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The concept of Neighbor Balanced Block (NBB) designs is defined for the experimental situation where the treatments are combinations of levels of two factors and only one of the factors exhibits a neighbor effect. Methods of constructing complete NBB designs for two factors in a plot that is strongly neighbor balanced for one factor are obtained. These designs are variance balanced for estimating the direct effects of contrasts pertaining to combinations of levels of both the factors. An incomplete NBB design for two factors is also presented and is found to be partially variance balanced with three associate classes.

Key words: Circular design, neighbor balanced, strongly neighbor balanced, variance balanced, partially variance balanced.

Introduction

In many agricultural experiments, the response from a given plot is affected by treatments applied to neighboring plots provided the plots are adjacent with no gaps. For example, when treatments are varieties, neighbor effects may be caused due to differences in height or date of germination, especially on small plots. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbor effects. In order to avoid the bias in comparing the effects of treatments in such a situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbor. Neighbor Balanced Designs, wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbors, are used for these situations. These designs permit the estimation of direct and neighbor effects of treatments.

Azais, et al. (1993) developed a series of circular neighbor balanced block (NBB) designs for single factor experiments. A NBB design for a single factor with border plots is circular if the treatment in the left border is the same as the treatment in the right-end inner plot and the treatment in the right border is the same as the treatment in the left-end inner plot. Tomar, et al. (2005) also obtained some incomplete NBB designs for single factor experiments.

In certain experimental situations, the treatments are the combination of levels of two factors and only one of the factors exhibits neighbor effects. For example, agroforestry experiments consist of tree and crop combination in a plot and, because trees are much taller than the crop, it is suspected that the tree species of one plot may affect the response from the neighboring plots. The effect of the crop species in neighboring plots is assumed to be negligible. Under this situation, it is therefore desirable that designs allowing the estimation of direct effects of treatment combinations free of

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neighbor effects are developed. Langton (1990) advocated the use of both NBBs and guard areas in agroforestry experiments. Monod and Bailey (1993) presented two factor designs balanced for the neighbor effect of one factor.

NBB designs are defined for the experimental situation where the treatments are the combinations of levels of two factors and only one of the factors exhibits a neighbor effect in a block design with no gaps or guard areas between the plots. Some methods of constructing these designs balanced for the effects of one factor in the adjacent neighboring plots are presented.

Model

Let F_1 and F_2 be two factors in an experiment with f_1 and f_2 levels, respectively. The f_1 levels are represented as (*1, 2, ...*) and the f_2 levels as (*a, b, ...*). Consider an inner plot i (i = 1, 2, …, k) in the block θ(i) [= 1, 2, …, b] of a block design with a left neighbor plot i−1 and a right neighbor plot i+1. Let ϕ(i) and φ(i) denote the levels of F_1 and F_2, respectively, on i. The general fixed effects model (Monod and Bailey, 1993) for Y_i, the response from plot i considered is

$$Y_i = \mu + \beta_{\theta(i)} + \tau_{\phi(i),\varphi(i)} + \delta_{\phi(i-1)} + \rho_{\phi(i+1)} + e_i \tag{1}$$

where μ is the general mean, $\beta_{\theta(i)}$ is the effect of block θ(i) to which plot i belongs, $\tau_{\phi(i),\varphi(i)}$ is the direct effect of the treatment combination ϕ(i)φ(i), $\delta_{\phi(i-1)}$ is the left neighbor effect of ϕ(i-1), $\rho_{\phi(i+1)}$ is the right neighbor effect of ϕ(i+1) and e_i is a random error term assumed to be identically and independently distributed with mean zero and constant variance.

Definitions

The following definitions for a block design with two factors in a plot and with neighbor effects for one factor (for example, F_1) from adjacent neighboring plots are provided.

Definition 1: circular block design. A block containing plots with treatment combinations and border plots is said to be left circular if the level of F_1 on the left border is the same as the level of F_1 on the right end inner plot. It is right circular if the level of F_1 on the right border is the same as the level of F_1 on the left end inner plot. A circular block is a left as well as right circular. A design with all circular blocks is called a circular block design. Note that the observations are not recorded from the border plots; these plots are taken only to have the neighbor effects of factor F_1.

Definition 2: strongly neighbor balanced. A circular block design with two factors F_1 and F_2 is called strongly neighbor balanced for factor F_1 if every combination of the two factors has each of the levels of factor F_1 as a right as well as a left neighbor a constant number of times, for example, μ_1'.

Definition 3: neighbor balanced. A circular block design with two factors F_1 and F_2 is neighbor balanced for factor F_1 if every combination of the two factors has levels of factor F_1 (except the level appearing in the combination) appearing μ_1'' times as a right and as a left neighbor.

Definition 4: variance balanced. A block design for two factors with left and right neighbor effects of factor F_1 is said to be variance balanced if the contrasts in the direct effects of $f_1 \times f_2$ combinations are estimated with the same variance, for example, V.

Definition 5: partially variance balanced. A block design for two factors with neighbor effects is partially variance balanced following some association scheme if the contrasts pertaining to the $f_1 \times f_2$ combinations from F_1 and F_2 factors are estimated with different variances, depending upon the order of association scheme.

Methodology

Complete NBB Designs for Two Factors: Method 1

Let f_1 be a prime number with its primitive root as x and $f_2 = f_1 - s$, $s = 1, 2, \ldots, f_1 - 2$. Obtain a basic array of f_2 columns each of size f_1 from the following initial sequence for values of $i = 1, 2, \ldots, f_2$:

$$\begin{array}{c} f_1 \\ x^i \\ x^{i+1} \\ . \\ . \\ . \\ x^{i+f_1-2} \end{array}$$

Develop the columns of this array cyclically, mod f_1 to obtain f_1 sets of f_2 columns each. Allocate f_2 symbols denoted by *a*, *b*, … to each of the sets in such a way that symbol *a* occurs with all entries of column 1 in each set, *b* with all entries of column 2 of each set and so on. Considering the rows as blocks and making the blocks circular by adding appropriate border plots results in a complete block design for f_1f_2 treatment combinations with f_1 blocks each of size f_1f_2 which is strongly neighbor balanced for factor F_1. It is observed that each of the f_1f_2 combinations of factor F_1 and F_2 has each level of factor F_1 as left and right neighbor once, that is, $\mu_1' = 1$, and the design is complete in the sense that all the f_1f_2 combinations appear in a block. The designs obtained are variance balanced for estimating the direct effects of contrasts in f_1f_2 treatment combinations as the corresponding information matrix ($\mathbf{C}_\tau$) is:

$$\mathbf{C}_\tau = f_1\mathbf{I} - \frac{1}{f_2}\mathbf{J}, \qquad (2)$$

where **I** is an identity matrix of order f_1f_2 and **J** is the matrix of all unities.

Example 1

Let $f_1 = 5$ be the number of level of first factor F_1 represented by 1, 2, 3, 4, 5. Further let $s = 3$ resulting in $f_2 = f_1 - s = 2$ level of second factor denoted by *a*, *b*. If the rows represent the blocks and 5×2 (= 10) treatment combinations in rows the block contents, then the following arrangement forms a circular complete block design for 10 treatment combinations in five blocks each of size 10 strongly neighbor balanced for five levels of F_1:

4	*5a 5b*	*1a 1b*	*2a 2b*	*3a 3b*	*4a 4b*	*5*
3	*2a 4b*	*3a 5b*	*4a 1b*	*5a 2b*	*1a 3b*	*2*
2	*4a 3b*	*5a 4b*	*1a 5b*	*2a 1b*	*3a 2b*	*4*
5	*3a 1b*	*4a 2b*	*5a 3b*	*1a 4b*	*2a 5b*	*3*
1	*1a 2b*	*2a 3b*	*3a 4b*	*4a 5b*	*5a 1b*	*1*

It may be observed from the above that all the 10 combinations of factor F_1 and F_1 are balanced for factor F_1 as each combination has each of the levels of factor F_1 as left and right neighbor exactly once.

Example 2

If $f_1 = 5$ and $s = 2$, then $f_2 = 3$ and the design for 15 treatment combinations is as follows:

4	*5a 5b 5c*	*1a 1b 1c*	*2a 2b 2c*	*3a 3b 3c*	*4a 4b 4c*	*5*
2	*2a 4b 3c*	*3a 5b 4c*	*4a 1b 5c*	*5a 2b 1c*	*1a 3b 2c*	*2*
5	*4a 3b 1c*	*5a 4b 2c*	*1a 5b 3c*	*2a 1b 4c*	*3a 2b 5c*	*4*
1	*3a 1b 2c*	*4a 2b 3c*	*5a 3b 4c*	*1a 4b 5c*	*2a 5b 1c*	*3*
3	*1a 2b 4c*	*2a 3b 5c*	*3a 4b 1c*	*4a 5b 2c*	*5a 1b 3c*	*1*

Table 1 presents a list of designs consisting of the variance of contrast between different treatments combinations (V) along with other parameters for number of level of first factor (F_1) $\leq$ 13.

Complete NBB Designs for Two Factors: Method 2

Let f_1 be an even number and $f_2 = 2$. Obtain a square array **L** of order f_1 by developing the following initial sequence mod f_1 (replacing *0* by f_1):

$$1 \quad f_1 \quad 2 \quad f_1\text{-}1 \ldots \frac{f_1}{2} \quad \frac{f_1}{2}+1$$

Table 1: Parameters and Variance of Strongly Complete NBB Designs for Two Factors

f_1	s	f_2	$k = f_1 f_2$	$b = f_1$	μ_1'	V
5	1	4	20	5	1	0.40
	2	3	15		1	0.40
	3	2	10		1	0.40
7	1	6	42	7	1	0.29
	2	5	35		1	0.29
	3	4	28		1	0.29
	4	3	21		1	0.29
	5	2	14		1	0.29
11	1	10	110	11	1	0.18
	2	9	99		1	0.18
	3	8	88		1	0.18
	4	7	77		1	0.18
	5	6	66		1	0.18
	6	5	55		1	0.18
	7	4	44		1	0.18
	8	3	33		1	0.18
	9	2	22		1	0.18
13	1	12	156	13	1	0.15
	2	11	143		1	0.15
	3	10	130		1	0.15
	4	9	117		1	0.15
	5	8	104		1	0.15
	6	7	91		1	0.15
	7	6	78		1	0.15
	8	5	65		1	0.15
	9	4	52		1	0.15
	10	3	39		1	0.15
	11	2	26		1	0.15

Juxtapose the mirror image $\mathbf{L}'$ of **L** to the right hand side of **L** to obtain an arrangement of f_1 rows and $2f_1$ columns, and allocate the first level of F_2 to all the units of **L** and second level to all the units of $\mathbf{L}'$. Considering the rows as blocks and making the blocks circular results in a complete NBB design with block size $2f_1$ which is strongly neighbor balanced for factor F_1. Each of the $2f_1$ combinations of factor F_1 and F_2 have each level of factor F_1 as left and right neighbor exactly once, that is, $\mu_1'=1$ and the design is also variance balanced.

In general, for any even number of levels of F_2 ($f_2 = 2n$), the squares may be juxtaposed in the following manner:

$$\mathbf{L} \quad \mathbf{L}' \quad \mathbf{L} \quad \mathbf{L}' \quad \ldots$$

Allocating the first level of F_2 to each unit in **L**, second level to units in $\mathbf{L}'$, third level to units in **L** again and so on, a complete NBB design in f_1 blocks of size $2f_1n$ balanced for factor F_1 is obtained. The designs obtained are also variance balanced for estimating the direct effects of contrasts in $2f_1n$ treatment combinations as the corresponding information matrix ($\mathbf{C}_\tau$) is of the following form:

$$\mathbf{C}_\tau = f_1\,\mathbf{I} - \frac{1}{2n}\mathbf{J}. \tag{3}$$

Example 3

Let $f_1 = 6$ and $f_2 = 2$. Figure 1 shows a circular complete NBB block design for 6×2 (= 12) combinations in six blocks of size 12 balanced for six levels of F_1. For $f_2 = 4$, the design obtained for 6×4 (= 24) combinations in six blocks of size 24 strongly balanced for six levels of F_1 is shown in Figure 2.

Figure 1: Circular Complete NBB Block Design for 6×2 (= 12) Combinations

	L						**L′**						
1	*1a*	*6a*	*2a*	*5a*	*3a*	*4a*	*4b*	*3b*	*5b*	*2b*	*6b*	*1b*	*1*
2	*2a*	*1a*	*3a*	*6a*	*4a*	*5a*	*5b*	*4b*	*6b*	*3b*	*1b*	*2b*	*2*
3	*3a*	*2a*	*4a*	*1a*	*5a*	*6a*	*6b*	*5b*	*1b*	*4b*	*2b*	*3b*	*3*
4	*4a*	*3a*	*5a*	*2a*	*6a*	*1a*	*1b*	*6b*	*2b*	*5b*	*3b*	*4b*	*4*
5	*5a*	*4a*	*6a*	*3a*	*1a*	*2a*	*2*	*1b*	*3b*	*6b*	*4b*	*5b*	*5*
6	*6a*	*5a*	*1a*	*4a*	*2a*	*3a*	*3*	*2b*	*4b*	*1b*	*5b*	*6b*	*6*

Figure 2: Block design for 6×4 (= 24) combinations in six blocks of size 24

1	*1a*	*6a*	*2a*	*5a*	*3a*	*4a*	*4b*	*3b*	*5b*	*2b*	*6b*	*1b*
2	*2a*	*1a*	*3a*	*6a*	*4a*	*5a*	*5b*	*4b*	*6b*	*3b*	*1b*	*2b*
3	*3a*	*2a*	*4a*	*1a*	*5a*	*6a*	*6b*	*5b*	*1b*	*4b*	*2b*	*3b*
4	*4a*	*3a*	*5a*	*2a*	*6a*	*1a*	*1b*	*6b*	*2b*	*5b*	*3b*	*4b*
5	*5a*	*4a*	*6a*	*3a*	*1a*	*2a*	*2b*	*1b*	*3b*	*6b*	*4b*	*5b*
6	*6a*	*5a*	*1a*	*4a*	*2a*	*3a*	*3b*	*2b*	*4b*	*1b*	*5b*	*6b*
	L						**L′**					

1c	*6c*	*2c*	*5c*	*3c*	*4c*	*4d*	*3d*	*5d*	*2d*	*6d*	*1d*	*1*
2c	*1c*	*3c*	*6c*	*4c*	*5c*	*5d*	*4d*	*6d*	*3d*	*1d*	*2d*	*2*
3c	*2c*	*4c*	*1c*	*5c*	*6c*	*6d*	*5d*	*1d*	*4d*	*2d*	*3d*	*3*
4c	*3c*	*5c*	*2c*	*6c*	*1c*	*1d*	*6d*	*2d*	*5d*	*3d*	*4d*	*4*
5c	*4c*	*6c*	*3c*	*1c*	*2c*	*2d*	*1d*	*3d*	*6d*	*4d*	*5d*	*5*
6c	*5c*	*1c*	*4c*	*2c*	*3c*	*3d*	*2d*	*4d*	*1d*	*5d*	*6d*	*6*
L						**L′**						

Incomplete NBB Designs for Two Factors

Let f_1 be a prime or prime power and be denoted by $1,2,\ldots$. Develop f_1 - 1 mutually orthogonal Latin squares (MOLS) of order f_1. Juxtapose these MOLS so that we obtain an arrangement of f_1 symbols in $f_1(f_1 - 1)$ rows and f_1 columns. Delete the last q columns (q =0, 1, 2, …, f_1 - 4) and consider rows as blocks along with border plots, to make the blocks circular. To all the units in l^{th} column ($l = 1, \ldots, f_1 - q$) of this arrangement attach the f_2 ($f_2 = a, b, \ldots$) levels of F_2, i.e. a to column 1, b to column 2 and so on. Considering the rows as blocks and making the blocks circular results in an incomplete NBB design in $f_1(f_1 - 1)$ blocks of size $f_1 - q$ each and $\mu_1'' = 1$ balanced for factor F_1. The design is incomplete because all the combinations are not appearing in a block. For q = 0, the design has all the levels of F_1 and F_2 appearing in all the blocks. The design obtained is combinatorially neighbor balanced but in the terms of variance, the design is partially balanced with three associate class association scheme.

Example 4

For $f_1 = f_2 = 5$ i.e. $q = 0$, NBB design in 25 combinations is as follows:

5	*1a*	*2b*	*3c*	*4d*	*5e*	*1*
1	*2a*	*3b*	*4c*	*5d*	*1e*	*2*
2	*3a*	*4b*	*5c*	*1d*	*2e*	*3*
3	*4a*	*5b*	*1c*	*2d*	*3e*	*4*
4	*5a*	*1b*	*2c*	*3d*	*4e*	*5*
4	*1a*	*3b*	*5c*	*2d*	*4e*	*1*
5	*2a*	*4b*	*1c*	*3d*	*5e*	*2*
1	*3a*	*5b*	*2c*	*4d*	*1e*	*3*
2	*4a*	*1b*	*3c*	*5d*	*2e*	*4*
3	*5a*	*2b*	*4c*	*1d*	*3e*	*5*
3	*1a*	*4b*	*2c*	*5d*	*3e*	*1*
4	*2a*	*5b*	*3c*	*1d*	*4e*	*2*
5	*3a*	*1b*	*4c*	*2d*	*5e*	*3*
1	*4a*	*2b*	*5c*	*3d*	*1e*	*4*
2	*5a*	*3b*	*1c*	*4d*	*2e*	*5*
2	*1a*	*5b*	*4c*	*3d*	*2e*	*1*
3	*2a*	*1b*	*5c*	*4d*	*3e*	*2*
4	*3a*	*2b*	*1c*	*5d*	*4e*	*3*
5	*4a*	*3b*	*2c*	*1d*	*5e*	*4*
1	*5a*	*4b*	*3c*	*2d*	*1e*	*5*

After randomization the design may have the following layout:

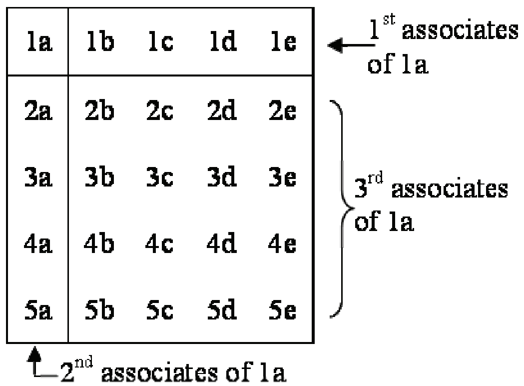
2	*3c*	*4d*	*5e*	*1a*	*2b*	*3*
2	*3b*	*4c*	*5d*	*1e*	*2a*	*3*
4	*5c*	*1d*	*2e*	*3a*	*4b*	*5*
1	*2d*	*3e*	*4a*	*5b*	*1c*	*2*
4	*5a*	*1b*	*2c*	*3d*	*4e*	*5*
3	*5c*	*2d*	*4e*	*1a*	*3b*	*5*
1	*3d*	*5e*	*2a*	*4b*	*1c*	*3*
3	*5b*	*2c*	*4d*	*1e*	*3a*	*5*
2	*4a*	*1b*	*3c*	*5d*	*2e*	*4*
2	*4c*	*1d*	*3e*	*5a*	*2b*	*4*
5	*3e*	*1a*	*4b*	*2c*	*5d*	*3*
2	*5b*	*3c*	*1d*	*4e*	*2a*	*5*
1	*4c*	*2d*	*5e*	*3a*	*1b*	*4*
5	*3d*	*1e*	*4a*	*2b*	*5c*	*3*
5	*3b*	*1c*	*4d*	*2e*	*5a*	*3*
2	*1a*	*5b*	*4c*	*3d*	*2e*	*1*
1	*5c*	*4d*	*3e*	*2a*	*1b*	*5*
1	*5d*	*4e*	*3a*	*2b*	*1c*	*5*
1	*5e*	*4a*	*3b*	*2c*	*1d*	*5*
1	*5a*	*4b*	*3c*	*2d*	*1e*	*5*

Association Scheme

Two treatment combinations $\phi\varphi$ and $\phi'\varphi'$ are said to be first associates if $\phi = \phi'$ i.e. the combinations with same F_1 level and different F_2 level are first associates. Two treatment combinations $\phi\varphi$ and $\phi'\varphi'$ are said to be second associate if $\varphi = \varphi'$ i.e. the combinations with same F_2 level and different F_1 level are second associates, and remaining are third associates.

For the Example 4, the arrangement of 25 treatment combinations arising from 5 levels of the first factor and 5 levels of second factor are shown in Figure 3. For the given association scheme $v = f_1 f_2$, number of first associates = $f_2 - 1$, number of second associates = $f_1 - 1$ and number of third associates = $f_1 f_2 - f_1 - f_2 + 1$. The two treatment combinations that are first and second associates do not appear together in the design whereas the third associates appear once in the design. The above association scheme may be also called a rectangular association scheme.

Figure 3: 25 Treatment Combinations Arising From 5 Levels of the First Factor and 5 Levels of the Second Factor



The information matrix for estimating twenty five combinations of the above design obtained using SAS (PROC IML) is shown in (4). The matrix has three distinct off -diagonal elements due to the three class association scheme. The design obtained by Monod (1992) becomes a special case of this for $q = 0$.

Example 5

For $f_1 = 5$, $q = 1$ and $f_2 = 4$, that is, a NBB design in $f_1 f_2 = 20$ combinations is as follows:

4	*1a*	*2b*	*3c*	*4d*	*1*
5	*2a*	*3b*	*4c*	*5d*	*2*
1	*3a*	*4b*	*5c*	*1d*	*3*
2	*4a*	*5b*	*1c*	*2d*	*4*
3	*5a*	*1b*	*2c*	*3d*	*5*
2	*1a*	*3b*	*5c*	*2d*	*1*
3	*2a*	*4b*	*1c*	*3d*	*2*
4	*3a*	*5b*	*2c*	*4d*	*3*
5	*4a*	*1b*	*3c*	*5d*	*4*
1	*5a*	*2b*	*4c*	*1d*	*5*
5	*1a*	*4b*	*2c*	*5d*	*1*
1	*2a*	*5b*	*3c*	*1d*	*2*
2	*3a*	*1b*	*4c*	*2d*	*3*
3	*4a*	*2b*	*5c*	*3d*	*4*
4	*5a*	*3b*	*1c*	*4d*	*5*
3	*1a*	*5b*	*4c*	*3d*	*1*
4	*2a*	*1b*	*5c*	*4d*	*2*
5	*3a*	*2b*	*1c*	*5d*	*3*
1	*4a*	*3b*	*2c*	*1d*	*4*
2	*5a*	*4b*	*3c*	*2d*	*5*

Variances of all estimated elementary contrasts pertaining to direct effects of various treatment combinations that are mutually first associate (V_1), second associates (V_2) and third associate (V_3) were computed using a SAS program developed in IML. A list of designs consisting of these variances along with other parameters is shown in Table 2 for a practical range of parameter values, that is, for the number of level of first factor (F_1) and second factor (F_2) ≤ 13.

$$\mathbf{C} = \begin{bmatrix} 3.20\mathbf{I}_5 - 0.11\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 \\ 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 3.20\mathbf{I}_5 - 0.11\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 \\ 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 3.20\mathbf{I}_5 - 0.11\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 \\ 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 3.20\mathbf{I}_5 - 0.11\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 \\ 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 0.20\mathbf{I}_5 - 0.17\mathbf{J}_5 & 3.20\mathbf{I}_5 - 0.11\mathbf{J}_5 \end{bmatrix} \quad (4)$$

Table 2: Parameters and Variances of Incomplete NBB Designs for Two Factors

f_1	f_2	$k = f_2$	b	μ_1''	V_1	V_2	V_3	$\overline{V}$
5	5	5	20	1	6.40	6.13	6.53	6.44
5	4	4	20	1	6.00	5.37	5.87	5.79
7	7	7	42	1	10.29	10.17	10.46	10.40
7	6	6	42	1	10.00	9.81	10.14	10.10
7	5	5	42	1	9.60	9.23	9.63	9.56
7	4	4	42	1	9.00	8.13	8.62	8.55
8	8	8	56	1	12.25	12.17	12.42	12.40
8	7	7	56	1	12.00	11.87	12.16	12.10
8	6	6	56	1	11.67	11.44	11.78	11.70
8	5	5	56	1	11.20	10.77	11.17	11.10
8	4	4	56	1	10.50	9.50	10.00	9.94
9	9	9	72	1	15.97	15.95	15.94	15.90
9	8	8	72	1	14.00	13.91	14.16	14.10
9	7	7	72	1	13.71	13.57	13.85	13.80
9	6	6	72	1	13.33	13.08	13.42	13.40
9	5	5	72	1	12.80	12.32	12.72	12.70
9	4	4	72	1	12.00	10.87	11.37	11.30
11	11	11	110	1	18.18	17.98	18.17	18.20
11	10	10	110	1	17.89	17.47	17.99	17.90
11	9	9	110	1	17.72	17.58	17.75	17.70
11	8	8	110	1	17.44	17.19	17.45	17.40
11	7	7	110	1	17.07	16.72	17.04	17.00
11	6	6	110	1	16.51	16.07	16.49	16.40
11	5	5	110	1	15.84	15.06	15.57	15.50
11	4	4	110	1	14.23	13.33	13.73	13.70
13	13	13	156	1	22.15	22.13	22.17	22.20
13	12	12	156	1	22.00	22.00	22.17	22.10
13	11	11	156	1	21.82	21.77	21.95	21.90
13	10	10	156	1	21.60	21.53	21.73	21.70
13	9	9	156	1	21.33	21.24	21.46	21.40
13	8	8	156	1	21.00	20.86	21.12	21.10
13	7	7	156	1	20.57	20.35	20.64	20.60
13	6	6	156	1	20.00	19.64	19.97	19.90
13	5	5	156	1	19.20	18.51	18.91	18.90
13	4	4	156	1	18.00	16.37	16.87	16.80

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