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On the BLUE of the Population Mean for Location and Scale Parameters of Distributions Based on Moving Extreme Ranked Set Sampling

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The best linear unbiased estimator (BLUE) for the population mean under moving extreme ranked set sampling (MERSS) is derived for general location and scale parameters of distributions which generalizes Al-Odat and Al-Saleh (2001). It is compared with the sample mean of simple random sampling (SRS). The efficient sample size under the MERSS for which the BLUE estimator dominates the usual sample mean under SRS for estimating the population mean is also computed for several distributions.

Key words: Best linear unbiased estimator; location parameter; scale parameter; moving extreme ranked set sampling, simple random sampling.

Introduction

Ranked set sampling (RSS) as introduced by McIntyre (1952) is useful for cases when the variable of interest can be more easily ranked than quantified. The aim of RSS is to increase the efficiency of the sample mean as an estimator for the population mean μ . Takahasi and Wakimoto (1968) established a very important statistical foundation for the theory of RSS. They showed that the mean of the RSS is an unbiased estimator for the population mean and has smaller variance than the mean of SRS. Dell and Clutter (1972) studied the effect of ranking error on the procedure. The RSS has many statistical applications in biological and environmental studies and reliability theory (e.g. Dell & Clutter, 1972; Stokes, 1977, 1980; Mode et al., 1999; Barabesi & El-Sharaawi, 2001; Al-Saleh & Zheng, 2002; & Al-Saleh & Al-Omary, 2002). Sinha, et al., (1996) explored the concept

of RSS when the population is partially known using the parameters of normal and exponential distributions. They found that the use of knowledge of the distribution along with RSS provides improvement in estimation over SRS, as well as over nonparametric RSS. Li and Chuiv (1997) discussed the issue of the efficiency of RSS compared to SRS in many parametric estimation problems. They found an improvement in estimation of many common parameters of interest with smaller numbers of measurements compared to SRS.

RSS has been investigated extensively (see for example, Stokes, 1977; Stokes & Sager, 1988; Lam, et al., 1994; Barabesi & El-Sharaawi, 2001). Al-Saleh and Al-Kadiri (2000) introduced Double RSS to increase the efficiency of RSS estimates without increasing the set size m and Al-Saleh and Al-Omary (2002) generalized it to multistage RSS. Samawi, et al., (1996) used extreme ranked set sample (ERSS), which is easier to use than the usual RSS procedure, when the set size is large to estimate the population mean in the case of symmetric distributions. Al-Odat and Al-Saleh (2001) introduced the concept of varied set size RSS, which is coined here as Moving Extreme Ranked Set Sampling (MERSS). They investigated this modification non-parametrically and found that the

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procedure can be more efficient and applicable than the simple random sampling technique (SRS). The MERSS procedure is as follows:

1. Select m random samples of size 1, 2, 3, ..., m respectively.
2. Identify the maximum of each set by eye or by some other relatively inexpensive method without actually measuring the characteristic of interest.
3. Measure accurately the selected judgment identified maximum.
4. Repeat steps 1, 2, 3, but for the minimum.
5. Repeat the above steps r times until the desired sample size, $n = 2rm$ is obtained.

Clearly, the procedure of MERSS is easier to use than the usual RSS procedure.

Methodology

The BLUE of the Mean for Distributions with a Location Parameter

Let $\{X_{i1}^1, X_{i2}^1, \dots, X_{ii}^1\}$ and

$\{X_{i1}^2, X_{i2}^2, \dots, X_{ii}^2\}$ be simple random samples each of size i , for $i = 1, 2, \dots, m$ from a population with distribution function F and a probability density function f . Let μ and σ^2 be the mean and variance of the population respectively. If

$$y_{i1} = \text{Min}\{X_{i1}^1, X_{i2}^1, \dots, X_{ii}^1\},$$

and

$$y_{i2} = \text{Max}\{X_{i1}^2, X_{i2}^2, \dots, X_{ii}^2\},$$

$$i = 1, 2, \dots, m,$$

then

$$\{y_{11}, y_{21}, \dots, y_{m1}, y_{12}, y_{22}, \dots, y_{m2}\}$$

is a MERSS of size $2m$.

The BLUE for μ for a population can be derived with a pdf of the form:

$$f(x - \theta), \quad -\infty < \theta < \infty, \quad (2.1)$$

where f is a pdf.

Result 1

Let y_1, y_2, \dots, y_{2m} be $2m$ independent ordered statistics of simple random samples each of size less than m from an underlying distribution with a pdf as in (2.1). Then the BLUE of the population μ is then given by:

$$\hat{\mu}_{Blue} = \sum_{i=1}^{2m} \frac{1}{2\sigma_i^2 d} \{k - bt + bw c_i - tc_i\} y_i \quad (2.2)$$

where

$$k = \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2}, \quad t = \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2}, \quad w = \sum_{i=1}^{2m} \frac{1}{\sigma_i^2},$$

$$d = \sum_{i=1}^{2m} \frac{1}{\sigma_i^2} \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} - \left(\sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right)^2,$$

and C_i and σ_i^2 are the mean and the variance of Z_i respectively, where $Z_i = y_i - \theta$ and $\mu = E_{\theta} X = \theta + b$. (Note that y_1, y_2, \dots, y_{2m} are not necessarily identically distributed.)

Proof

Starting with a class of unbiased linear estimators of μ of the form

$$\hat{\mu} = \sum_{i=1}^{2m} a_i y_i, \quad (2.3)$$

implies that

$$E(\hat{\mu}) = \theta \sum_{i=1}^{2m} a_i + \sum_{i=1}^{2m} a_i c_i = \mu = \theta + b,$$

which, in turn, implies that

$$\sum_{i=1}^{2m} a_i = 1$$

and

$$\sum_{i=1}^{2m} a_i c_i = b. \quad (2.4)$$

Applying the method of the Lagrange multiplier to minimize

$$Var(\hat{\mu}) = \sum_{i=1}^{2m} a_i^2 \sigma_i^2,$$

subject to (2.4), results in:

$$a_i^* = \frac{\lambda_1 + \lambda_2 c_i}{2\sigma_i}, \quad \sum_{i=1}^{2m} a_i^* = 1, \quad \sum_{i=1}^{2m} a_i^* c_i = b,$$

$$\lambda_1 = \frac{\left\{ -\sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} + b \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}}{\left(\sum_{i=1}^{2m} \frac{1}{\sigma_i^2} \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} - \left\{ \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}^2 \right)},$$

$$\lambda_2 = \frac{\left\{ -b \sum_{i=1}^{2m} \frac{1}{\sigma_i^2} + \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}}{\left(\sum_{i=1}^{2m} \frac{1}{\sigma_i^2} \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} - \left\{ \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}^2 \right)},$$

where λ_1 and λ_2 are the Lagrange multipliers and

$$a_i^* = \frac{1}{2\sigma_i^2 d} \{k - bt + c_i bw - c_i t\}. \quad (2.5)$$

Then

$$\hat{\mu}^* = \sum_{i=1}^{2m} \frac{1}{2\sigma_i^2 d} \{k - bt + c_i bw - c_i t\} y_i, \quad (2.6)$$

is the BLUE of μ with variance

$$Var(\hat{\mu}^*) = \sum_{i=1}^{2m} \left(\frac{1}{2d\sigma_i^2} \{k - bt + c_i bw - c_i t\} \right)^2 \sigma_i^2. \quad (2.7)$$

If $y_i = y_{i1}$, for $i = 1, 2, \dots, m$ and $y_i = y_{i2}$, for $i = m + 1, m + 2, \dots, 2m$.

If $E(y_i) = E(y_{i1}) = c_{i1} + \theta$, $i = 1, 2, \dots, m$

$E(y_i) = E(y_{i2}) = c_{i2} + \theta$, $i = m + 1, m + 2, \dots, 2m$

$Var(y_i) = Var(y_{i1}) = \sigma_{i1}^2$, $i = 1, 2, \dots, m$

and

$Var(y_i) = Var(y_{i2}) = \sigma_{i2}^2$, $i = m + 1, m + 2, \dots, 2m$,

where $c_{i1} = E(u_i)$, $Var(u_i) = \sigma_{i1}^2$, and u_i is the minimum of a SRS of size i , and $c_{i2} = E(w_i)$, $Var(w_i) = \sigma_{i2}^2$ and w_i is the maximum of a SRS of size i , under $\theta = 0$. It then follows that:

$$\begin{aligned} \hat{\mu}_{MEBLUE} &= \sum_{i=1}^m \frac{1}{2\sigma_{i1}^2 d} \{k - bt + c_{i1} bw - c_{i1} t\} y_{i1} \\ &+ \sum_{i=m+1}^{2m} \frac{1}{2\sigma_{i2}^2 d} \{k - bt + c_{i2} bw - c_{i2} t\} y_{i2} \end{aligned} \quad (2.8)$$

and

$$\begin{aligned} Var(\hat{\mu}_{MEBLUE}) &= \sum_{i=1}^m \frac{1}{4d^2 \sigma_{i1}^2} (k - bt + c_{i1} bw - c_{i1} t)^2 \\ &+ \sum_{i=m+1}^{2m} \frac{1}{4d^2 \sigma_{i2}^2} (k - bt + c_{i2} bw - c_{i2} t)^2 \end{aligned} \quad (2.9)$$

Al-Odat and Al-Saleh (2001) introduced MERSS and studied the linear estimators of the form: $\sum_{i=1}^m a_i (y_{i1} + y_{i2})$. They derived the

BLUE among such linear combinations for the population mean. The BLUE derived by Al-Odat and Al-Saleh (2001) is not the BLUE estimator based on $(y_{11}, y_{21}, \dots, y_{m1}, y_{12}, y_{22}, \dots, y_{m2})$,

but the BLUE based on (k_1, k_2, \dots, k_m) where

$k_i = y_{i1} + y_{i2}$ for $i=1,2,\dots, m$. If the underlying distribution is symmetric about its mean μ , then (2.9) coincides with the results obtained by Al-Odat and Al-Saleh (2001).

The BLUE estimator based on MERSS, obtained with the sample mean based on SRS in case of uniform $U(\theta, \theta + 1)$ and $Exp(\theta, 1)$ distributions are compared. The first is symmetric about its mean $\theta + \frac{1}{2}$ and the second

is skewed to the right with mean $\theta + 1$. Both families are location parameter families of distributions, so the BLUE's are the same as given in (2.8), with $b = \frac{1}{2}$ for $U(\theta, \theta + 1)$ and

$b=1$ for $Exp(\theta, 1)$. Balakrishnan and Cohen (1990) computed the variances of the estimators in this case and in the following cases.

The estimators compared are both unbiased for μ . Therefore, they will be compared through their variances. The efficiency between two estimators $\hat{\mu}_1$ and $\hat{\mu}_2$ is defined as:

$$eff(\hat{\mu}_2, \hat{\mu}_1) = Var(\hat{\mu}_1) [Var(\hat{\mu}_2)]^{-1}$$

The larger the efficiency, the better the estimator $\hat{\mu}_2$ will be. The efficiency of $\hat{\mu}_{MEBlue}$ with respect to the sample mean under SRS was computed for both distributions for $m = 2, \dots, 10$. The results are summarized in Tables 1 and 2. From these tables, it may be concluded that the variance of the BLUE decreases as m increases and $eff(\hat{\mu}_{MEBlue}, \bar{X}_{2m}) \geq 1$ for both distributions. Also, the efficiency is more than 2 for $m \geq 4$ in the uniform case and for $m \geq 9$ in the exponential case.

Efficiency of $\hat{\mu}_{MEBlue}$ with respect to \bar{X}_{2m}

Table 1		Table 2	
U($\theta, \theta+1$)		Exp($\theta, 1$)	
m	$eff(\hat{\mu}_{MEBlue}, \bar{X}_{2m})$	m	$eff(\hat{\mu}_{MEBlue}, \bar{X}_{2m})$
2	1.333	2	1.167
3	1.765	3	1.333
4	2.200	4	1.483
5	2.863	5	1.639
6	3.150	6	1.647
7	3.683	7	1.8397
8	4.288	8	1.996
9	4.932	9	2.087
10	5.620	10	2.177

The BLUE of the mean for distributions with a scale parameter

Let $\{y_{11}, y_{21}, \dots, y_{m1}, y_{12}, y_{22}, \dots, y_{m2}\}$ be a MERSS from a population with a pdf of the form:

$$\frac{1}{\theta} f\left(\frac{x}{\theta}\right), \theta > 0 \tag{3.1}$$

where f is a pdf. Then as shown previously, if

$$y_{i1} = \theta \min\left\{\frac{X_{i1}^1}{\theta}, \frac{X_{i2}^1}{\theta}, \dots, \frac{X_{ii}^1}{\theta}\right\},$$

then

$$E(y_{i1}) = \theta \text{Min}\left\{\frac{X_{i1}^1}{\theta}, \frac{X_{i2}^1}{\theta}, \dots, \frac{X_{ii}^1}{\theta}\right\} = C_{i1} \theta$$

where $C_{i1} = E(U_i)$ and U_i is the first order statistic of a SRS of size i from the pdf in (3.1), under $\theta = 1$. Similarly, $E(y_{i2}) = C_{i2} \theta$, for $C_{i2} = E(W_i)$ where W_i is the maximum order statistic of a SRS of size i from the pdf in (3.1),

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under $\theta = 1$. Also, $\text{Var}(y_{i1}) = \theta^2 \sigma_{i1}^2$ and $\text{Var}(y_{i2}) = \theta^2 \sigma_{i2}^2$ where σ_{i1}^2 and σ_{i2}^2 are the variances of u_i and w_i respectively, for $i = 1, 2, \dots, m$. (The BLUE of the mean of the population with pdf (3.1) proof is similar to that of Result (1) and therefore is omitted.)

Result 2

Let y_1, y_2, \dots, y_{2m} be $2m$ independent order statistics each of size less than m from an underlying distribution with a pdf as in (3.1). Then the BLUE of the population μ is given by:

$$\hat{\mu}_{Blue} = \frac{\sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} y_i}{\sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2}} \quad (3.2)$$

with variance

$$\text{Var}(\hat{\mu}_{Blue}) = \frac{\theta^2}{\sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2}} \quad (3.3)$$

where $\mu = b\theta$ and $b = E_{\theta=1} X$.

The BLUE of μ using MERSS is given by:

$$\hat{\mu}_{MEBlue} = \frac{\sum_{i=1}^m \frac{c_{i1}}{\sigma_{i1}^2} y_{i1} + \sum_{i=m+1}^{2m} \frac{c_{i2}}{\sigma_{i2}^2} y_{i2}}{\sum_{i=1}^m \frac{c_{i1}}{\sigma_{i1}^2} + \sum_{i=m+1}^{2m} \frac{c_{i2}}{\sigma_{i2}^2}} \quad (3.4)$$

and

$$\text{Var}(\hat{\mu}_{MEBlue}) = \frac{\theta^2}{\sum_{i=1}^m \frac{c_{i1}}{\sigma_{i1}^2} + \sum_{i=m+1}^{2m} \frac{c_{i2}}{\sigma_{i2}^2}} = \frac{\theta^2}{\left\{ \sum_{i=1}^m \frac{c_{i1}}{\sigma_{i1}^2} + \sum_{i=m+1}^{2m} \frac{c_{i2}}{\sigma_{i2}^2} \right\}} \quad (3.5)$$

Comparing the BLUE estimator based on MERSS with the sample mean based on SRS in

case of uniform $\text{Exp}(\theta)$ and $U(0, \theta)$ distributions. The first is skewed to the right with mean θ and the second is symmetric about its mean $\frac{\theta}{2}$. So, the BLUE's are the same as given in (3.2). The estimators are unbiased and therefore are compared using their variances for $m = 2 \dots 10$. The results are summarized in Tables (3) and (4). Similar conclusions to those presented for Tables (1) and (2) can be given.

Efficiency of $\hat{\mu}_{MEBlue}$ with respect to \bar{X}_{2m}

Table 3		Table 4	
Exp(θ)		U(0, θ)	
m	$eff(\hat{\mu}_{MEBlue}, \bar{X}_{2m})$	m	$eff(\hat{\mu}_{MEBlue}, \bar{X}_{2m})$
2	1.200	2	1.331
3	1.380	3	1.815
4	1.540	4	2.264
5	1.690	5	2.955
6	1.820	6	3.574
7	1.950	7	4.593
8	2.070	8	5.713
9	2.190	9	6.935
10	2.300	10	8.261

Saving by using MERSS to estimate the population mean

Measuring the units of a sample costs money, time, and effort. The previous tables show that the BLUE for estimating the population mean μ under MERSS is more efficient (less variance) than the sample mean of SRS, which is usually used for estimating μ . Therefore, $\hat{\mu}_{MEBlue}$ will be as good as \bar{X}_{2m} by using a smaller number of observations which will result in saving time, money and effort. Table (5) shows the smallest $2m$ such that the variance of the BLUE under MERSS using $2m$ observations is smaller than the variance of the sample mean of SRS using a specified sample size in case of the normal, logistic, uniform, and exponential distributions. The first two

distributions are location parameter families of distributions while the other two are scale parameter families.

Table (5), shows how the BLUE, under MERSS for estimating the population mean, requires a smaller number of observations than \bar{X}_{2m} based on SRS. This indicates a reduction in the sample size required for estimating the mean. As m increases then the savings will be greater for all the cases studied. According to Table (5), the savings in sample sizes range from 0% to 70%. For example, $\hat{\mu}_{MEBlue}$ based on 12 observations is better than \bar{X}_{2m} based on 40 observations in the case of $U(\theta, \theta + 1)$ for estimating the mean, resulting in saving 70% of the sample size from using the MERSS compared to SRS.

Conclusion

If ordering the data can be done more easily than quantifying it, then the BLUE under MERSS can be used instead of the mean of SRS for estimating the population mean because the BLUE under MERSS provides better results than the mean of SRS with fewer numbers of observations.

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Table 5: Efficiency of the Smallest Number of Observations for MERSS Compared to the SRS of Size 2m

SRS 2m	MERSS					
	$N(\theta,1)$	$L(\theta,1)$	$Exp(\theta,1)$	$Exp(\theta)$	$U(\theta, \theta+1)$	$U(0, \theta)$
2	2	2	2	2	2	2
4	4	4	4	4	4	4
6	6	6	6	6	6	6
8	6	6	8	6	6	6
10	8	8	8	8	6	6
12	8	10	10	8	8	8
14	10	10	10	10	8	8
16	10	12	10	10	8	8
18	12	12	12	12	10	10
20	12	14	12	12	10	10
22	14	14	14	14	10	10
24	14	16	14	14	10	10
26	14	16	16	14	10	10
28	16	18	16	16	10	10
30	16	20	16	16	12	12
32	16	20	18	16	12	12
34	18	21	18	18	12	12
36	18	21	18	18	12	12
38	19	22	20	18	14	12
40	19	22	20	20	14	12