[Journal of Modern Applied Statistical Methods](#)

Volume 7 | Issue 1 Article 20

5-1-2008

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Journal of Modern Applied Statistical Methods
May, 2008, Vol. 7, No. 1, 246-252

1538 – 9472/08/$95.00

Robustness of Some Estimators of Linear Model with Autocorrelated Error Terms When Stochastic Regressors are Normally Distributed

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Performances of estimators of the linear model under different level of autocorrelation (ρ) are known to be affected by different specifications of regressors. The robustness of some methods of parameter estimation of linear model to autocorrelation are examined when stochastic regressors are normally distributed. Monte Carlo experiments were conducted at both low and high replications. Comparison and preference of estimator(s) are based on their performances via bias, absolute bias, variance and more importantly the mean squared error of the estimated parameters of the model. Results show that the performances of the estimators improve with increased replication. In estimating all the parameters of the model, the Ordinary Least Square (OLS) estimator is more efficient than any of the Generalized Least Square (GLS) estimators considered when $-0.25 < \rho \leq 0.25$; and the Maximum Likelihood (ML) and the Hildreth and LU (HILU) estimators are robust.

Key words: Robustness, Stochastic regressors, linear model with autocorrelated error, OLS estimator, Feasible GLS Estimators.

Introduction

The Ordinary Least Square (OLS) estimator is unbiased but inefficient in estimating the parameters of the linear model with autocorrelated error terms, and its predicted values are inefficient if the variance of the autocorrelated error terms are underestimated (Johnston, 1984; Fomby et. al, 1984; Maddala, 2002). Consequently, the Generalized Least Square (GLS) estimator was developed.

Aitken (1935) had shown that the GLS estimator given by

$$\hat{\beta} = (X^1\Omega^{-1}X)^{-1}X^1\Omega^{-1}Y \qquad (1)$$

with the variance – covariance matrix

$$V\left(\hat{\beta}\right) = \sigma^2\left(X^1\Omega^{-1}X\right)^{-1} \qquad (2)$$

is efficient among the class of linear unbiased estimator provided Ω is known. Consider the linear model where the error terms follow AR (1) process

$$Y = X\beta + U \qquad (3)$$

where

$$U_t = \rho U_{t-1} + \varepsilon_t \qquad |\rho| < 1 \qquad t = 1,2,\ldots,n \qquad \varepsilon_t \sim N\left(0,\sigma^2\right)$$

Then the inverse of Ω is given as

$$\Omega^1 = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & -\rho & 0 & \ldots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \ldots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \ldots & 0 & 0 \\ . & . & . & \ldots & . & . \\ . & . & . & \ldots & . & . \\ . & . & . & \ldots & . & . \\ 0 & 0 & 0 & \ldots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \ldots & -\rho & 1 \end{bmatrix}_{n\times n}$$

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Cochrane and Orcutt (1949) pointed out that the presence of antocorrelated error terms in Linear Model requires some modifications of the usual least square method of estimation. They suggested a transformation that uses the matrix

$$P = \begin{bmatrix} -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ 0 & 0 & -\rho & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix}_{(n-1) \times n}$$

which ignores the first observation of the error terms. Paris & Winstein (1954) showed that the appropriate transformation required for the transformation is

$$Q = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix}_{nxn}$$

which retains the first observation. The difference in the usage of P and Q can be negligible when n is large, but in small sample investigation such as in this study, the difference may be major. However, they both require ρ to be known before they can be used.

Fomby et al. (1984) and others emphasized that in practice ρ (and hence Ω) is usually unknown but has to be estimated. They indicated that many consistent estimators $\hat{\rho}$ of ρ (and hence $\hat{\Omega}$ of Ω) can be estimated to have the Feasible Generalized Least Square estimators. Some of the Feasible GLS estimators available in literatures are the Cochrane and Orcutt estimator (1949), Hildreth and Lu estimator (1960), Paris & Winstein estimator (1954), Thornton estimator (1982), Durbin estimator (1960), Theil's estimator (1971), the Maximum Likelihood estimator and the Maximum Likelihood Grid estimator (Beach and Mackinnon, 1978), some of which use either the P or Q transformation matrix. Furthermore, some have also been incorporated into White's SHAZAM program (White, 1978) and the new version of the time series processor (TSP, 2005).

However, these estimators are known to be asymptotically equivalent but the question on which is to be preferred in small samples is another matter. The question at what value of ρ does the OLS estimator become inefficient when compared with the feasible generalized least square estimators arises, and what transformation is to be preferred are still of concern (Johnson, 1984; Fomby et. al, 1984). Therefore, the finite properties of these estimators are studied through Monte Carlo methods.

Chipman (1979), Kramer (1980), Kleiber (2001) and others observed that the efficiency of these estimators depends on the structure of the regressors that are used. Rao and Griliches (1969) conducted one of the earliest Monte Carlo studies on the performances of some of these estimators with autoregressive stochastic regressor. They observed that the OLS estimator is only more efficient than any of the GLS estimators considered when $|\rho| < 0.3$; and that the performances of the GLS estimators are not far apart. Park and Mitchell (1980) observed that when regressors are trended, the estimator that uses the P transformation (Paris & Winstein) is more efficient than the one that uses the Q transformation (Cochrane – Orcutt) and that the latter should even be avoided since it is less efficient than the OLS estimator.

More recently, Nwabueze (2005a) examined the performance of some of these estimators with exponential independent variable. His result, among other things, show that the OLS estimator compares favorably with the Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGD) estimators for small value of ρ but it appears to be superior to Cochrane – Orcutt (CORC) and the Hidreth and Lu (HILU) especially when ρ is large. Some other recent works that are done with different specification of regressors include that of

Iyaniwura and Nwabuwze (2004a), Iyaniwura and Nwabuwze (2004b), Nwabuwze (2005b), Nwabuwze (2005c), and Olaomi and Iyaiwura (2006).

Consequently, without lost of generality, the purpose of this article is to find out if any or some of these estimators would be robust to autocorrelation when stochastic regressors are normally distributed.

Methodology

Consider the GLS model with stochastic regressors and AR (1) of the form

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t \qquad (4)$$

where $u_t = \rho u_{t-1} + \varepsilon_t$

$$|\rho| < 1 \quad t = 1, 2, ..., n \qquad \varepsilon_t \sim N\left(0, \sigma^2\right).$$

Its parameter estimations can be done using the OLS and the (feasible) GLS estimators. Thus, the performances of the OLS estimator and the following feasible GLS estimators are studied: CORC, HILU, ML and the MLGD estimators. The CORC and HILU estimators use the P transformation while the ML and MLGD estimator use the Q transformation.

Monte Carlo experiments were performed for $n = 20$, a small sample size representative of many time series study (Park and Mitchell, 1980) with four replication (R) levels (R = 10, 40, 80, 120) and nine various degree of autocorrelation ($\rho =$ -0.99,-0.75,-0.5,… 0.99). At a particular choice of ρ and R (a scenario), the first replication was obtained by generating $e_t \sim N(0,1)$ and hence u_t. Assuming the process start from infinite past and continue to operate, the initial value of U (i.e u_1) was thus drawn from a normal population with mean zero and variance $\frac{1}{1-\rho_1^2}$. Hence

$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho_1^2}} \qquad (6)$$

$$u_t = \rho_1 u_{t-1} + \varepsilon_t \quad t = 2, 3, \ldots, 20 \qquad (7)$$

Furthermore, $x_{1t} \sim N(0,1)$ and $x_{2t} \sim N(0,1)$ were generated. Hence, the values of y_t in equation (1) were also calculated by setting the true regression coefficients as $\beta_0 = \beta_1 = \beta_2 = 1$. This process continued until all replications in this scenario were obtained. Another scenario then started until all the scenarios were completed.

Evaluation and comparison of estimators were examined using the finite sampling properties of estimators which include bias (B), absolute bias (AB), and variance (Var) and the more importantly the mean squared error (MSE) criteria. For any estimator $\hat{\beta}_i$ of β_i of model (4)

$$\bar{\hat{\beta}}_i = \frac{1}{R}\sum_{j=1}^{R} \hat{\beta}_{ij} \qquad (7)$$

$$B\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right) = \bar{\hat{\beta}}_i - \beta_i \qquad (8)$$

$$AB\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left|\hat{\beta}_{ij} - \beta_i\right| \qquad (9)$$

$$Var\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \bar{\hat{\beta}}_i\right)^2 \qquad (10)$$

$$MSE\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right)^2$$

$$= Var\left(\hat{\beta}_i\right) + \left[B\left(\hat{\beta}_i\right)\right]^2 \qquad (11)$$

for i = 0, 1, 2 and j= 1,2,…,R.

For each of the estimation methods, a computer program was written using TSP software to estimate all the model parameters and to evaluate the criteria. The four replication levels were further grouped into low (R =10, 40) and high (R = 40, 80) and the effect of autocorrelation on the performances of the methods (estimators) were examined via the Analysis of Variance of the criteria of each of

Table 1. Summary of the ANOVA TABLE showing the sum of squares of the model parameters based on the criteria in the two replication groups.

Parameter	Replication Group	Source	d.f	Type III Sum of squares: Bias	Absolute Bias	Variance	Mean Squared Error
		R	8	22.290***	293.867***	21568.681***	25967.186***
	LOW	M	4	.137	7.470E-02	3.560	7.428
		R*M	32	.619	.876	29.330	68.430
		Error	45	22.356	2.308	87.452	591.721
β_0		Total	89	45.403	297.126	21689.023	26634.766
		R	8	.319***	258.096***	20470.474***	20505.716***
	HIGH	M	4	4.858E-03	4.201E-02**	1.596**	1.583*
		R*M	32	1.862E-02	.228***	10.494***	10.380***
		Error	45	2.367E-02	.105	4.651	4.688
		Total	89	.366	258.471	20487.216	20522.367
		R	8	7.462E-02***	.206***	1.698***	1.763***
	LOW	M	4	1.940E-02*	.459**	1.445***	1.539***
		R*M	32	.162***	1.271***	7.632***	8.005***
		Error	45	5.861E-02	2.786E-02	.540	.542
		Total	89	.315	1.964	11.315	11.848
β_1		R	8	9.684E-03***	.291***	3.657***	3.678***
	HIGH	M	4	2.858E-03*	.399***	2.367***	2.393***
		R*M	32	1.917E-02**	1.451***	14.913***	15.036***
		Error	45	9.472E-03	1.861E-03	3.527E-03	5.605E-03
		Total	89	4.118E-02	2.143	20.940	21.113
		R	8	.264***	.407***	6.920***	8.095***
	LOW	M	4	.200***	.550***	4.463***	5.245***
β_2		R*M	32	.310***	1.727***	27.988***	32.496***
		Error	45	5.493E-02	9.957E-02	1.060	.825
		Total	89	.829	2.783	40.431	46.661
		R	8	4.806E-02***	.323***	5.885***	6.164***
	HIGH	M	4	3.751E-02***	.420***	3.580***	3.780***
		R*M	32	7.292E-02**	1.667***	24.255**	25.385***
		Error	45	3.841E-02	2.413E-03	.388	.267
		Total	89	.197	2.412	34.109	35.595

* $\Rightarrow$ Computed F value is significant at $\alpha = 0.05$. ** $\Rightarrow$ Computed F value is significant at $\alpha = 0.01$.

*** $\Rightarrow$ Computed F value is significant at $\alpha = 0.001$. ρ = R $\Rightarrow$ Autocorrelation levels

M $\Rightarrow$ Methods (Estimators)

the model parameters in the two replication groups. Because at least one of the estimators (CORC) is biased in small samples (Rao & Griliches, 1969), and that the mean squared error is known to replace the absolute bias (Kruthkoff, 1970) and also comprises variance and bias; therefore a further test on significant interaction effect of autocorrelation by method was performed on the basis of the mean squared error criterion. The LSD test of the estimated marginal mean was done at each level of autocorrelation.

At a particular level of autocorrelation, estimators were preferred if their estimated marginal means are not significantly different from the most preferred one. An estimator is most preferred if its estimated marginal mean is the smallest. Estimators that are preferred at all the levels of autocorrelation are said to be robust to autocorrelation; and if estimators are robust to autocorrelation in all the model parameters, the estimators are simply said to be robust.

Simulation Results and Discussion

The summary of findings on the performances of the estimators based on the criteria for each of the model parameters in the two replication groups is given in Table 1. It is observed that the error sum of square and hence the mean square error (if estimated) in all the criteria reduce with increased replications. Thus, the performances of the estimators in estimating all the parameters of the model improve with increased replication.

In estimating β_0 , the interaction effect of autocorrelation and method is only significant at the high replication group in all the criteria except bias. Thus, the performances of the methods are not affected by autocorrelation in bias criterion but in others criteria they do. The estimated marginal means based on the mean squared error criterion are shown in appendix. From the appendix, it is observed that as ρ decreases from zero, the estimated marginal means of the GLS estimators decrease while that of the OLS estimator first decreases before it starts to increase. As ρ increases from zero, the estimated marginal means of all the methods increase. Furthermore, the OLS estimator is observed to be more efficient than any of the GLS estimators when $-0.25 < \rho \leq 0.25$. It is also noted that the ML and the HILU estimators are robust to autocorrelation in estimating β_0.

In estimating β_1 and β_2, the interaction effect of autocorrelation and method is significant at the two replication groups in all the criteria. Thus, the performances of the estimators are affected by autocorrelation in all the criteria under the two replication groups. The estimated marginal means based the mean squared error of the estimated parameters under the two replication groups are given in appendix. The estimated marginal mean of the OLS estimator increases as $|\rho|$ increases while that of the GLS estimators decrease as $|\rho|$ increases, although this is not consistently the situation in β_2 especially when replication is low. Furthermore, it is observed that in estimating β_1 the OLS estimator is only more efficient than any of the GLS estimators at the two replication groups when $|\rho| \leq 0.25$ while in estimating β_2 OLS is more efficient when $|\rho| < 0.25$ at the low replication and when $-0.25 < \rho \leq 0.25$ at high replication. Moreover, the GLS estimators are robust in estimating β_1 and β_2 of the Linear Model.

Conclusion

Because performances of the estimators improve with increased replication, it can therefore be concluded that in estimating all the parameters of the model the ML and HILU estimators are robust; and that OLS estimator is more efficient than any of the GLS estimators considered when $-0.25 < \rho \leq 0.25$.

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Appendix

+⇒ Estimate that is significantly different from the most preferred one at α = 0.05

ρ_1	M	Replication = High Estimated Marginal Means: β_0	Replication = Low Estimated Marginal Means: β_1	R replication = High Estimated Marginal Means: β_1	Replication = Low Estimated Marginal Means: β_2	R replication = High Estimated Marginal Means: β_2
	OLS	.467	2.455+	3.343+	4.876+	4.309+
	COCR	1.495E-02	3.846E-02	4.990E-02	3.269E-02	4.547E-02
-.99	HILU	1.474E-02	3.801E-02	4.961E-02	3.195E-02	4.450E-02
	ML	1.491E-02	3.882E-02	5.013E-02	3.217E-02	4.430E-02
	MLGD	1.505E-02	4.066E-02	5.180E-02	3.132E-02	4.318E-02
	OLS	3.889E-02	.156	.154	.180	.170
	COCR	1.961E-02	5.127E-02	5.972E-02	3.624E-02	5.597E-02
-.75	HILU	1.965E-02	5.135E-02	5.991E-02	3.587E-02	5.573E-02
	ML	1.930E-02	4.711E-02	5.899E-02	3.496E-02	5.329E-02
	MLGD	1.931E-02	4.732E-02	5.898E-02	3.446E-02	5.293E-02
	OLS	3.352E-02	8.808E-02	8.571E-02	7.802E-02	9.500E-02
-.5	COCR	2.667E-02	8.085E-02	6.966E-02	3.786E-02	6.443E-02
	HILU	2.664E-02	8.121E-02	6.939E-02	3.831E-02	6.472E-02
	ML	2.555E-02	5.732E-02	6.496E-02	3.717E-02	6.206E-02
	MLGD	2.545E-02	5.614E-02	6.438E-02	3.733E-02	6.205E-02
	OLS	3.828E-02	6.348E-02	6.027E-02	4.305E-02	6.943E-02
-.25	COCR	3.862E-02	.110	7.478E-02	3.613E-02	7.206E-02
	HILU	3.848E-02	.110	7.427E-02	3.643E-02	7.208E-02
	ML	3.605E-02	7.062E-02	6.652E-02	3.866E-02	6.864E-02
	MLGD	3.592E-02	7.074E-02	6.641E-02	3.933E-02	6.864E-02
	OLS	5.191E-02	5.803E-02	5.130E-02	2.983E-02	5.654E-02
0	COCR	5.858E-02	.112	7.607E-02	3.812E-02	7.242E-02
	HILU	5.877E-02	.113	7.621E-02	3.832E-02	7.241E-02
	ML	5.299E-02	7.658E-02	6.765E-02	3.039E-02	6.447E-02
	MLGD	5.306E-02	7.595E-02	6.731E-02	3.002E-02	6.405E-02
	OLS	8.332E-02	6.888E-02	5.651E-02	2.732E-02	5.110E-02
.25	COCR	9.563E-02	.103	6.786E-02	4.053E-02	6.463E-02
	HILU	9.649E-02	.103	6.807E-02	3.995E-02	6.518E-02
	ML	8.512E-02	7.588E-02	6.108E-02	2.549E-02	5.632E-02
	MLGD	8.523E-02	7.998E-02	6.181E-02	2.547E-02	5.688E-02
	OLS	.169	.102	8.018E-02	4.270E-02	5.670E-02
.5	COCR	.204	7.069E-02	4.837E-02	3.784E-02	5.399E-02
	HILU	.214	7.025E-02	4.845E-02	3.799E-02	5.443E-02
	ML	.169	5.501E-02	4.556E-02	2.237E-02	4.720E-02
	MLGD	.168	5.247E-02	4.464E-02	2.206E-02	4.688E-02
	OLS	.571	.180	.133	.126	9.479E-02
	COCR	.936	4.010E-02	3.969E-02	3.493E-02	4.611E-02
.75	HILU	.861	3.989E-02	3.975E-02	3.485E-02	4.577E-02
	ML	.549	2.681E-02	3.800E-02	2.284E-02	4.071E-02
	MLGD	.546	2.550E-02	3.752E-02	2.284E-02	4.036E-02
	OLS	48.841+	.320+	.208+	.329+	.202+
.99	COCR	49.897+	3.654E-02	3.466E-02	4.112E-02	3.995E-02
	HILU	46.888	3.479E-02	3.396E-02	3.954E-02	3.967E-02
	ML	47.537	2.156E-02	3.373E-02	2.539E-02	3.758E-02
	MLGD	47.727+	2.171E-02	3.391E-02	2.542E-02	3.746E-02