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Using Monte Carlo methods, the properties of Granger causality test in stable VAR models are studied under the presence of different magnitudes of GARCH effects in the error terms. Analysis reveals that substantial GARCH effects influence the size properties of the Granger causality test, especially in small samples. The power functions of the test are usually slightly lower when GARCH effects are imposed among the residuals compared with the case of white noise residuals.

Key words: Causality test, GARCH, size and power.

Introduction

One of the most important issues in the subject of time series econometrics is the ability to statistically perform causality test. By causality it is meant causality in the Granger (1969) sense. That is, one would like to know if one variable precedes the other variable or if they are contemporaneous. The Granger approach to the question whether a variable say y_1 causes another variable say y_2 is to see how much of the current value of the second variables can be explained by past values of the first variable. Y_2 is said to be Granger-caused by y_1 if y_1 helps in the prediction of y_2, or equivalently, if the coefficients of the lagged y_1 are statistically significant in a regression of y_2 on y_1. Empirically, one way to test for causality in Granger sense is by means of vector autoregressive (VAR) model.

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The main purpose of this article is to investigate the properties of the Granger causality test in stationary and stable VAR models under conditions when there exists some kind of volatility among the error terms, more specifically, Generalised Autoregressive Conditional Heteroscedasticity (GARCH) effects. It is well known that the analysis of causality is very sensitive to model specification and is almost only valid under conditions when the error terms are fairly close to white noise. At the same time it is also known that a considerable proportion of the time series variables follow some type of GARCH process. Hence, it is important to investigate the properties of this commonly used causality test under the presence of generalized conditional heteroscedasticity.

The Model and the Monte Carlo Experiment

Consider the data-generating process (DGP) consists of a two dimensional time series generated by a stabile VAR(p) process:

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + \varepsilon_t \qquad (1)$$

where $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{kt})'$ is a zero mean independent white noise process with nonsingular covariance matrix Σ_ε and, for j = 1, ... , k, $\mathrm{E}|\varepsilon_{jt}|^{2+\tau} < \infty$ for some $\tau > 0$. The order p of the process is assumed to be known. Let

$\alpha_p = \text{vec}\left[\mathrm{A}_1, \cdots, \mathrm{A}_\mathrm{p}\right]$ be the vector of the true parameters, where vec[.] denotes the vectorization operator that stacks the columns of the argument matrix. Now, suppose that one is interested in testing q independent linear restrictions:

$$H_o: \mathrm{R}\alpha_p = \mathrm{s}$$

vs. (2)

$$H_1: \mathrm{R}\alpha_p \neq \mathrm{s}$$

where q and s are fixed (q x 1) vectors and R is a fixed [q x $k^2(p)$] matrix with rank q.

The process $\{y_t\}$ is generated by the VAR(p) process in (1), with the $\hat{A}_i$ (i = 1, … p) the Ordinary Least Squares (OLS) estimators and $\hat{\alpha}_p^{p-1}$ the $\left[k^2(p-1)\right]$ dimensional vector, consisting of the $k^2(p-1)$ elements of $\hat{\alpha} = vec\left[\hat{A}_1, \cdots, \hat{A}_p\right]$, that are obtained by deleting the matrix $\hat{A}_i$ $i \in \{1, \ldots p\}$. Then:

$$T^{1/2}\left(\hat{\alpha}_p - \alpha_p\right) \Rightarrow N\left(0, \Sigma_p\right) \qquad (3)$$

where $\Rightarrow$ denotes weak convergence in distribution and the [$k^2(p)$ x $k^2(p)$] covariance matrix Σ_p is non-singular. The α_p is the $\left[k^2(p)\right]$ dimensional vector of the true parameters. Moreover given a consistent estimator $\hat{\Sigma}_p$, then the Wald test of the null hypothesis in (2):

$$\lambda_w = T(R\hat{\alpha}_p - s)'(R\Sigma_p R')^{-1}(R\hat{\alpha}_p - s) \quad (4)$$

has an asymptotic $\chi^2(q)$-distribution under the null hypothesis. And with y_t portioned in (m) and (k-m) dimensional sub vectors y_t^1 and y_t^2, and A_i matrices portioned conformably, then y_t^2 does not Granger-cause the y_t^1 if the following hypothesis is true:

$$H_0 = \mathrm{A}_{12,\mathrm{i}} = 0$$

for

$$\mathrm{i} = 1, \cdots, p\text{-}1. \qquad (5)$$

The error components $\left(\varepsilon_{1t}, \varepsilon_{2t}\right)'$ in (1) and (2) are generated by GARCH(1,1) models, i.e.,

$$\begin{aligned} \varepsilon_{it} &= h_{it}\upsilon_{it} \quad i = 1,2 \\ \upsilon_{it} &\text{ i.i.d., } \mathrm{E}(\upsilon_{it}) = 0, \mathrm{E}(\upsilon_{\mathrm{it}}^2) = 1 \\ h_{it}^2 &= \gamma_i + \phi_i h_{it-1}^2 + \varphi_i \varepsilon_{it-1}^2 \end{aligned} \qquad (6)$$

and $\mathrm{Cov}\left(\varepsilon_{1t} \ \varepsilon_{2t}\right) = 0$. The condition for finite variance is $\phi_i + \varphi_i < 1$ and the condition for finite fourth moment is $3\phi_{i1}^2 + 2\phi_i\varphi_i + \varphi_i^2 < 1$. Furthermore, if $\gamma_i > 0$ and $\phi_i + \varphi_i < 1$, then the unconditional variance of the ε_i exist and equals $\sigma_{\varepsilon i}^2 = \left(\gamma_i / 1 - \phi_i - \varphi_i\right)$. Note that when $\phi = \varphi = 0$, the ε_{it} is reduced to iid white noises.

To illustrate and study the possible effects of a GARCH(1,1) process on the Granger-causality test in a stable VAR(1) system Monte Carlo methods. The estimated size is calculated by simply observing how many times the null is rejected in repeated samples under conditions where the null is true. To judge the reasonability of the results use an approximated 95% confidence interval for the actual size (π):

$$\hat{\pi} \pm 2\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{N}} \qquad (7)$$

where $\hat{\pi}$ is the estimated size and N is the number of replications.

The Monte Carlo experiment has been performed by generating data according to the model defined by (1) and (2),

$$y_t = \begin{bmatrix} 0.02 \\ 0.03 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.3 \\ T^{-1/2}\lambda & 0.5 \end{bmatrix} y_{t-1} + \varepsilon_t \qquad (8)$$

If λ= 0, y_{1t} is Granger-non-causal for y_{2t} and if $\lambda \neq$ 0, y_{1t} causes y_{2t}. Therefore, the λ = 0 is used to study the size of the test.

Three GARCH versions are simulated with a) high persistence, HP, (0.01, 0.09, 0.9), b) medium persistence, MP, (0.05, 0.05, 0.9) and c) low persistence, LP, (0.20,0.05,0.75). The processes includes a constant term and fit a VAR(1) : $y_t = v + A_1 y_{t-1} + \varepsilon_t$.

This means that order p of the process is assumed to be known and since this assumption might be too optimistic, however, also fit a VAR (2) : $y_t = v + A_1 y_{t-1} + A_2 y_{t-2} + \varepsilon_t$.

For each model perform 10 000 replications and use three different nominal sizes, namely 1%, 5% and 10%. However, different authors have put forward reasons for using both larger and smaller significance levels. Maddala (1992) suggests using significance levels of as high as 25% in diagnostic testing, while MacKinnon (1992) suggest going in the other direction to avoid mass significance. To reduce this problem, in this study, also use graphical methods that may provide more information about the size and the power of the test. Simple graphical methods are used, developed and illustrated by Davidson and MacKinnon (1998), which are based on the empirical distribution function (EDF) of the P-values and are easy to interpret. The P value plot is used to study the size and the Size-Power curves to study the power of the test.

Furthermore, to judge the reasonability of the results use a 95% confidence interval for the actual size (π) as: $\pi_0 \pm 2\sqrt{\dfrac{\pi_0(1-\pi_0)}{N}}$, where N is the number of replications. Results that lie between these bounds will be considered satisfactory.

Several factors are expected to affect the size and power properties of causality tests. Samples typical for small, medium, large and very large sizes have been investigated. For each time series 20 pre-sample values are generated with zero initial conditions, and with net sample sizes of T = 50, 100, 200, 500, 1000. Table 1 shows the different parameters of our Monte Carlo design. The number of replications per model is 10 000 for the size, and 1000 for the power of the test. The calculations were performed using GAUSS 6.0.

Results of the Size of the Test

Presented in this section are the most important results of our Monte Carlo experiment concerning the size of the test. Regarding the P value plots, under the condition when the distribution used to compute the p_s is correct, each of the p_s should be distributed as uniform (0,1) and therefore the resulting graph should be close to the 45^o line as in Figure 1a below.

Size of the test for the VAR (1), given that the true model is a VAR (1)

In this sub-section the results are presented when the estimated and the true model is a VAR (1). As can be seen from the results, in Table 1a in the Appendix, the calculated sizes of the test over estimate the nominal sizes in all situations more or less regardless whether there exist low, medium or high GARCH effects. This is the case when a small sample of 50 observations are studied. This is also confirmed when the P-value plots are observed in Figure 1a, in the Appendix, in which one only presents the size when white noise and high GARCH effects are imposed. Here one can see that in both cases the test over rejects the size, but that the calculated sizes still lay near to the 95% confidence interval for nominal size with a slightly higher over rejection when the high GARCH magnitudes are present.

When the sample size increases to 100 observations, as is illustrated in Table 2a and Figure 2a, the properties of the test become better but there still some over rejection present. When enlarging the sample size to 200 observations the test performs well in all cases except for the case with high GARCH effect. In this case the test slightly over rejects the nominal size, as can be seen in Table 3a. Figure 3a shows that the over rejection become more severe for larger nominal sizes.

Table 1 Monte Carlo Parameters of the GARCH Effects

	Λ	γ	ϕ	φ
High Persistence	0	0.01	0.09	0.90
Medium	0	0.05	0.05	0.90
Low	0	0.20	0.05	0.75
High Persistence	2	0.01	0.09	0.90
Medium	2	0.05	0.05	0.90
Low	2	0.20	0.05	0.75

The same is also true when the sample size is equal to 500 observations, as is illustrated in Table and Figure 4a in the Appendix.

In a very large sample, i.e. 1000 observations in Table and Figure 5a, the test performs satisfactorily in almost all situations, but with one exception in the case when a high GARCH effect is present

Size of the test for the extra lag; VAR (2), given that the real model is a VAR (1)

Here the results are presented when the estimated model contains an extra lag, i.e. a VAR(2), while the true model is a VAR(1). In this case to investigate the effect of possible over parameterization of the true model is what is desired. Table and Figure 1b in the Appendix, the sizes of the test, as in the previous sub-section, over estimate the nominal sizes in all situations almost regardless whether there exist low, medium or high GARCH effects. In Figure 1b, the clear over rejection is illustrated for both white noise and high GARCH effects.

However, as the results confirm in Table 2b and Figure 2b, the over rejection become less severe when the number of observations increases to 100 observations. The results are almost similar when increase the sample size is increased to 200 observations, see Table 3b and Figure 3b in the Appendix.

When the sample size increases to 500 observations, as in Table 4b and Figure 4b, the test performs well in almost all situations except for in the case of high GARCH effects. Finally, in Table 5b and Figure 5b, the results show that the test performs satisfactorily but still with a slight over rejection in the case of high GARCH effects.

In general, the results from these two sub-sections are generally similar. Moreover, one could not find the over rejection to be that severe even in the case of the existence of high GARCH effects in comparison with that of the white noise. The test is consistent and converges slowly to its nominal size as the sample size increases.

Analysis of the Power of the Test

In this section the results of the Monte Carlo experiment regarding the power of the Granger-causality test are discussed. The power of the test was analyzed using sample sizes of 50, 100, 200, 500 and 1000 observations. The power functions have been calculating for the test in the case of white nose and under different GARCH effects. The power functions have shown to be fairly similar in the cases of the white noise, low persistence and medium persistence GARCH. Based on this and since one could not find any noticeable differences in the performances of the test between these combinations regarding the size properties, only show and compare the power functions of the white nose and the high GARCH.

The power functions are estimated by calculating the rejection frequencies in 1000 replications using values of the λ coefficients in equation (8) equal to 2. The estimated power functions of the test have been compared only graphically. One may follow the same procedure as for the size investigation to evaluate the EDF's denoted $\hat{F}^{\oplus}\left(x_j\right)$, by using the same sequence of random numbers as in the case of the size of the test. For plotting the estimated power functions against the nominal size, there are the Size-Power Curves. Presented is the power of the test in cases when the model is exactly identified, i.e. the true and estimated models are VAR (1) and in the case when the model is over parameterized, i.e. the estimated model is VAR (2) while the true is VAR (1).

The power of the Granger causality test, as expected, depends on how well the model is specified. This can be seen when comparing the power functions in the upper and lower parts of Figures 6-10 in the Appendix. This is the effect of over parameterization.

Moreover, from the figures it can be seen that the power functions satisfy the expected properties of increasing with the sample size. Lower powers are observed when the samples are small and higher when the samples are large. A closer examination of the figures shows, that most frequently, the power functions are slightly lower in the case of the GARCH residuals (the dashed lines) than the white noise.

Conclusion

The results regarding the size of the tests have been presented both in form of tables and P-value plots. Our analysis revealed that the Granger-causality test slightly over rejects the nominal sizes in small samples and under the existence of high GARCH effects. This over rejection becomes even lower when the sample size increases and when the GARCH effects are not high. These results are similar in both of the exactly parameterized VAR (1) model and the over parameterized VAR (2) model. Moreover, the test is consistent in the sense that the size of the test converges slowly to its nominal size as the sample size increases.

The power functions have been presented only graphically. As expected, the analysis of the power indicates that these power functions increase with an increasing sample size. Furthermore, most of the times these power functions are slightly lower in the case of the GARCH residuals than under white noise. The power of the test, as expected, becomes lower when including an extra lag in the VAR model, i.e. in the case of VAR(2).

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APPENDIX

Table 1a. Size of the test for 50 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0160	0.0151	0.0156	0.0152
0.05	0.0642	0.0643	0.0658	0.0668
0.10	0.1169	0.1222	0.1231	0.1225

Table 2a. Size of the test for 100 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0133	0.0126	0.0126	0.0141
0.05	0.0584	0.0579	0.0578	0.0593
0.10	0.1093	0.1069	0.1051	0.1087

Table 3a. Size of the test for 200 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0112	0.0119	0.0119	0.0146
0.05	0.0546	0.0528	0.0527	0.0584
0.10	0.1056	0.1036	0.1054	0.1109

Table 4a. Size of the test for 500 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0095	0.0107	0.0108	0.0141
0.05	0.0558	0.0544	0.0535	0.0639
0.10	0.1068	0.1031	0.1038	0.1121

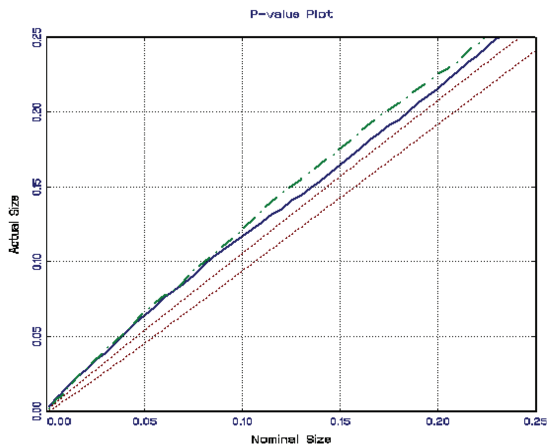
Table 5a. Size of the test for 1000 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0096	0.0083	0.0084	0.0150
0.05	0.0476	0.0479	0.0496	0.0628
0.10	0.0979	0.1034	0.0997	0.1183

P-value plots HP (GARCH)

Figure 1a. 50 observations

Figure 2a. 100 observations



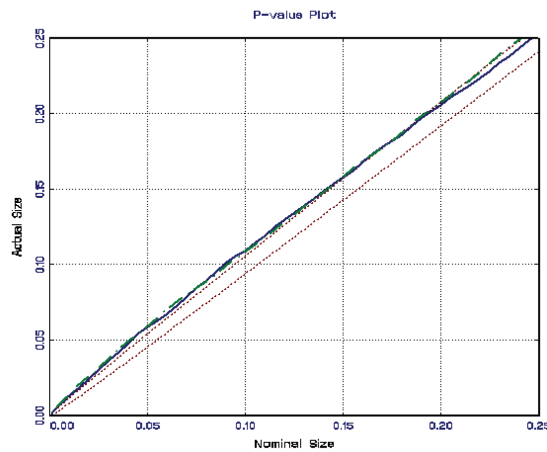
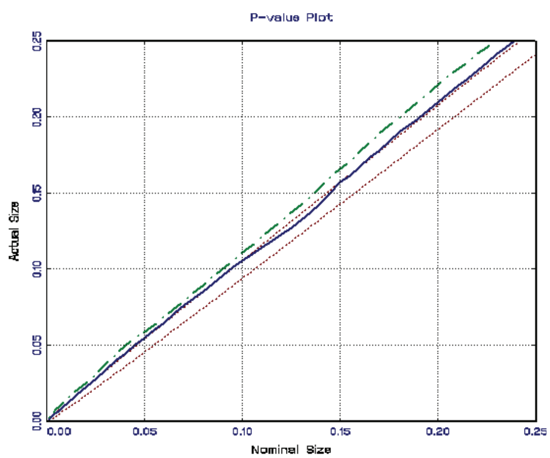


Figure 3a. 200 observations

Figure 4a. 500 observations



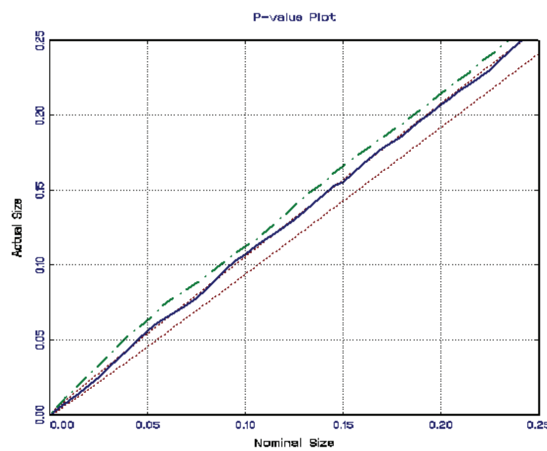
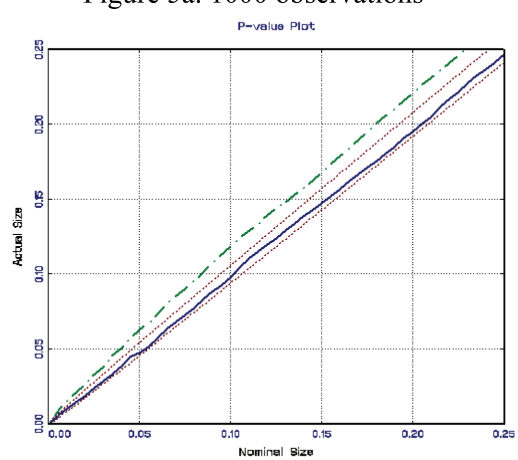


Figure 5a. 1000 observations



Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Table 1b. Size of the test for 50 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0169	0.0151	0.0179	0.0185
0.05	0.0633	0.0643	0.0648	0.0671
0.10	0.1192	0.1222	0.1221	0.1248

Table 2b. Size of the test for 100 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0125	0.0115	0.0109	0.0119
0.05	0.0542	0.0566	0.0566	0.0593
0.10	0.1067	0.10890.1089	0.1095	0.1126

Table 3b. Size of the test for 200 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0138	0.0122	0.0129	0.0143
0.05	0.0582	0.0542	0.0542	0.0611
0.10	0.1098	0.1053	0.1062	0.1118

Table 4b. Size of the test for 500 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0111	0.0111	0.0111	0.0125
0.05	0.0524	0.0532	0.0529	0.0568
0.10	0.1006	0.1017	0.1026	0.1127

Table 5b. Size of the test for 1000 observations

Nominal	White Noise	GARCH(1,1) LP	GARCH(1,1) MP	GARCH(1,1) HP
0.01	0.0092	0.0095	0.0091	0.0130
0.05	0.0449	0.0487	0.0484	0.0581
0.10	0.0950	0.0969	0.0943	0.1084

P-value plots HP (GARCH)

Figure 1b. 50 observations

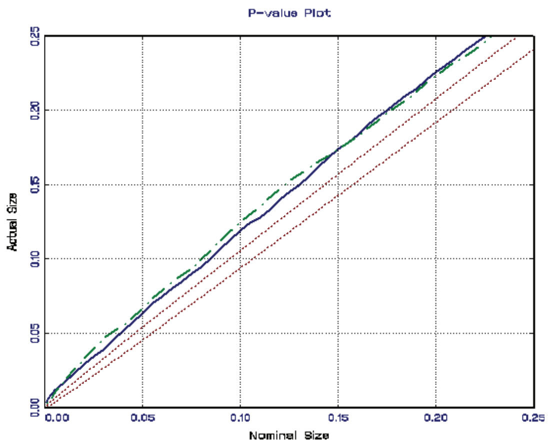


Figure 2b. 100 observations

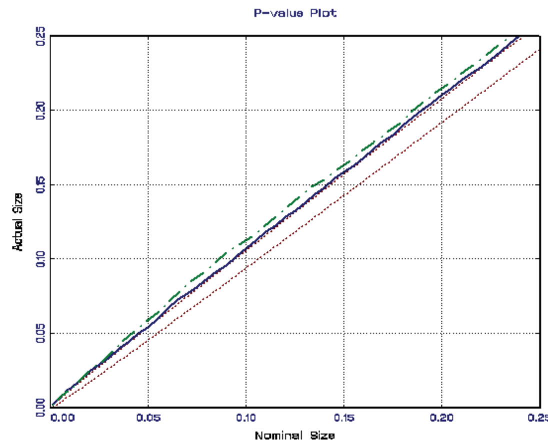


Figure 3b. 200 observations

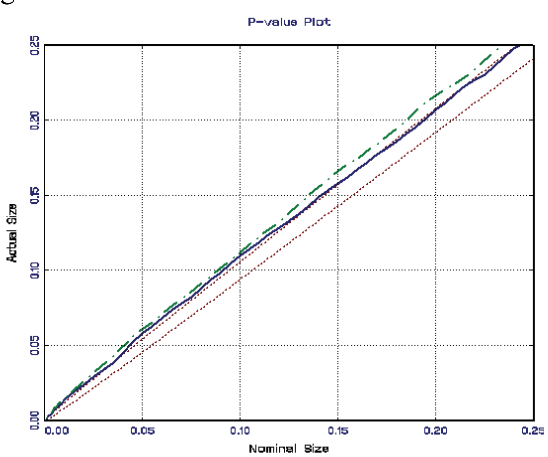


Figure 4b. 500 observations

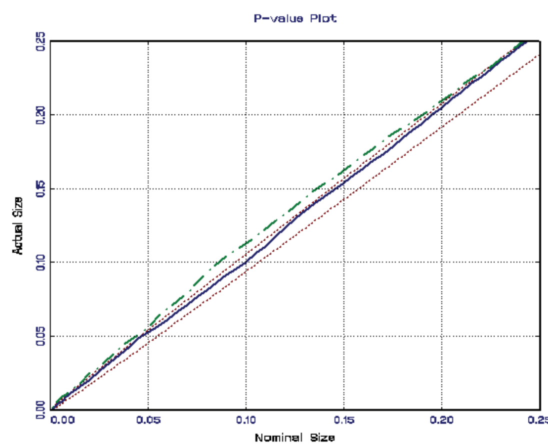
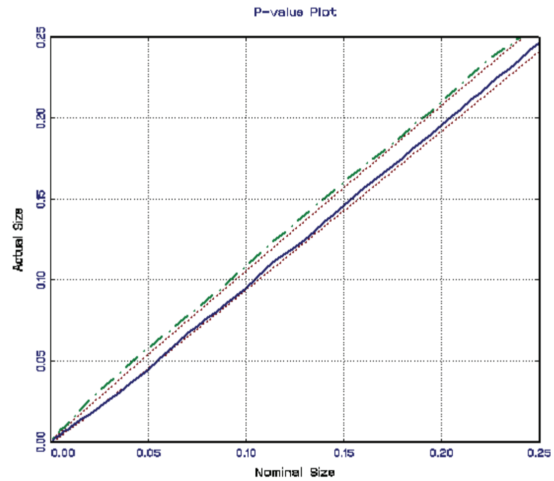
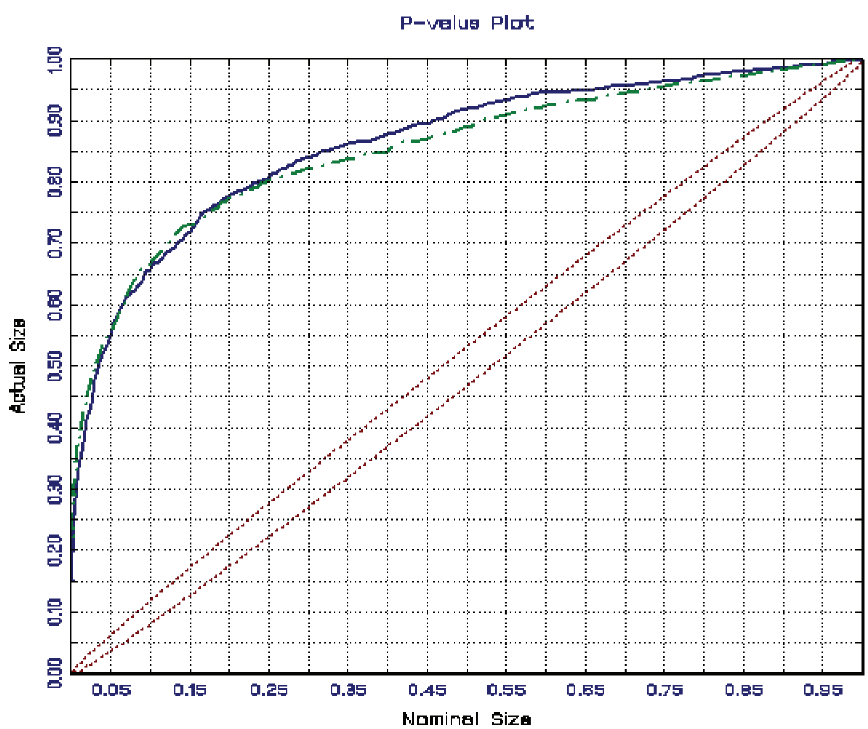


Figure 5b. 1000 observations



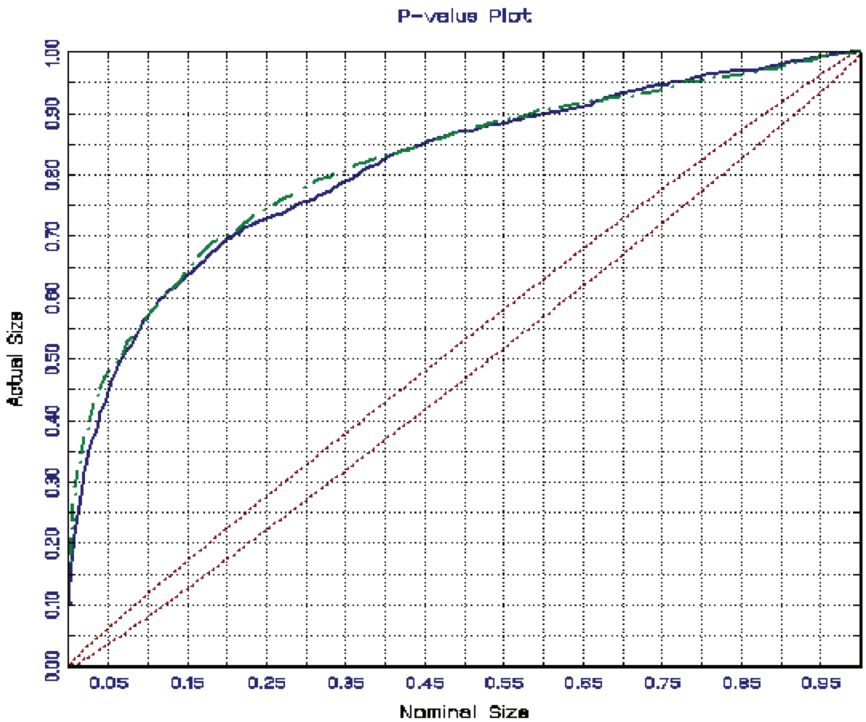
Solid lines = White noise. Dot dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 6a. Power–Size plots of the Granger-causality test for 50 observations
VAR(1)



Solid lines = White noise. Dash line = GARCH. Dot lines = 95% confidence interval for nominal size.

Figure 6b Power–Size plots of the Granger-causality test for 50 observations VAR(2)



Solid lines = White noise. Dash line = GARCH. Dot lines = 95% confidence interval for nominal size.