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Double Acceptance Sampling Plans Based on Truncated Life Tests for Marshall-Olkin Extended Lomax Distribution

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Double Acceptance Sampling Plans (DASP) is developed for a truncated life test when the lifetime of an item follows the Marshall-Olkin extended Lomax distribution. Probability of Acceptance (PA) is calculated for different consumer's confidence levels fixing the producer's risk at 0.05. Probability of acceptance and producer's risk are illustrated with examples.

Key words: Marshall-Olkin extended Lomax distribution, double acceptance sampling plan, probability of acceptance, consumer's risk, producer's risk, truncated life test.

Introduction

The main goal of competitive enterprises in a global business market is how to maintain and improve the quality of their products. A high quality product has the high probability of acceptance. Two important tools for ensuring quality are statistical quality control and acceptance sampling (AS). Acceptance sampling plans are concerned with accepting or rejecting a submitted large sized lot of products on the basis of the quality of the products inspected in a small sample taken from the lot.

A single acceptance sampling plan (SASP) is a specified plan that establishes the minimum sample size to be used for testing. In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. On the basis of information obtained from this first sample a lot is either accepted or rejected. If a good lot is rejected on the basis of this information, its probability is called the type-1 error (producer's risk) and it is denoted by α. The probability of accepting a bad lot is known as the type-2 error (consumer's risk) and it is denoted as β. If the product is electronic components or has failure mechanisms a random sample of the lot is tested and the entire lot is accepted if no more than c (acceptance sampling number) failures occur during the experiment time. Recently, Aslam (2007) proposed the double acceptance sampling plan based on a truncated life test when the lifetime of an item follows the Rayleigh distribution.

Single acceptance sampling plans based on truncated life tests for a variety of distributions are discussed by Epstein (1954), Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Kantam and Rosaiah (1998), Kantam, et al. (2001), Baklizi (2003), Baklizi and El Masri (2004), Rosaiah and Kantam (2005), Rosaiah, et al. (2006, 2007, 2007), Tsai and Wu (2006), Balakrishnan, et al. (2007), Aslam and Kantam (2008) and Rao, et al. (2008, 2009).

This article proposes a double acceptance sampling plan (DASP) based on truncated life tests when the lifetime of a product follows the Marshall-Olkin extended Lomax distribution with known shape parameter as introduced by Ghitany, et al. (2007) and to determine the probability of acceptance (PA).

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The probability density function (pdf) and cumulative distribution function (cdf) of the Marshall-Olkin extended Lomax distribution are given by

$$g(t;\nu,\sigma,\theta)=\frac{\frac{\nu\theta}{\sigma}(1+t/\sigma)^{\theta-1}}{\left[(1+t/\sigma)^{\theta}-\bar{\nu}\right]^{2}},$$

$$t>0,\ \nu,\sigma,\theta>0,\ \bar{\nu}=1-\nu \tag{1.1}$$

and

$$G_T(t;\nu,\sigma,\theta)=\frac{(1+t/\sigma)^{\theta}-1}{\left[(1+t/\sigma)^{\theta}-\bar{\nu}\right]},$$

$$t>0,\nu,\sigma,\theta>0,\ \bar{\nu}=1-\nu \tag{1.2}$$

respectively, where σ is a scale parameter, θ is a shape parameter and ν is an index parameter. The mean of this distribution is given by μ = 1.570796σ when $\nu=2,\theta=2$. Rao, et al. (2008, 2009) studied single acceptance sampling plans based on the Marshall-Olkin extended Lomax distribution.

The Double Acceptance Sampling Plan (DASP) for Life Tests

DASP is used to minimize the producer's risk because it provides another opportunity for acceptance of the product. In DASP a sample of size n_1 items is taken from a lot which is called the sample first; this sample first is then put on tests. Let c_1 and c_2 be the acceptance number for sample first and the sample second respectively. The experiment is terminated if no more than c_1 failures occur during the experiment time (t_0), i.e., the lot is accepted or rejected on the basis of sample first if more than c_2 failures occur or if time of experiment ends (whichever is earlier). If $(c_1+1)^{\text{th}}$ failures occur in sample first, then all possibilities for sample second are given as shown in Table 1.

Table 1: Possibilities for Sample Second Failure Based on Sample First Failure

Sample First	Sample Second
$(c_1+1)^{\text{th}}$ failures occur in sample 1	less than $(c_2-1)^{\text{th}}$ must be occurred in this sample 2 for acceptance
$(c_1+2)^{\text{th}}$ failures occur in sample 1	less than $(c_2-2)^{\text{th}}$ must be occurred in this sample 2 for acceptance
$(c_1+n)^{\text{th}}$ failures occur in sample 1	less than $(c_2-n)^{\text{th}}$ must be occurred in this sample for acceptance

Let μ represent the true average life of a product and μ_0 denote the specified life of an item, under the assumption that the lifetime of an item follows the Marshall-Olkin extended Lomax distribution. A product is considered good and accepted for consumer use if the sample information supports the hypothesis $H_0:\mu\geq\mu_0$; if the sample does not support this hypothesis, the lot of product is rejected. In acceptance sampling schemes, this hypothesis is tested based on the number of failures from a sample in a pre-fixed time: If the number of failures exceeds the action limit *c* the lot is rejected. The lot will be only be accepted if there is enough evidence that $\mu\geq\mu_0$ at a certain level of consumer risk, otherwise the lot will be rejected.

When determining the parameters of a proposed sampling plan the consumer's risk is used, and often, the consumer's risk is expressed by the consumer's confidence level. If the confidence level is p^*, then the consumer's risk will be $\beta=1-p^*$. In this study the consumer's risk was fixed not to exceed $1-p^*$ and it satisfied the inequality:

$$\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i}\leq 1-p^{*} \tag{2.1}$$

where *p* is the probability that an item fails before the termination time.

Consider a life testing experiment having the measurements: number of items from sample first put on test (n_1), acceptance number for sample first (c_1 = 0), number of items from sample second put on test (n_2) and accept the lots if no more than two failures occur in sample second (c_2 = 2).

In this life experiment if no failure occurs when sample first of n_1 items is put on test the lot is accepted. If the true - but unknown - life of the product deviates from the specified life of the product it should result in a considerable change in the probability of acceptance of the lot based on the sampling plan. Hence, the probability of acceptance (PA) can be regarded as a function of the deviation of a specified average from the true average. This function is called the operating characteristic (OC) function of the sampling plan. The PA for sample first using the Marshall-Olkin extended Lomax distribution with $\nu = 2, \theta = 2$ is shown in Table 2. The probability of acceptance for $L(p_1)$ and $L(p_2)$ for the sampling plan $(n_1, c_1, \frac{t}{\sigma_0})$ and $(n_2, c_2, \frac{t}{\sigma_0})$ is calculated using equations

$$L(p_1) = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n_1 - i} \quad (2.2)$$

and

$$L(p_2) = \sum_{i=0}^{c_2=2} \binom{n_2}{i} p^i (1-p)^{n_2 - i} \quad (2.3)$$

respectively, where

$$p = G_T(t;\sigma) = G_T(\frac{t}{\sigma_0}\frac{\sigma_0}{\sigma}),$$

is given in (1.2).

The probability of acceptance for DASP can be obtained by using (2.2) and (2.3) as:

$$\begin{aligned} P(A) = {} & P(\text{no failures occur in sample 1}) \\ & + P(\text{1 failure occurs in samples 1 and 0, and 1 failure occurs in sample 2}) \\ & + P(\text{2 failures occur in sample 1, and 0 failures occur in sample 2}). \end{aligned}$$

The values of probability of acceptance for DASP are determined at p^* = 0.75, 0.90, 0.95, 0.99 and t/σ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.142, 3.927, 4.712 with $\nu = 2, \theta = 2$ and are shown in Table 3. It is important to note that in sample first and sample second, p is function of the cdf of the Marshall-Olkin extended Lomax distribution. These choices are consistent with Gupta and Groll (1961), Gupta (1962), Kantam, et al. (2001), Baklizi and EI Masri (2004), Balakrishnan, et al. (2007) and Rao, et al. (2008).

Results

Suppose that the lifetime of a product follows the Marshall-Olkin extended Lomax distribution with $\nu = 2, \theta = 2$ and an experimenter wants to establish that the true unknown mean life is at least 1,000 hours with confidence 0.75. The acceptance numbers for this experiment would be c_1 = 0 and c_2 = 2 with sample sizes n_1 = 6 and n_2 = 8. The lot will be accepted if no failure is observed in a sample of 6during 628 hours. The probability of acceptance for this single sampling from Table 2 is 0.1558. The PA for the same measurements using double acceptance sampling from Table 3 is 0.29276. In the DASP scheme as σ / σ_0 increases PA also increases.

For the above sampling plan, PA is 0.97289 when the ratio of unknown average life to specified average life is 12. As the time of experiment increases, the probability of acceptance for the double acceptance sampling plan decreases. From Table 3, it is clear that when the time of experiment is 4,712 hours, the PA for ratio σ / σ_0 =2 is 0.04906. For this same experiment time, as σ / σ_0 increases, PA also increases. It is important to note that the double acceptance sampling scheme minimizes the

Table 2: Operating Characteristics Values for Sample First for the Sampling Plan $(n_1, c_1, \frac{t}{\sigma_0})$ when $c_1 = 0$ Marshall-Olkin Extended Lomax Distribution with $\nu = 2, \theta = 2$

p^*	n_1	t/σ_0	σ/σ_0					
			2	4	6	8	10	12
0.75	6	0.628	0.15576	0.39119	0.53422	0.62466	0.68622	0.73062
	4	0.942	0.15968	0.39271	0.53488	0.62500	0.68641	0.73074
	4	1.257	0.08995	0.28921	0.43485	0.53462	0.60556	0.65817
	3	1.571	0.10891	0.31489	0.45928	0.55663	0.62527	0.67589
	3	2.356	0.04222	0.18266	0.31496	0.41760	0.49594	0.55670
	2	3.142	0.06907	0.22806	0.36256	0.46285	0.53784	0.59527
	2	3.927	0.04180	0.16475	0.28637	0.38514	0.46289	0.52449
	2	4.712	0.02660	0.12125	0.22812	0.32193	0.39946	0.46292
0.90	8	0.628	0.08381	0.28610	0.43348	0.53398	0.60527	0.65804
	6	0.942	0.06381	0.24610	0.39119	0.49411	0.56869	0.62466
	5	1.257	0.04926	0.21209	0.35312	0.45715	0.53420	0.59282
	4	1.571	0.05201	0.21423	0.35435	0.45789	0.53467	0.59315
	3	2.356	0.04222	0.18266	0.31496	0.41760	0.49594	0.55670
	3	3.142	0.01815	0.10891	0.21831	0.31489	0.39444	0.45928
	3	3.927	0.00855	0.06687	0.15325	0.23902	0.31493	0.37985
	3	4.712	0.00434	0.04222	0.10896	0.18266	0.25247	0.31496
0.95	9	0.628	0.06147	0.24467	0.39047	0.49371	0.56845	0.62451
	7	0.942	0.04034	0.19482	0.33455	0.43933	0.51763	0.57755
	5	1.257	0.04926	0.21209	0.35312	0.45715	0.53420	0.59282
	5	1.571	0.02484	0.14575	0.27339	0.37666	0.45720	0.52054
	4	2.356	0.01470	0.10364	0.21430	0.31214	0.39256	0.45796
	3	3.142	0.01815	0.10891	0.21831	0.31489	0.39444	0.45928
	3	3.927	0.00855	0.06687	0.15325	0.23902	0.31493	0.37985
	3	4.712	0.00434	0.04222	0.10896	0.18266	0.25247	0.31496
0.99	12	0.628	0.02426	0.15303	0.28540	0.39020	0.47089	0.53381
	9	0.942	0.01612	0.12208	0.24467	0.34732	0.42886	0.49371
	7	1.257	0.01477	0.11406	0.23286	0.33426	0.41570	0.48094
	6	1.571	0.01186	0.09916	0.21093	0.30984	0.39096	0.45682
	5	2.356	0.00512	0.05880	0.14580	0.23331	0.31072	0.37673
	4	3.142	0.00477	0.05201	0.13145	0.21423	0.28927	0.35435
	4	3.927	0.00175	0.02714	0.08201	0.14833	0.21427	0.27509
	4	4.712	0.00071	0.01470	0.05204	0.10364	0.15957	0.21430

Table 3: Operating Characteristics Values for the Double Sampling Plan $(n_2, c_2, \frac{t}{\sigma_0})$ when $c_1 = 0$ and $c_2 = 2$ Marshall-Olkin Extended Lomax Distribution with $\nu = 2, \theta = 2$

p^*	n_1	n_2	t/σ_0	σ/σ_0					
				2	4	6	8	10	12
0.75	6	8	0.628	0.29276	0.50105	0.86425	0.92779	0.95741	0.97289
	4	6	0.942	0.28692	0.49834	0.85658	0.92304	0.95436	0.97084
	4	5	1.257	0.18319	0.38854	0.79087	0.88279	0.92856	0.95349
	3	5	1.571	0.17606	0.39214	0.76899	0.86779	0.91832	0.94632
	3	4	2.356	0.07833	0.24442	0.63350	0.77292	0.85228	0.89938
	2	4	3.142	0.09425	0.27216	0.61281	0.75306	0.83604	0.88665
	2	3	3.927	0.08244	0.22838	0.59443	0.73854	0.82510	0.87845
	2	3	4.712	0.04906	0.16496	0.49061	0.65090	0.75619	0.82513
0.90	8	10	0.628	0.14865	0.36654	0.76759	0.86826	0.91912	0.94708
	6	8	0.942	0.10420	0.31013	0.70705	0.82753	0.89150	0.92779
	5	6	1.257	0.09101	0.28114	0.68727	0.81386	0.88208	0.92116
	4	6	1.571	0.07686	0.26266	0.63949	0.77821	0.85639	0.90249
	3	5	2.356	0.05706	0.21770	0.56209	0.71656	0.81006	0.86785
	3	4	3.142	0.02910	0.14104	0.46313	0.63339	0.74510	0.81789
	3	4	3.927	0.01211	0.08352	0.32825	0.50281	0.63346	0.72651
	3	4	4.712	0.00560	0.05094	0.22930	0.39104	0.52766	0.63350
0.95	9	12	0.628	0.09469	0.30286	0.69561	0.81978	0.88620	0.92407
	7	9	0.942	0.06148	0.24378	0.62870	0.77176	0.85233	0.89979
	5	7	1.257	0.07461	0.26273	0.64649	0.78411	0.86090	0.90588
	5	6	1.571	0.04055	0.18977	0.56103	0.71960	0.81391	0.87145
	4	5	2.356	0.02212	0.13247	0.45228	0.62671	0.74100	0.81528
	3	5	3.142	0.02157	0.12425	0.38835	0.56197	0.68478	0.76899
	3	4	3.927	0.01211	0.08352	0.32825	0.50281	0.63346	0.72651
	3	4	4.712	0.00560	0.05094	0.22930	0.39104	0.52766	0.63350
0.99	12	16	0.628	0.03081	0.18223	0.52412	0.69017	0.79171	0.85476
	9	11	0.942	0.02141	0.15000	0.48085	0.65448	0.76426	0.83390
	7	9	1.257	0.01890	0.13770	0.45168	0.62824	0.74308	0.81729
	6	8	1.571	0.01449	0.11744	0.40429	0.58423	0.70674	0.78834
	5	6	2.356	0.00648	0.07208	0.30745	0.48783	0.62337	0.71970
	4	6	3.142	0.00516	0.05738	0.23006	0.39485	0.53317	0.63949
	4	5	3.927	0.00200	0.03154	0.16512	0.31441	0.45222	0.56520
	4	5	4.712	0.00077	0.01645	0.09793	0.21434	0.33887	0.45228

producer's risk, but this scheme also exerts pressure on the producer to improve the quality level of the product. At 4,712 hours, σ/σ_0 =12, p^* =0.75 and the PA is 0.82513. The producer's risk for the sample first and double sampling are placed for p^* =0.75 in Table 4. For σ/σ_0 = 2 (the unknown average life is twice that of the specified average life), the producer's risk when time of experiment is 628 hours and 4,712 hours are 0.70724 and 0.95094 respectively. Thus, the producer's risk decreases as the quality level of the product increases with p^* =0.75. (See Table 4 and Figure 1.)

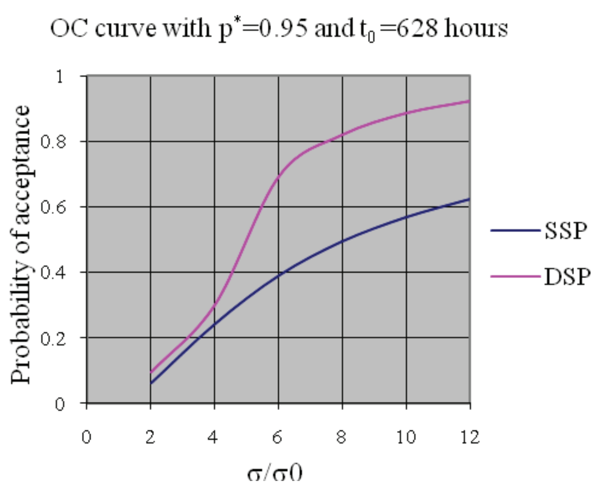
Conclusion

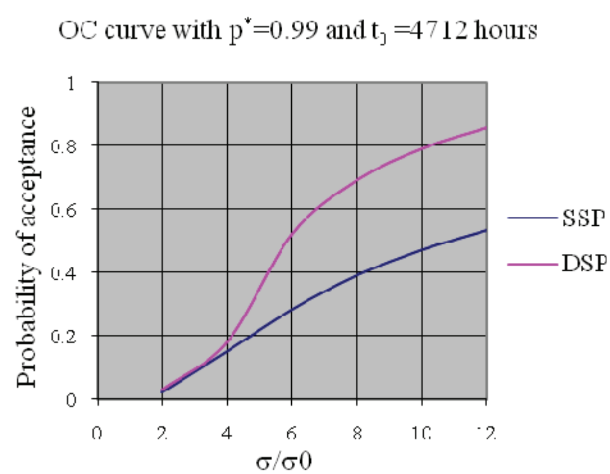
This study established the acceptance sampling plans for various values of σ/σ_0 and an experiment time assuming a life test follows the Marshall-Olkin extended Lomax distribution. This distribution provides a high probability for $\sigma/\sigma_0 > 6$.

Table 4: Producer's Risk with Respect to Time of Experiment for Double Sampling (p^* =0.75)

σ/σ_0	2	4	6	8	10	12
$(c_1 = 0, c_2 = 2, n_1 = 6, n_2 = 8, t/\sigma_0 = 0.628)$	0.70724	0.49895	0.13575	0.07221	0.04259	0.02711
$(c_1 = 0, c_2 = 2, n_1 = 2, n_2 = 3, t/\sigma_0 = 4.712)$	0.95094	0.83504	0.50939	0.34910	0.24381	0.17487

Figure1: OC Curve with p^* =0.95, t_0 =628 and p^* =0.99, t_0 =4712 for Single and Double Acceptance Sampling Plans (SSP & DSP).





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