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The Normal and Laplace's methods of approximation for posterior density based on the location-scale family of distributions in terms of the numerical and graphical simulation are examined using S-PLUS and R Software.

Key words: Bayesian analysis, location-scale family, logistic distribution, Newton-Raphson iteration, normal and Laplace's approximation, S-PLUS, R software.

Introduction

A parametric location-scale model for a random variable *y* on (-∞ ,∞) is distributed with pdf of the form

$$p(y;\mu,\sigma)=\frac{1}{\sigma}f\left(\frac{y-\mu}{\sigma}\right),\quad -\infty<y<\infty \tag{1.1}$$

where μ $(-\infty<\mu<\infty)$ is a location parameter and $\sigma>0$ is a scale parameter (not necessarily mean and standard deviation). This family can also be written as

$$y=\mu+\sigma z \tag{1.2}$$

where $z=\dfrac{y-\mu}{\sigma}$ is the standardized variate with density *f(z)*, David (1981). A few important models, namely, normal, logistic and extreme value are some important members of the location-scale family.

Bogdanoff and Pierce (1973) analyzed an extreme value model treating non informative priors for location and scale parameters. Stavrakakis and Drakopoulos (1995) and Galanis, et al. (2002) deal with an extreme value model with Bayesian statistics. Sinha (1986) and Khan (1997) also cite several references for non-normal *f(z)*.

Bayesian Analysis when Both Parameters μ and σ Are Unknown

Suppose that n observations $y^T=(y_1,y_2,\ldots,y_n)$ can be regarded as a random sample from a location-scale family of models in (1.2), but both μ *and* σ are unknown; in terms of general notation $\theta^T=(\mu,\sigma)$, the likelihood function is given by

$$p(y\mid\mu,\sigma)=\prod_{i=1}^{n}p(y_i\mid\mu,\sigma)$$

The log-likelihood is defined as

$$\begin{aligned}l(\mu,\sigma)&=\log\prod_{i=1}^{n}p(y_i\mid\mu,\sigma)\\&=\log\prod_{i=1}^{n}\sigma^{-1}f\left(\frac{y-\mu}{\sigma}\right)\\&=\sum_{i=1}^{n}\log f(z_i)-n\log\sigma\end{aligned}$$

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or equivalently

$$l(\mu,\sigma)=\sum_{i=1}^{n} l_i \qquad (2.1)$$

where $l_i = \log f(z_i) - \log\sigma$ and $z_i = \dfrac{y_i-\mu}{\sigma}$.

Following the standard approach of Box and Tiao (1973), assume that a priori μ and σ are approximately independent, so that

$$p(\mu,\sigma) \cong p(\mu)p(\sigma) \qquad (2.2)$$

where $p(\mu)$ and $p(\sigma)$ are priors for μ and σ, respectively. Using Bayes theorem, the posterior density of $p(\mu,\sigma \mid y)$ is given by

$$p(\mu,\sigma \mid y) \quad \alpha \quad \prod_{i=1}^{n} p(y_i \mid \mu,\sigma)\, p(\mu)\, p(\sigma)$$

or

$$p(\mu,\sigma \mid y) \quad \alpha \quad \left[\prod_{i=1}^{n} \sigma^{-1} f(z_i)\right] p(\mu)\, p(\sigma) \qquad (2.3)$$

The joint posterior density of μ and σ is assumed to contain all information required in the statistical analysis (e.g., Box & Tiao, 1973), therefore, the main job remains to study the different features of $p(\mu,\sigma \mid y)$. The posterior mode can be obtained by maximizing (2.3) with respect to μ and σ. To formalize this, define

$$l^*(\mu,\sigma) = \log p(\mu,\sigma \mid y)$$

thus,

$$l^*(\mu,\sigma) = l(\mu,\sigma) + \log p(\mu) + \log p(\sigma). \qquad (2.4)$$

The maximization of $p(\mu,\sigma \mid y)$ is equivalent to maximizing $l^*(\mu,\sigma)$ with respect to (μ,σ). To apply the Newton-Raphson technique, partial derivatives of $l^*(\mu,\sigma)$ are needed and some notations must be defined for simplification purposes. For example

$$l_\mu = \frac{\partial l}{\partial \mu},\quad l_\sigma = \frac{\partial l}{\partial \sigma},\quad l_{\mu\sigma} = \frac{\partial^2 l}{\partial\mu\partial\sigma},\quad l_{\sigma\mu} = \frac{\partial^2 l}{\partial\sigma\partial\mu},$$

$$l_{\mu\mu} = \frac{\partial^2 l}{\partial \mu^2},\ and \quad l_{\sigma\sigma} = \frac{\partial^2 l}{\partial \sigma^2}.$$

Similarly, define $l_\mu^* = l_\mu + \dfrac{p'(\mu)}{p(\mu)}$, $l_\sigma^* = l_\sigma + \dfrac{p'(\sigma)}{p(\sigma)}$, $l_{\mu\sigma}^* = l_{\mu\sigma}$, $l_{\sigma\mu}^* = l_{\sigma\mu}$,

$$l_{\mu\mu}^* = l_{\mu\mu} + \left[\frac{p'(\mu)}{p(\mu)}\right]',$$

and

$$l_{\sigma\sigma}^* = l_{\sigma\sigma} + \left[\frac{p'(\sigma)}{p(\sigma)}\right]'$$

where $f'(x) = Df(x)$ and $[f'(x)]' = D^2 f(x)$, D stands for differential operator. Consequently, the score vector of log-posterior

$$U(\mu,\sigma) = (l_\mu^*, l_\sigma^*)^T$$

and Hessian matrix of log-posterior, that is,

$$H(\mu,\sigma) = \begin{bmatrix} l_{\mu\mu}^* & l_{\mu\sigma}^* \\ l_{\sigma\mu}^* & l_{\sigma\sigma}^* \end{bmatrix}$$

thus, the posterior mode $(\hat{\mu},\hat{\sigma})$ can be obtained from iteration scheme

$$\begin{bmatrix} \hat{\mu} \\ \hat{\sigma} \end{bmatrix} = \begin{bmatrix} \mu_0 \\ \sigma_0 \end{bmatrix} - H^{-1}(\mu_0,\sigma_0)\begin{bmatrix} l_\mu^* \\ l_\sigma^* \end{bmatrix} \qquad (2.5)$$

Consequently, the modal variance Σ can be obtained as

$$I^{-1}(\hat{\mu},\hat{\sigma}) = -H^{-1}(\hat{\mu},\hat{\sigma}).$$

For drawing an inference about μ and σ simultaneously, the joint posterior $p(\mu,\sigma \mid y)$ is used. It is preferable to use approximations to this posterior as given below:

Normal Approximations

A bivariate normal approximation of $p(\mu,\sigma \mid y)$, is

$$p(\mu,\sigma \mid y) \cong N_2\left((\hat{\mu},\hat{\sigma})^T, I^{-1}(\hat{\mu},\hat{\sigma}) \right) \tag{2.6}$$

Similarly, the Bayesian analog of likelihood ratio criterion is

$$-2[l^*(\mu,\sigma) - l^*(\hat{\mu},\hat{\sigma})] \approx \chi_2^2 \tag{2.7}$$

where the symbol $\approx$ means approximately distributed as. Defining $W(\mu,\sigma) = -2[l^*(\mu,\sigma) - l^*(\hat{\mu},\hat{\sigma})]$ using $W(\mu,\sigma)$ as a test criterion in hypothesis testing and construction of the credible region (confidence interval in non-Bayesian terminology).

Laplace's Approximation

Laplace's approximation of $p(\mu,\sigma \mid y)$ can also be written as

$$p(\mu,\sigma \mid y) \cong (2\pi)^{-1} \mid I(\hat{\mu},\hat{\sigma}) \mid^{\frac{1}{2}} exp[l^*(\mu,\sigma) - l^*(\hat{\mu},\hat{\sigma})] \tag{2.8}$$

The Marginal Inference

The marginal Bayesian inference about μ and σ is based on marginal posterior densities of these parameters. The marginal posterior for μ can be obtained after integrating out $p(\mu,\sigma \mid y)$ with respect to σ, that is,

$$p(\mu \mid y) = \int_0^{\infty} p(\mu,\sigma \mid y)\, d\sigma$$

Similarly, marginal posterior of σ can be obtained as

$$p(\sigma \mid y) = \int_{-\infty}^{\infty} p(\mu,\sigma \mid y)\, d\mu.$$

For normal likelihood $p(\mu,\sigma \mid y)$ and non-informative prior $p(\mu,\sigma)\ \alpha\ \frac{1}{\sigma}$, it can be shown that $p(\sigma \mid y)$ is the inverted χ-distribution (Box & Tiao, 1973; Zellener, 1971). But if either assumption of normality is extended to other members of location scale family or the prior is changed then closed form expressions cannot be obtained and approximations must be relied upon (Khan, 1997). In practice, the Gauss-Hermite quadrature (Naylor & Smith, 1982) can be used to find accurate approximations of $p(\mu \mid y)$ and $p(\sigma \mid y)$, however, following simple approximations is recommended.

Normal Approximation

The normal approximation of marginal posterior $p(\mu \mid y)$ is:

$$p(\mu \mid y) = N_1(\hat{\mu}, I_{11}^{-1}) \tag{3.1}$$

In addition, the Bayesian analog of likelihood ratio criterion can also be defined as a test criterion based on (3.1) as

$$(\mu - \hat{\mu})^T I_{11} (\mu - \hat{\mu}) \approx \chi_1^2 \tag{3.2}$$

Laplace's Approximation

The marginal posterior density $p(\mu \mid y)$ can alternatively be approximated by

$$p(\mu \mid y) \cong \left[\frac{\mid I(\hat{\mu},\hat{\sigma}) \mid}{2\pi \mid I(\mu,\hat{\sigma}(\mu)) \mid} \right]^{\frac{1}{2}} exp[l^*(\mu,\hat{\sigma}(\mu)) - l^*(\hat{\mu},\hat{\sigma})] \tag{3.3}$$

Similarly, $p(\sigma \mid y)$ can be approximated and results corresponding to normal and Laplace's approximation can be written as

$$p(\sigma \mid y) = N_1(\hat{\sigma}, I_{22}^{-1}) \tag{3.4}$$

or equivalently,

$$(\sigma-\hat{\sigma})^T I_{22}(\sigma-\hat{\sigma}) \approx \chi_1^2 \qquad (3.5)$$

$$p(\sigma\,|\,y) \cong \left[\frac{|I(\hat{\mu},\hat{\sigma})|}{2\pi\,|I(\hat{\mu}(\sigma),\sigma)|}\right]^{\frac{1}{2}} exp[l^*(\hat{\mu}(\sigma),\sigma)-l^*(\hat{\mu},\hat{\sigma})] \qquad (3.6)$$

Bayesian Analysis of Logistic Distribution

The pdf of the logistic distribution is given by

$$p(y;\mu,\sigma)=\frac{e^{\frac{(y-\mu)}{\sigma}}}{\sigma\left(1+e^{\frac{(y-\mu)}{\sigma}}\right)^2},$$

$$-\infty<y<\infty,$$
$$\sigma>0$$

The likelihood function is given by

$$p(y\,|\,\mu,\sigma)=\prod_{i=1}^{n} p(y_i\,|\,\mu,\sigma)$$

And the log-likelihood is defined as

$$l(\mu,\sigma)=log\prod_{i=1}^{n} p(y_i\,|\,\mu,\sigma)$$
$$=\sum_{i=n}^{n}(z_i-2\,log(1+e^{z_i}))-n\,log\,\sigma \qquad (4.1)$$

where $z_i=\frac{y_i-\mu}{\sigma}$.

Taking partial derivatives with respect to μ and σ

$$l_\mu=\frac{\partial l}{\partial\mu}=\frac{1}{\sigma}\sum_{i=1}^{n}\left(\frac{e^{z_i}-1}{e^{z_i}+1}\right)$$

$$l_\sigma=\frac{\partial l}{\partial\sigma}=\frac{1}{\sigma}\sum_{i=1}^{n}z_i\left(\frac{e^{z_i}-1}{e^{z_i}+1}\right)-\frac{n}{\sigma}$$

$$l_{\mu\sigma}=\frac{\partial^2 l}{\partial\mu\partial\sigma}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(\frac{e^{2z_i}+2z_ie^{z_i}-1}{(e^{z_i}+1)^2}\right)$$

$$l_{\sigma\mu}=\frac{\partial^2 l}{\partial\sigma\partial\mu}=-\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(\frac{e^{2z_i}+2z_ie^{z_i}-1}{(e^{z_i}+1)^2}\right)$$

$$l_{\mu\mu}=\frac{\partial^2 l}{\partial\mu^2}=-\frac{2}{\sigma^2}\sum_{i=1}^{n}\left(\frac{e^{z_i}}{(e^{z_i}+1)^2}\right)$$

$$l_{\sigma\sigma}=\frac{\partial^2 l}{\partial\sigma^2}=-\frac{2}{\sigma^2}\sum_{i=1}^{n}z_i\left(\frac{e^{2z_i}+z_ie^{z_i}-1}{(e^{z_i}+1)^2}\right)+\frac{n}{\sigma^2}$$

Following the standard approach of Box and Tiao (1973), Gelman, et al. (1995), it is assumed that the prior μ and σ are approximately independent so that

$$p(\mu,\sigma)\cong p(\mu)p(\sigma) \qquad (4.2)$$

where $p(\mu)p(\sigma)$ and $p(\sigma)$ are priors for μ and σ. Using Bayes theorem, the posterior density $p(\mu,\sigma\,|\,y)$ is

$$p(\mu,\sigma\,|\,y)\quad \alpha \quad \prod_{i=1}^{n}p(y_i\,|\,\mu,\sigma)\,p(\mu)\,p(\sigma) \qquad (4.3)$$

and the log-posterior is given by

$$\log p(\mu,\sigma \mid y) = \log \prod_{i=1}^{n} p(y_i \mid \mu,\sigma) + \log p(\mu) + \log p(\sigma) \tag{4.4a}$$

or

$$l^*(\mu,\sigma) = l(\mu,\sigma) + \log p(\mu) + \log p(\sigma) \tag{4.4b}$$

For a prior $p(\mu,\sigma) \cong p(\mu)p(\sigma) = 1$, $l_\mu^* = l_\mu$, $l_\sigma^* = l_\sigma$, $l_{\mu\sigma}^* = l_{\mu\sigma}, l_{\sigma\mu}^* = l_{\sigma\mu}, l_{\mu\mu}^* = l_{\mu\mu}$ and $l_{\sigma\sigma}^* = l_{\sigma\sigma}$. The posterior mode is obtained by maximizing (4.4) with respect to μ and σ. The score vector of the log posterior is given by

$$U(\mu,\sigma) = (l_\mu^*, l_\sigma^*)^T$$

and the Hessian matrix of the log posterior is

$$H(\mu,\sigma) = \begin{bmatrix} l_{\mu\mu}^* & l_{\mu\sigma}^* \\ l_{\sigma\mu}^* & l_{\sigma\sigma}^* \end{bmatrix}$$

Posterior mode $(\hat{\mu},\hat{\sigma})$ can be obtained from iteration scheme

$$\begin{bmatrix} \hat{\mu} \\ \hat{\sigma} \end{bmatrix} = \begin{bmatrix} \mu_0 \\ \sigma_0 \end{bmatrix} - H^{-1}(\mu_0,\sigma_0) \begin{bmatrix} l_\mu^* \\ l_\sigma^* \end{bmatrix}$$

consequently, the modal variance Σ can be obtained as

$$\hat{I}^{-1}(\hat{\mu},\hat{\sigma}) = -H^{-1}(\hat{\mu},\hat{\sigma}).$$

For drawing inferences about μ and σ simultaneously, the joint posterior (7.3) is used.

Using normal approximation, a bivariate normal approximation of (7.3) can be written as

$$p(\mu,\sigma \mid y) \cong N_2\left((\hat{\mu},\hat{\sigma})^T, I^{-1}(\hat{\mu},\hat{\sigma}) \right)$$

Similarly, a Bayesian analog of likelihood ratio criterion is

$$-2\left[l^*(\mu,\sigma) - l^*(\hat{\mu},\hat{\sigma}) \right] \approx \chi_2^2$$

Using Laplace's approximation, $p(\mu,\sigma \mid y)$ can be written as

$$p(\mu,\sigma \mid y) \cong (2\pi)^{-1} \mid I(\hat{\mu},\hat{\sigma}) \mid^{\frac{1}{2}} \exp[l^*(\mu,\sigma) - l^*(\hat{\mu},\hat{\sigma})]$$

The marginal Bayesian inferences about μ and σ are based on the marginal posterior densities of these parameters, and the marginal posterior for μ can be obtained after integrating out $p(\mu,\sigma \mid y)$ with respect to σ, that is

$$p(\mu \mid y) = \int_0^\infty p(\mu,\sigma \mid y)\, d\sigma$$

Similarly, the marginal posterior of σ can be obtained as

$$p(\sigma \mid y) = \int_{-\infty}^{\infty} p(\mu,\sigma \mid y)\, d\mu,$$

thus, normal approximation of the marginal posterior $p(\mu \mid y)$ is

$$p(\mu \mid y) = N_1(\hat{\mu}, I_{11}^{-1}).$$

The Bayesian analog of likelihood ratio criterion can also be defined as a test criterion as

$$(\mu - \hat{\mu})^T I_{11} (\mu - \hat{\mu}) \approx \chi_1^2$$

and Laplace's approximation of marginal posterior density $p(\mu \mid y)$ can be given by

$$p(\mu \mid y) \cong \left[\frac{|I(\hat{\mu},\hat{\sigma})|}{2\pi |I(\mu,\hat{\sigma}(\mu))|}\right]^{\frac{1}{2}} exp[l^*(\mu,\hat{\sigma}(\mu)) - l^*(\hat{\mu},\hat{\sigma})]$$

Similarly, $p(\sigma \mid y)$ can be approximated with results corresponding to normal and Laplace's approximation can be written as

$$p(\sigma \mid y) = N_1(\hat{\sigma}, I_{22}^{-1})$$

or equivalently,

$$(\sigma - \hat{\sigma})^T I_{22}(\sigma - \hat{\sigma}) \approx \chi_1^2$$

$$p(\sigma \mid y) \cong \left[\frac{|I(\hat{\mu},\hat{\sigma})|}{2\pi |I(\hat{\mu}(\sigma),\sigma)|}\right]^{\frac{1}{2}} exp[l^*(\hat{\mu}(\sigma),\sigma) - l^*(\hat{\mu},\hat{\sigma})]$$

Numerical and Graphical Illustrations

Numerical and graphical illustrations are implemented using both S-PLUS and R software for Logistic distribution. These illustrations are intended for the purpose of showing the strength of Bayesian methods in practical situations. The posterior mode and standard errors of parameters μ and σ of logistic distribution are presented in Table 4. A graphical display for comparing the posterior of μ using the Normal and Laplace approximations are shown in Figures 1 to 3 and a comparison for the posterior of σ is displayed in Figures 4 to 6. The graph shows that the two approximations are in close agreement.

Table 1: A Summary of Derivatives of Log Likelihoods

	Distributions		
Derivatives	Normal	Extreme-Value	Logistic
l_μ	$\frac{1}{\sigma}\sum_{i=1}^{n} z_i$	$-\frac{1}{\sigma}\sum_{i=1}^{n}(1-e^{z_i})$	$\frac{1}{\sigma}\sum_{i=1}^{n}\left(\frac{e^{z_i}-1}{e^{z_i}+1}\right)$
l_σ	$\frac{1}{\sigma}\sum_{i=1}^{n} z_i^{\,2} - \frac{n}{\sigma}$	$-\frac{1}{\sigma}\sum_{i=1}^{n} z_i(1-e^{z_i}) - \frac{n}{\sigma}$	$\frac{1}{\sigma}\sum_{i=1}^{n} z_i\left(\frac{e^{z_i}-1}{e^{z_i}+1}\right) - \frac{n}{\sigma}$
$l_{\mu\sigma}$	$-\frac{2}{\sigma^2}\sum_{i=1}^{n} z_i$	$-\frac{1}{\sigma^2}\sum_{i=1}^{n}(z_i e^{z_i} + e^{z_i} - 1)$	$-\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(\frac{e^{2z_i} + 2z_i e^{z_i} - 1}{(e^{z_i}+1)^2}\right)$
$l_{\sigma\mu}$	$-\frac{2}{\sigma^2}\sum_{i=1}^{n} z_i$	$-\frac{1}{\sigma^2}\sum_{i=1}^{n}(z_i e^{z_i} + e^{z_i} - 1)$	$-\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(\frac{e^{2z_i} + 2z_i e^{z_i} - 1}{(e^{z_i}+1)^2}\right)$
$l_{\mu\mu}$	$-\frac{n}{\sigma^2}$	$-\frac{1}{\sigma^2}\sum_{i=1}^{n} e^{z_i}$	$-\frac{2}{\sigma^2}\sum_{i=1}^{n}\left(\frac{e^{z_i}}{(e^{z_i}+1)^2}\right)$
$l_{\sigma\sigma}$	$-\frac{3}{\sigma^2}\sum_{i=1}^{n} z_i^{\,2} + \frac{n}{\sigma^2}$	$-\frac{1}{\sigma^2}\sum_{i=1}^{n}(z_i^{\,2} e^{z_i} + 2z_i e^{z_i} - 2z_i) + \frac{n}{\sigma^2}$	$-\frac{2}{\sigma^2}\sum_{i=1}^{n} z_i\left(\frac{e^{2z_i} + z_i e^{z_i} - 1}{(e^{z_i}+1)^2}\right) + \frac{n}{\sigma^2}$

where $z_i = \frac{y_i - \mu}{\sigma}, \quad i = 1,2,...,n$.

Table 2: A Summary of Prior Densities for Location Parameter μ

Name of Density	$p(\mu)$	$\frac{p'(\mu)}{p(\mu)}$	$\left[\frac{p'(\mu)}{p(\mu)}\right]'$
Non-Informative	Constant	Zero	Zero
Normal	$c\sigma_0^{-1}\exp\left(-\frac{D^2}{2}\right)$	$-D\sigma^{-1}$	$-\sigma^{-2}$
Logistic	$c\exp(D)[1+\exp(D)]^{-2}$	$\sigma_0^{-1}[1-2F(D)]$	$\frac{2}{\sigma_0^2}F(D)[1-F(D)]$
Extreme-Value	$c\exp[D-\exp(D)]$	$\sigma_0^{-1}[1-F(D)]$	$-\sigma_0^{-2}\exp(D)$

where $D=\frac{\mu-\mu_0}{\sigma_0}$, $F(D)=\frac{e^D}{1+e^D}$, and c is the normalizing constant.

Table 3: A Summary of Prior Densities for σ

Name of Prior	$p(\sigma)$	$\frac{p'(\sigma)}{p(\sigma)}$	$\left[\frac{p'(\sigma)}{p(\sigma)}\right]'$
Non-Informative	$\frac{1}{\sigma}$	$-\frac{1}{\sigma}$	$\frac{1}{\sigma^2}$
Inverted Gamma	$c\sigma^{-(\alpha_0+1)}\exp\left(-\frac{1}{\sigma\beta_0}\right)$	$\frac{1}{\beta_0\sigma^2}-\frac{\alpha_0+1}{\sigma}$	$\frac{\alpha_0+1}{\sigma^2}-\frac{2}{\beta_0\sigma^3}$
Lognormal	$c\sigma_0^{-1}\exp\left(-\frac{D^2}{2}\right)$	$-\frac{D}{\sigma_0\sigma}-\frac{1}{\sigma}$	$\frac{1}{\sigma^2}-\frac{1}{(\sigma_0\sigma)^2}-\frac{D}{\sigma_0\sigma^4}$
Gamma	$c\sigma^{(\alpha_0-1)}e^{-\sigma\beta_0^{-1}}$	$\frac{1}{\beta_0}+\frac{\alpha_0-1}{\sigma}$	$-\frac{\alpha_0-1}{\sigma^2}$
Weibull	$c\sigma^{(\beta_0-1)}e^{-(\sigma_0\sigma)^{\beta_0}}$	$\frac{\beta_0-1}{\sigma}-\alpha_0\beta_0(\alpha_0\sigma)^{\beta_0-1}$	$-\frac{\beta_0-1}{\sigma^2}-\alpha_0^{\beta_0}\beta_0(\beta_0-1)\sigma^{(\beta_0-2)}$

where c is the normalizing constant and $D=\frac{\log\sigma-\mu_0}{\sigma_0}$.

Table 4: Posterior Mode and Posterior Standard Error of Parameters of Logistic Distribution with Different Priors

Prior	Posterior Mode μ	Posterior Standard Error μ	Posterior Mode σ	Posterior Standard Error σ
1	168.63355	2.679672	58.65997	1.320980
1/sigma	168.62814	2.678635	58.63024	1.319912
1/(mu*sigma)	168.58558	2.678692	58.62837	1.319845
1/(mu*sigma)^2	168.53766	2.677714	58.59681	1.318714

Figures 1-3: Comparing Normal and Laplace's Approximation for μ of Logistic Distribution for Various Priors in S-PLUS and R

Figure 1: Comparison between Normal and Laplace Approximations

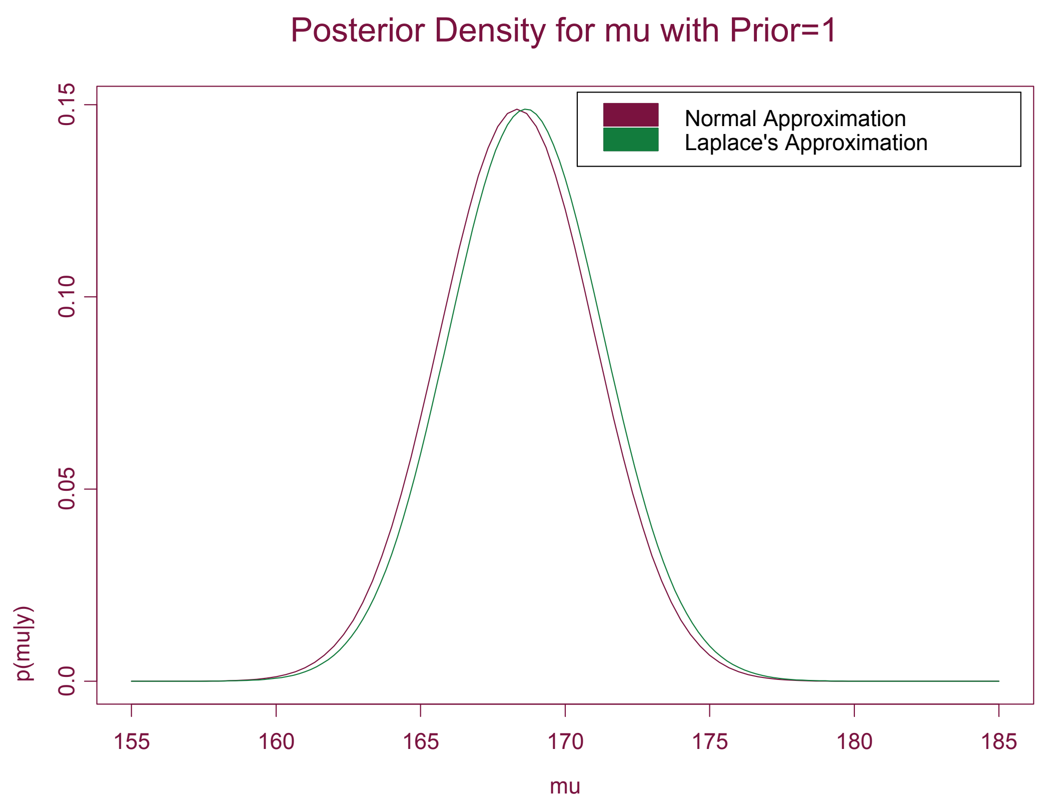


Figure 2: Comparison between Normal and Laplace Approximations

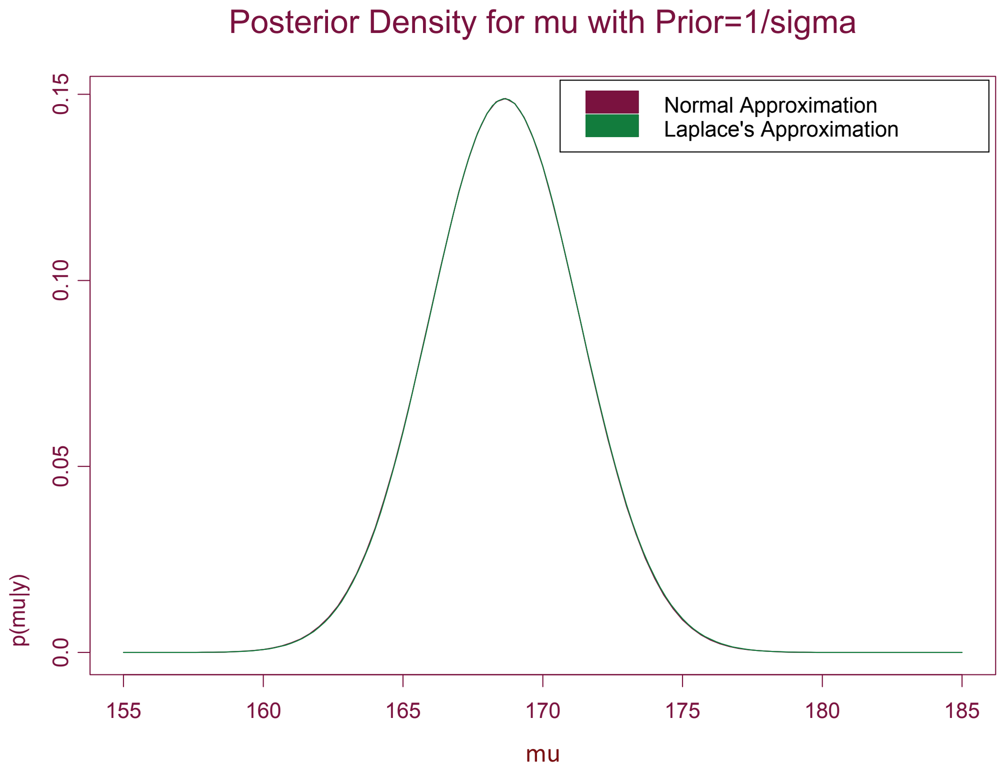
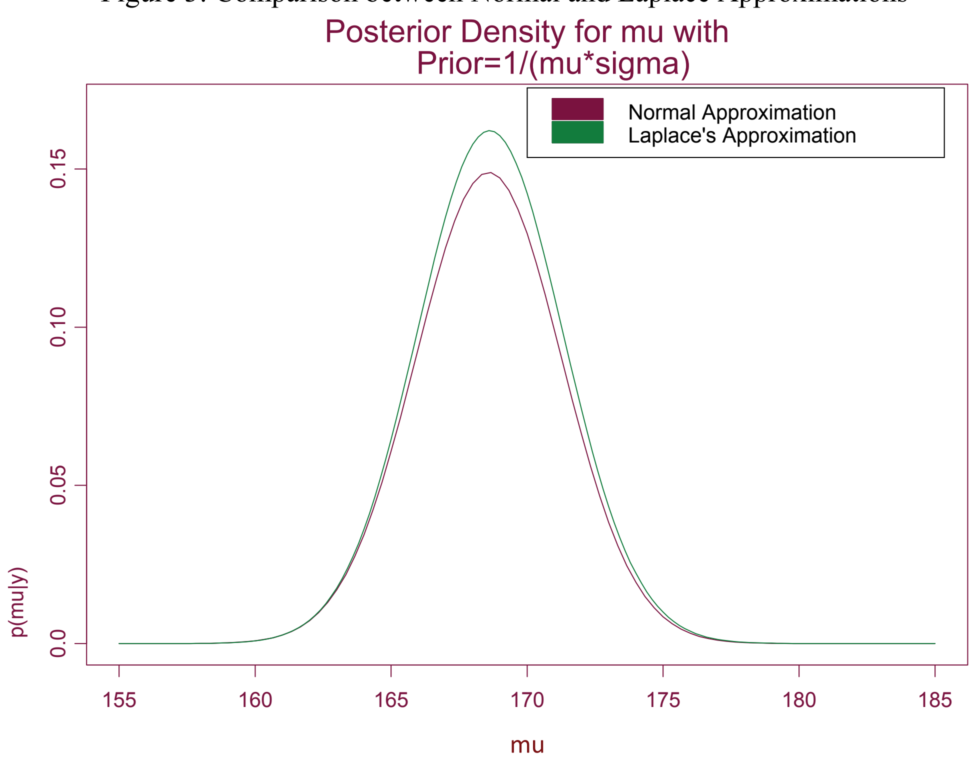


Figure 3: Comparison between Normal and Laplace Approximations



Figures 4-6: Comparing Normal and Laplace's Approximation for σ of Logistic Distribution for Various Priors in S-PLUS and R

Figure 4: Comparison between Normal and Laplace Approximations

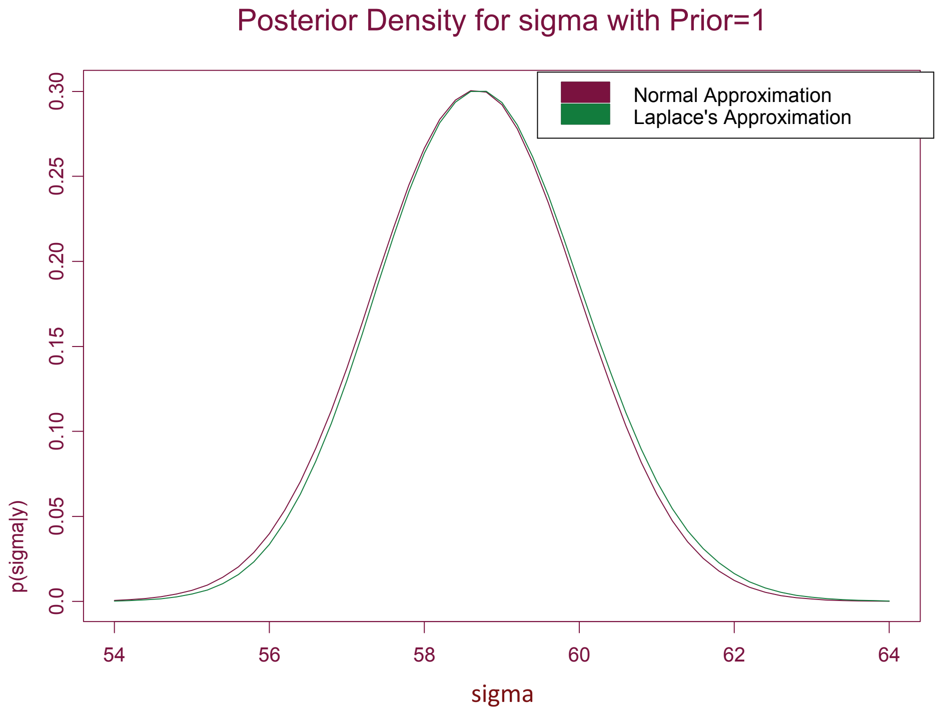


Figure 5: Comparison between Normal and Laplace Approximation

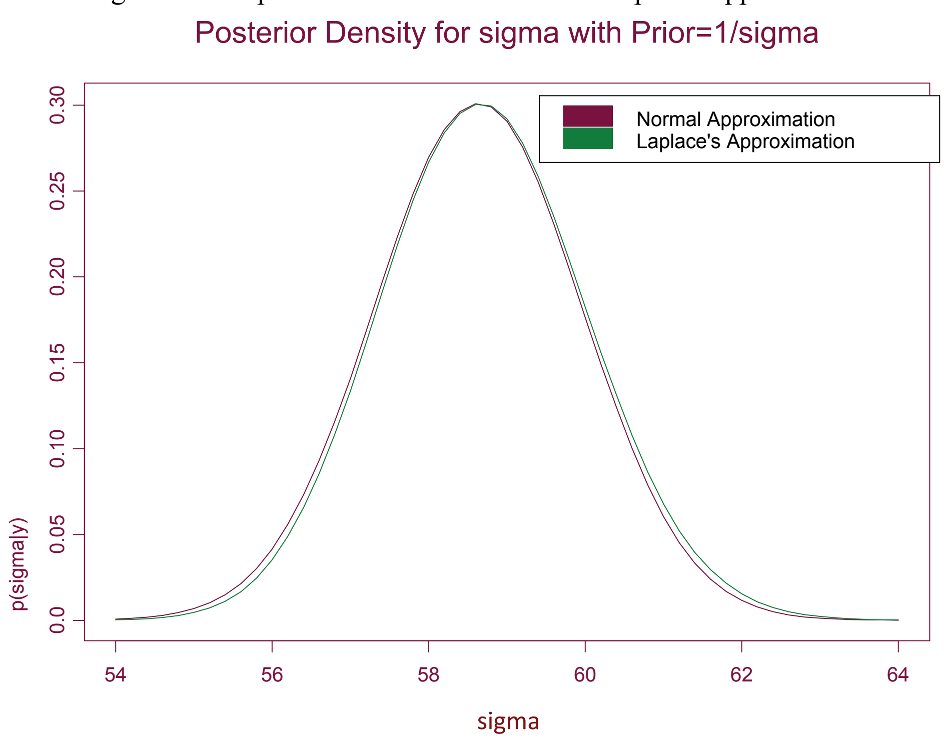
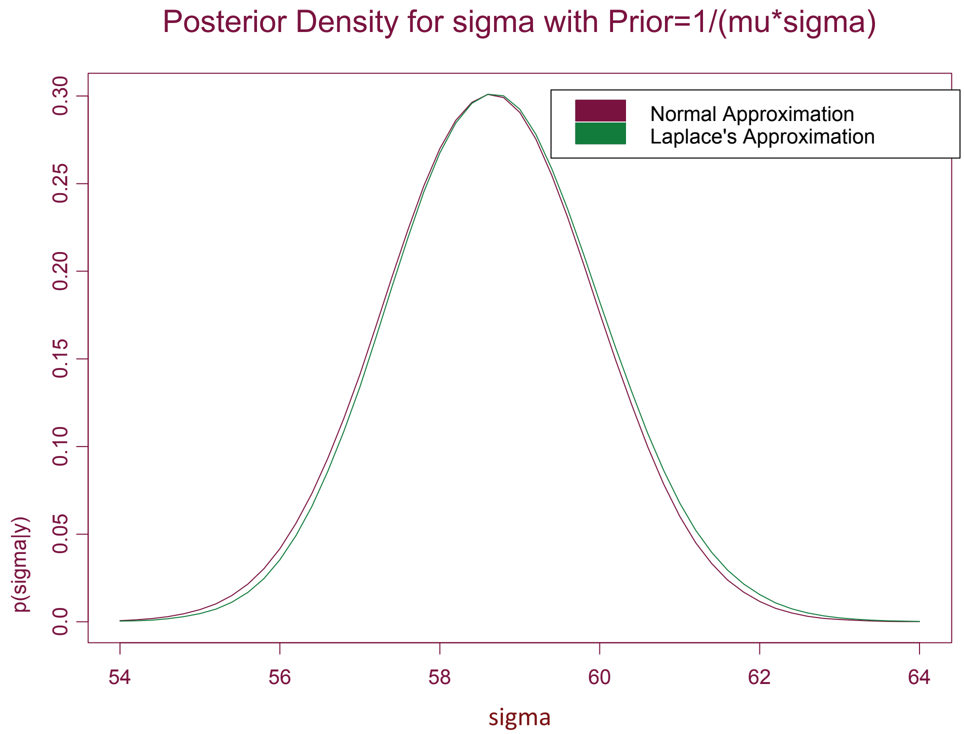


Figure 6: Comparison between Normal and Laplace Approximation



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