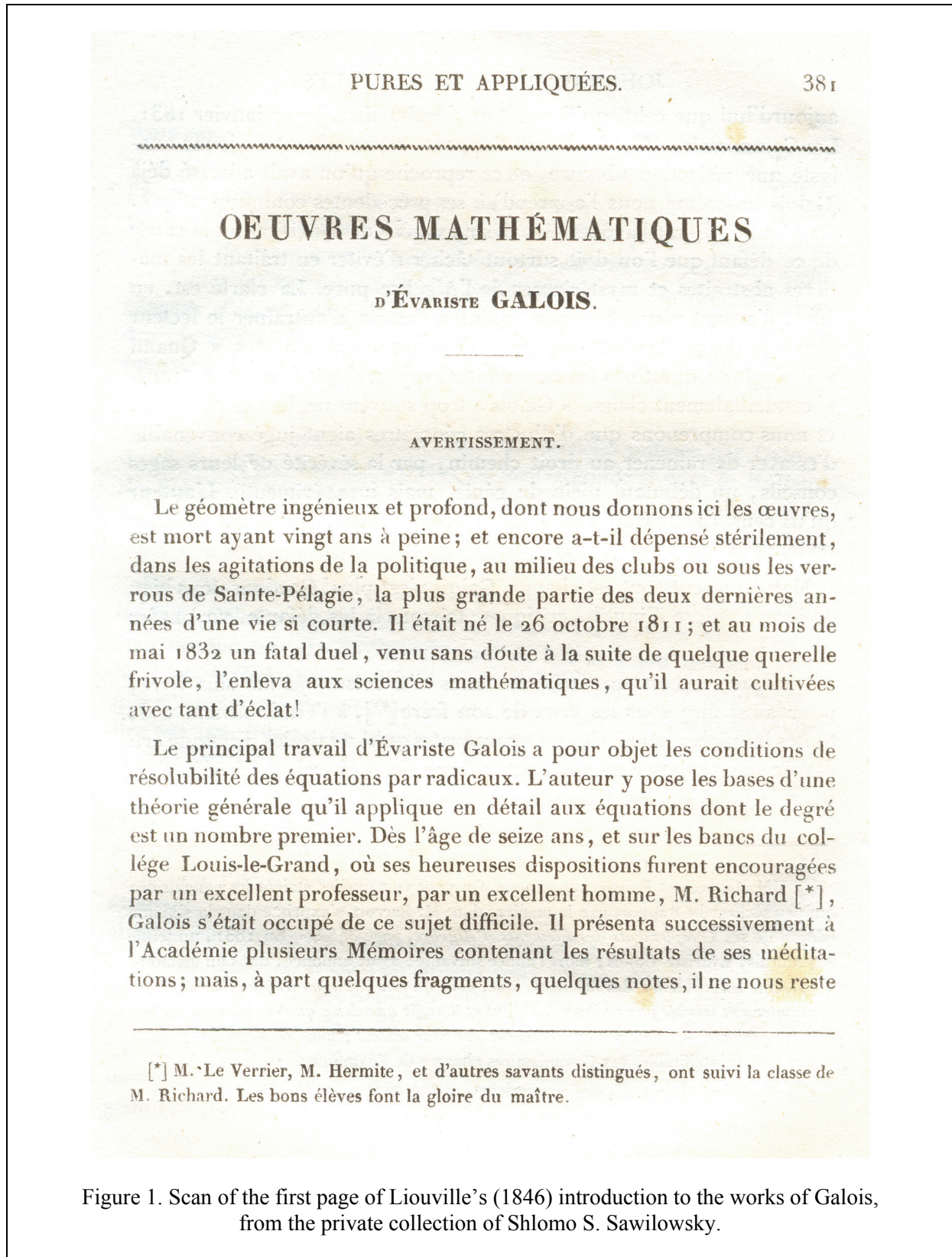


Figure 1. Scan of the first page of Liouville's (1846) introduction to the works of Galois, from the private collection of Shlomo S. Sawilowsky.