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Performance Ratings of an Autocovariance Base Estimator (ABE) in the Estimation of GARCH Model Parameters When the Normality Assumption is Invalid

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The performance of an autocovariance base estimator (ABE) for GARCH models against that of the maximum likelihood estimator (MLE) if a distribution assumption is wrongly specified as normal was studied. This was accomplished by simulating time series data that fits a GARCH model using the Log normal and t-distributions with degrees of freedom of 5, 10 and 15. The simulated time series was considered as the true probability distribution, but normality was assumed in the process of parameter estimations. To track consistency, sample sizes of 200, 500, 1,000 and 1,200 were employed. The two methods were then used to analyze the series under the normality assumption. The results show that the ABE method appears to be competitive in the situations considered.

Key words: Autocovariance Functions, Parameter Estimation, Garch, Normality.

Introduction

The assumption of constant variance in the traditional time series models of ARMA is a major impediment to their applications in financial time series data where heteroscedasticity is obvious and cannot be ignored. To solve this problem, Engle (1982) proposed the Autoregressive Conditional Heteroscedascity (ARCH) model. In his first application, however, Engle noted that a high order of ARCH is needed to satisfactorily model time varying variances and that many parameters in ARCH will create convergence problems for maximization routines. To address these difficulties, Bollerslev (1986) extended Engle's model, developing the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. GARCH models time-varying variances as a linear function of past square residuals and of its past value. It has proved useful in interpreting volatility clustering effects and has gained wide acceptance in measuring the volatility of financial markets. The ARCH and GARCH models are both known as symmetric models.

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Other extensions based on observed characteristics of financial time series data exist and include some asymmetric models. Examples of asymmetric models are Nelson's (1991) exponential GARCH (EGARCH) model, Glosten, Jaganathan and Runkle's (1993) GJR-GARCH and Zakoian's (1994) threshold model (T GARCH). These model and interpret leverage effects, where volatility is negatively correlated with returns. In addition, the Fractionally Integrated GARCH model (FIGARCH) (Baillie, Bollerslev & Mikeson, 1996) was introduced to model long memory via the fractional operator $(1\text{-}L)^d$, and the GARCH in mean model allows the mean to influence the variance.

These models are popularly estimated by the quasi-maximum likelihood method (QMLE) under the assumption that the distribution of one observation conditional to the past is normal. The asymptotic properties of the QMLE are well established. Weiss (1989) showed that QMLE estimates are consistent and asymptotically normal under fourth moment conditions. These were again shown by Ling and McAleer (2003) under second moment conditions. If the assumption of normality is satisfied by the data, then the method will produce efficient estimates; otherwise, inefficient estimates will be produced. Engle and Gonzalez-Rivera (1991) studied the loss of estimation efficiency inherent in QMLE and concluded it may be severe if the distribution density is heavy tailed.

The QMLE estimator requires the use of a numerical optimization procedure which depends on different optimization techniques for implementation. This potentially leads to different estimates, as shown by Brooks, Burke and Perssand (2001) and McCullough and Renfro (1999). Both studies reported different QMLE estimates across various packages using different optimization routines. These techniques estimate time-varying variances in different ways and may result in different interpretations and predictions with varying implications to the economy. To resolve these problems, Eni and Etuk (2006) developed an Autocovariance Base Estimator (ABE) for estimating the parameters of GARCH models through an ARMA transformation of the GARCH model equation. The purpose of this article is to rate the performance of the ABE when the normality assumption is violated.

The Autocovariance Base Estimator (ABE)

Consider the GARCH (p, q) equation

$$h_t = w_0 + \sum_{i=1}^{p} \boldsymbol{\alpha}_i e_{t-1}^2 + \sum_{j=1}^{q} \mathrm{B}_j h_{t-1}, \qquad (1)$$

or its ARMA (Max (p, q), q) transform

$$\boldsymbol{\varepsilon}_t^2 = w_0 + \sum_{i=1}^{\max(p,q)} (\boldsymbol{\alpha}_i + B_i)\boldsymbol{\varepsilon}_{t-1}^2 - \sum_{j=1}^{q} B_J a_{t-1} + B_0 a_t \qquad (2)$$

$\epsilon \sim N(0, \sigma^2)$.

To obtain the autoregressive parameters, consider that the variance, $Var(\varepsilon_t^2 \varepsilon_{t-i}^2)$ for i > q in equation (2) will not contain the moving average parameter B_i. Hence, i = q + 1 … q + p is used to obtain the estimator:

$$\begin{bmatrix} V_{q+1} \\ V_{q+2} \\ \vdots \\ V_{q+\max\ (p,q)} \end{bmatrix} = \begin{bmatrix} Vq & V_{q-1} & \cdots & V_{q-(p-1)} \\ V_{q-1} & V_q & \cdots & V_{q-(p-2)} \\ \vdots & \vdots & \vdots & \\ V_{q-(p-1)} & V_{q-(p-2)} & \cdots & Vq \end{bmatrix} \begin{bmatrix} (\boldsymbol{\alpha}_1 + \mathrm{B}_1) \\ (\boldsymbol{\alpha}_2 + \mathrm{B}_2) \\ \vdots \\ (\boldsymbol{\alpha}_p + \mathrm{B}q) \end{bmatrix} \qquad (3)$$

Where V_i is the set of variances associated with equation (2). The autoregressive parameters $\alpha_i + \mathrm{B}_i$ are obtained by solving (3).

Eni and Etuk (2006) have shown that the moving average parameters B_i can be obtained from

$$\sum_{i=0}^{p} f(\Phi_i) \begin{vmatrix} V_i & V_{i-1} & \cdots & V_{I-p} \\ V_{i+1} & V_i & \cdots & V_{i-(p-1)} \\ \vdots & \vdots & \vdots & \vdots \\ V_{i+q} & V_{i-q-1} & \cdots & V_{i+q-p} \end{vmatrix} \begin{bmatrix} \Phi_0 \\ -\Phi_1 \\ \vdots \\ -\Phi_p \end{bmatrix} =$$

$$\boldsymbol{\sigma}_a^2 \begin{bmatrix} B_0 & -B_1 & \cdots & -B_{q-1} & -B_q \\ -B_1 & -B_2 & \cdots & -B_q & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -B_{q-1} & -B_q & \cdots & 0 & 0 \\ -B_q & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} B_0 \\ -B_1 \\ \vdots \\ -B_{q-1} \\ -B_q \end{bmatrix}$$

or

$$\sum_{i=0}^{p} f(\Phi_i) V\Phi = \boldsymbol{\sigma}_a^2 B\, b \qquad (4)$$

where

$$f(\Phi_i) = -\Phi_i, f(\Phi_0) = \Phi_0, \Phi_i = (\boldsymbol{\alpha}_i + \mathrm{B}_i).$$

Note that the quantity $\sum_{i=0}^{p} f(\Phi_i) V\Phi$ is known, the variance V having been calculated from the data, and the autoregressive parameters Φ having been calculated from equation (3).

The moving average parameters Bi are found by solving the system:

$$F(B) = \sum_{i=0}^{p} f(\Phi_i) V\Phi - \sigma_r^2 B\, b = 0 \qquad (5)$$

Equation (5) is nonlinear and the solution can be found only through an iterative method. One procedure to consider is based on the Newton-Raphson algorithm, in this case, the B_{r+1} solution is obtained from the rth approximation according to

$$B_{r+1} = B_r - \{f'(B_r)\}^{-1} f(B_r) \qquad (7)$$

where $f(B_r)$ and $f'(B_r)$ represent the vector function (5) and its derivative evaluated at $B=B_r$. Note that

$$f'(B) = \sigma_a^2 \begin{bmatrix} B_0 & B_1 & \cdots & B_{q-1} & B_q \\ -B_1 & B_2 & \cdots & B_q & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ -B_{q-1} & B_q & \cdots & 0 & 0 \\ -B_q & 0 & \cdots & 0 & 0 \end{bmatrix} + \sigma_a^2 \begin{bmatrix} B_0 & B_1 & \cdots & B_{q-1} & B_q \\ 0 & -B_0 & \cdots & B_{q-z} & B_{q-1} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & -B_0 & B_1 \\ 0 & 0 & \cdots & 0 & -B_0 \end{bmatrix}$$

$$= \sigma_r^2 (D_1+D_2)$$

and (4) becomes

$$B_{r+1} = B_r + \{\sigma_a^2 (D_1 + D_2)\}^{-1} \left[\sum_{i=0}^{p} f(\Phi_1) V\, \Phi - \sigma_a^2 B\, b \right] \qquad (8)$$

The starting point for the iteration (8) is $\sigma_r^2 = 1$, $B_0 = V_0$, $B_i = 0$, i = 1 … q.

Having computed the Autoregressive parameters $\Phi_i = (\alpha_i + B_i)$ and the Moving average parameter B_i, it is simple to obtain the GARCH (p, q) parameters, α_i, and the constant parameter w_0, which is estimated using

$$W = E(\varepsilon_t^2)(1 - \sum_{i=1}^{p} \alpha_i - \sum_{i=1}^{B} B_i). \qquad (9)$$

Methodology

The data generating process (DGP) in this study involved the simulation of 1,500 data points with 10 replications using the random number generator in MATLAB 5. The random number generator in MATLAB 5 is able to generate all floating point numbers in the interval $\left[2^{-53},\ 1-2^{-53}\right]$. Thus, it can generate 2^{1492} values before repeating itself. Note that 1,500 data points are equivalent to $2^{10.55}$ and, with 10 replications; results in only $2^{13.87267}$ data points. Hence 1,500 data points with 10 replications were obtained without repetitions. Also, a program implementation was used for ARMA to find the QMLE (McLeod and Sales, 1983). Although normality would typically be assumed, the data points were simulated using the Log normal and the T-distribution with 5, 10 and 15 degrees of freedom.

Of the 1,500 data points generated for each process, the first 200 observations were discarded to avoid initialization effects, yielding a sample size of 12,000 observations, with results reported in sample sizes of 200, 500, 1,000 and 1,200. These sample presentations enable tracking of consistency and efficiency of the estimators. The relative efficiency of the autocovariances based estimator (ABE) and the quasi-maximum likelihood (QML) estimators were studied under this misspecification of distribution function. The selection criteria used was the Aikake information criteria (AIC).

For simulating the data points, the conditional variance equation for low persistence due to Engle and Ng (1993) was adopted.

$$h_t = 0.2 + 0.05\,\varepsilon_{t-1}^2 + 0.75 h_{t-1}$$

$$\varepsilon_{t-1}^2 = h_t Z_t^2$$

and Z_t^2 is any of $Z \sim t_5$ or $Z \sim t_{10}$ or Z~LN(0,1) or $Z \sim t_{15}$, where N = normality, t_V = t-distribution with V degree of freedom, and LN = log normal.

Results

Apart from the parameter setting in the DGP, selected studies of the parameter settings $(W,\alpha,B) = (0,\ 1,\ 0.15,\ 0.85)$ and $(W,\alpha,B) = (0,\ 1,\ 0.25,\ 0.65)$ (Lumsdaine, 1995), and $(W,\alpha,B) = (1, 0.3, 0.6)$ and $(W,\alpha,B) = (1, 0.05, 0.9)$ (Chen, 2002) were also studied. The results obtained agree with the results obtained from detailed studies of the DGP.

Table 1 shows the results from a sample size of 200 data points. The table reveals that the estimates are poor for QMLE and ABE. On the basis of the Aikate information criteria (AIC), however, the QMLE performed better than the ABE except under the log normal distribution where ABE performed better than QMLE.

Table 2 shows that the estimates using a sample size of 500 are better, although still poor. The performance bridge between QMLE and ABE appears to be closing. This is observed from the AIC of QMLE and ABE under the different probability distribution functions, with one exception in the case of the log normality. Surprisingly, the QMLE method failed to show consistency, but it is notable that the performance of both methods was enhanced under the t-distribution as the degrees of freedom increase.

Table 3 shows that both estimation models, QMLE and ABE, had equal performance ratings and gave consistent estimates in general. However, the ABE had an edge in its performance under $t_{(5)}$ and LN(0, 1) while QMLE had an edge under $t_{(10)}$ and $t_{(15)}$. The estimates under $t_{(15)}$ and $t_{(10)}$ were close to their true values for both estimation methods. Finally, the results shown in Table 3 are further confirmed by examining Table 4 where the two methods have nearly equal ratings based on the values of their AIC.

Conclusion

It is shown is this study that the ABE method is adequate in estimating GARCH model parameters and can perform as well as the maximum likelihood estimate for reasonably large numbers of data points when the distribution assumption is misspecified.

Table 1: Performance Rating of QMLE and ABE for Sample Size *n* = 200

	Estimation Method							
	QMLE				ABE			
Estimates	W	α	B	AIC	W	α	B	AIC
t (5)	0.16	0.01	0.77	-70.90	1.14	0.016	0.74	-65.312
t (10)	0.14	0.014	0.76	-140.36	1.138	0.012	0.75	-124.31
t (15)	0.15	0.17	0.76	-169.40	1.42	0.016	0.76	-157.21
Ln (0, 1)	9.3	-0.2	0.86	129.17	6.2	0.20	0.81	108.23

Table 2: Performance Rating of QMLE and ABE for Sample Size n = 500

	Estimation Method							
	QMLE				ABE			
Estimates	W	α	B	AIC	W	α	B	AIC
t (5)	0.115	0.02	0.739	-132.341	0.15	0.025	0.73	-151.24
t (10)	0.018	0.034	0.742	-1021.22	0.21	0.029	0.74	-956.31
t (15)	0.17	0.036	0.75	-1973.42	0.20	0.030	0.75	-1472.40
Ln (0, 1)	6.79	-0.15	0.88	289.39	3.94	0.08	0.80	108.21

Table 3: Performance Rating of QMLE and ABE for Sample Size n = 1,000

	Estimation Method							
	QMLE				ABE			
Estimates	W	α	B	AIC	W	α	B	AIC
t (5)	0.18	0.02	0.74	-137.12	0.19	0.03	0.73	-140.12
t (10)	0.193	0.029	0.75	-1141.62	0.22	0.03	0.74	-1094.72
t (15)	0.195	0.034	0.75	-1984.71	0.21	0.04	0.75	-1976.22
Ln (0, 1)	5.24	-0.40	0.86	119.72	3.50	0.07	0.79	101.13

Table 4: Performance Rating of QMLE and ABE for Sample Size n = 1,200

	Estimation Method							
	QMLE				ABE			
Estimates	W	α	B	AIC	W	α	B	AIC
t (5)	0.15	0.03	0.76	-162.11	0.18	0.039	0.73	-173.70
t (10)	0.018	0.039	0.743	-1391.30	0.19	0.041	0.74	-1350.11
t (15)	0.19	0.043	0.746	-2441.30	0.19	0.044	0.742	-2430.39
Ln (0, 1)	4.3	0.08	0.81	168.59	3.48	0.060	0.78	256.23

References

Baillie, R., Bollerslev, T., & Mikkelson, H. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *74*, 3-30.

Bollerslev, T. (1986). Generalized autoregressive heteroskedasticity. *Journal of Econometrics*, *31*, 307-327.

Brooks, I., Burke, S., & Perssand, G. (2001). Benchmarks and accuracy of GARCH model estimation. *International Journal of Forecasting*, *17*, 45-56.

Chen, Y. T. (2002). On the robustness of Ljung-Box and Mcleed-Li Q test. *Economics Bulletin*, *3*(*17*), 1-10.

Engle, F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation. *Econometrica*, *50*, 987-1008.

Engle, F., & Gonzalez-Revera. (1991). Semi parametric ARCH models. *Journal of Business and Economic Statistics*, *19*, 3-29.

Engle, F., & Ng, V. (1993). Measurement and testing the impact of news on volatility. *Journal of Finance*, *48*(*5*), 1749-1778.

Gosten, L., Jaganatan, R., & Runkle, D. (1993). On the relationship between the expected value and the volatility of the nominal excess return on stock. *Journal of Finance*, *48*, 1779-1801.

Eni, D., & Etuk, E. H. (2006). *Parameter estimation of GARCH models: An autocovariance approach*. Proceedings of International Conference on New Trends in the Mathematical & Computer Sciences with Applications to Real World Problems Held at Covenant University Ota, Nigeria, 357-368.

Ling & McAleer, M. (2003). Asymptotic theory for a vector GARCH (1.1) quasi-maximum likelihood estimator. *Economic Theory*, *19*, 280-310.

Lumsdain, R. L. (1995). Consistency and asymptotic normality of the quasi-maximum likelihood estimator. *Econometrica*, *64*, 575-596.

McCullough, B. D., & Renfro, C. G. (1999). Bench marks and software standard: A case study of GARCH procedure. *Journal of Economics and Social Measurement*, *25*, 59-71.

McLeod, A., & Sales, P. (1983). Algorithm for approximate likelihood calculation of ARMA and seasonal ARMA models. *Journal of Applied Statistics*, *32*, 211-2190

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, *59*, 347-370

Weiss, A. (1986). Asymptotic Theory of GARCH (1,1) model. *Economic Theory*, *2*, 107-131

Zakoian, J. M. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamic and Control*, *18*, 931-955.