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The Overall F-tests for Seasonal Unit Roots under Nonstationary Alternatives: Some Theoretical Results and a Monte Carlo Investigation

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In many empirical studies concerning seasonal time series, it has been shown that the whole set of unit roots associated with seasonal random walks are not present. This article focuses on the overall F-tests for seasonal unit roots under some nonstationary alternatives different from the seasonal random walk. The asymptotic theory of these tests is established for these cases using a new approach based on circulant matrix concepts. The simulation results joined to this theoretic analysis showed that the overall F-tests, as well as their augmented versions, maintained high power against the nonstationary alternatives.

Key words: Kunst test, nonstationary alternatives, Brownian motion, Monte Carlo Simulation.

Introduction

The stochastic nature of seasonality appears to be gaining ground in empirical studies. Several aspects related to seasonal unit root tests are treated in the literature. In this respect, the power of these tests against nonstationary alternatives is an important issue that recently acquired some concern. To the best of our knowledge, Ghysels, Lee and Noh (1994) are the first authors who studied this question. Using a Monte Carlo study, they showed that, against a nonseasonal random walk, the power of the tests of Dickey, Hasza and Fuller (1984) is much lower than that of the tests introduced by Hylleberg, Engle, Granger and Yoo (1990).

Ghysels, et al. (1994) suggested that “the Dickey et al. test may not separate unit roots at each frequency” (p. 432). The restriction behind the Dickey, et al. procedure is that all unit roots (conventional and seasonal roots) are inseparably present with equal modulus; thus, it is clear that the conventional random walk does not fulfil this requirement. However, Rodrigues and Osborn (1999) showed that if this restriction

holds, the power of the Dickey, et al. tests would have a proper superiority in finite samples as opposed to that of the tests of Hylleberg, et al. (1990). Taylor (2003) analyzed the large sample behaviour of the seasonal unit root tests of Dickey, et al. when the data generating process (DGP) is a conventional random walk, that is, when the series only admits a zero frequency unit root. In such a case (and as shown by Taylor, 2003), the Dickey, et al. statistics have nondegenerate limiting distributions. These results theoretically explain the empirical findings of Ghysels, Lee and Noh (1994). Furthermore, Taylor (2005) showed that asymptotically the statistics of the Dickey, et al. augmented test will also do not diverge.

In a similar context, del Barrio Castro (2006) generalized the results of Taylor (2003) to a set of nonstationary alternatives which include the non seasonal random walk. He found that the Dickey, et al. statistics did not have standard limiting distributions and did not diverge. Based on the same methodology, del Barrio Castro (2007) established the limit theory of the Fisher and Student statistics originally developed by the Hylleberg, Engle, Granger and Yoo (1990) procedure. In that case, del Barrio Castro derived the effect that can have one unit root asymptotically on the others at different frequencies. Following the terminology of Buseti and Taylor (2003), this situation may be said to have “unattended unit roots” (p. 33).

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However, del Barrio Castro (2007), in a large sample analysis, did not directly consider the effects of nonstationary alternatives on the overall F-type statistic of seasonal integration which is complementarily specified for the Hylleberg, Engle, Granger and Yoo (1990) procedure by Ghysels, Lee and Noh (1994). This article adopts the seasonal integration definition of Ghysels and Osborn (2001, p. 43).

In a recent article, Osborn and Rodrigues (2002) developed an appealing approach for deriving asymptotic results for test statistics in seasonal models with unit roots. Such an approach is based on the use of circulant matrices which could, in seasonal context, retrieve the limit theory of the involved statistics as well as conveniently traducing the dynamics of time series and its evolution across different seasons. In a similar vein, Haldrup, Montanes and Sanso (2005) have used this approach to show the effects of outliers on the limit theory of seasonal unit root tests.

This article focuses on the large sample properties of the overall F-tests of the seasonal integration when the observed series is generated from nonstationary alternatives treated by del Barrio Castro (2006). This task is accomplished using the circulant matrix-based approach of Osborn and Rodrigues (2002).

The Kunst Test

The Kunst test for quarterly time series is based on the following regression

$$\Delta_4 y_t = \alpha_1 y_{t-1} + \dots + \alpha_3 y_{t-3} + \delta y_{t-4} + \varepsilon_t, \quad t = 1, \dots, T, \quad (1)$$

which is an F-type test of the form

$$F_{\hat{\alpha}_1, \dots, \hat{\alpha}_3, \hat{\delta}}^* = (T - 4)(\hat{\varepsilon}'_0 \hat{\varepsilon}_0 - \hat{\varepsilon}' \hat{\varepsilon}) / (\hat{\varepsilon}' \hat{\varepsilon}), \quad (2)$$

where $\hat{\varepsilon}_0$ and $\hat{\varepsilon}_1$ are vectors of residuals estimated under the null $H_0 : \alpha_1 = \dots = \alpha_3 = \delta = 0$ and alternative hypotheses of the test. Assuming that, without any loss of generality, the initial values required by (1) are null. It should be noted that Kunst did not divide the numerator of the statistic (2) by 4

(the number of restrictions), as was done in this research to perform a conventional Fisher test.

The Hylleberg, Engle, Granger and Yoo (HEGY) Test

The basic regression for the HEGY test, without any augmentation and with no deterministic terms, is:

$$\Delta_4 y_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \varepsilon_t, \quad t = 1, \dots, T, \quad (3)$$

where

$$y_{1t} = (1 + L + L^2 + L^3)y_t,$$

$$y_{2t} = -(1 - L + L^2 - L^3)y_t, \quad (4)$$

$$y_{3t} = -(1 - L^2)y_t,$$

with L as the lag operator.

Ghysels, Lee and Noh (1994) extended the HEGY approach with a joint test statistic F_{1234} for the null hypothesis, $H_0 : \pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$, implying all unit roots in data are observed at quarterly frequency. H_0 is an overall hypothesis for seasonal integration SI (1) in accordance with the notation of Ghysels and Osborn (2001). Note that:

$$\begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-2} \\ y_{3t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ y_{t-3} \\ y_{t-4} \end{bmatrix}. \quad (5)$$

It may be deduced from (5) that the regressors of the Kunst test are non-singular linear transformations of those of the HEGY test. Consequently, the F-type statistics, F_{1234} and $F_{\hat{\alpha}_1, \dots, \hat{\alpha}_3, \hat{\delta}}^* / 4$, will have the same limit theory. Given that the two statistics are asymptotically related, the analysis is confined to that of Kunst in the sequel.

It can be observed that slight differences exist between the critical values of both statistics. In general, such critical values are tabulated assuming that the DGP of y_t is:

$$y_t = y_{t-4} + e_t. \quad (\text{A.0})$$

In this article it is assumed that the DGP of y_t is drawn from one of the following stochastic processes:

$$y_t = y_{t-1} + e_t, \quad (\text{A.1})$$

$$y_t = -y_{t-1} + e_t, \quad (\text{A.2})$$

$$y_t = y_{t-2} + e_t, \quad (\text{A.3})$$

$$y_t = -y_{t-2} + e_t, \quad (\text{A.4})$$

or

$$y_t = -y_{t-1} - y_{t-2} - y_{t-3} + e_t. \quad (\text{A.5})$$

Using the double subscript notation, the following annual vectors can be defined:

$$Y_n = (y_{1n}, y_{2n}, y_{3n}, y_{4n})',$$

and

$$E_n = (e_{1n}, e_{2n}, e_{3n}, e_{4n})',$$

where it is assumed that $n = 1, \dots, N$ and in T observations there are N years, thus, $T = 4N$. To keep matters tractable, suppose that $Y_0 = (y_{10}, y_{20}, y_{30}, y_{40})' = (0, 0, 0, 0)'$.

The error processes in the alternatives (A.1)-(A.5) follow a stationary AR(p)

$$\phi(L)e_{sn} = v_{sn},$$

where $\phi(z)e_{sn} = 1 - \sum_{i=1}^p \phi_i z^i$ and $s = 1, \dots, 4$.

The roots of $\phi(z) = 0$ all lie outside the unit circle $|z| = 1$. As for the error sequence $\{v_{sn}\}$, it depicts an innovation process with constant conditional variance σ^2 (see Spanos,

2003, p. 443). Similar to what has been conjectured by del Barrio Castro (2007) regarding the error structure in the nonstationary alternatives described above, suppose that the vector E_n has the following dynamics:

$$E_n = \sum_{j=0}^{\infty} \Gamma_j^* V_n,$$

where $v_n = (v_{1n}, v_{2n}, v_{3n}, v_{4n})'$, and the sequence of 4×4 matrices are defined as:

$$\Gamma_0^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \gamma_1 & 1 & 0 & 0 \\ \gamma_2 & \gamma_1 & 1 & 0 \\ \gamma_3 & \gamma_2 & \gamma_1 & 1 \end{bmatrix}$$

$$\Gamma_0^* = \begin{bmatrix} \gamma_{j4} & \gamma_{j4-1} & \gamma_{j4-2} & \gamma_{j4-3} \\ \gamma_{j4+1} & \gamma_{j4} & \gamma_{j4-1} & \gamma_{j4-2} \\ \gamma_{j4+2} & \gamma_{j4+1} & \gamma_{j4} & \gamma_{j4-1} \\ \gamma_{j4+3} & \gamma_{j4+2} & \gamma_{j4+1} & \gamma_{j4} \end{bmatrix},$$

for $j = 1, 2, \dots$, with

$$\gamma(z) = 1 - \sum_{j=1}^{\infty} \gamma_j z^j$$

being the inverse of $\phi(z)$. Finally, $\Gamma^*(1)$ is defined as:

$$\Gamma^*(1) = \sum_{j=0}^{\infty} \Gamma_j^*.$$

del Barrio Castro (2006) used the vector of moving average representation to express the alternatives (A.i), $i = 1, \dots, 5$, in a vector of quarters representations where the observations of each year are stacked in the above defined vectors Y_n et E_n , let

$$(1 - B)Y_n = (\Theta_0^i + \Theta_1^i B)E_n, \quad i = 1, 2, \dots, 5, \quad (6)$$

where B is the annual backward operator. The 4×4 matrices Θ_0^i and Θ_1^i (corresponding to the alternatives A.1-A.5) are defined as follows: for (A.1)

$$\Theta_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \Theta_1^1 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (7.1)$$

for (A.2)

$$\Theta_0^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix},$$

$$\Theta_1^2 = \begin{bmatrix} 0 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (7.2)$$

for (A.3)

$$\Theta_0^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \Theta_1^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (7.3)$$

for (A.4)

$$\Theta_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix},$$

$$\Theta_1^4 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (7.4)$$

and for (A.5)

$$\Theta_0^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$\Theta_1^5 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (7.5)$$

The following result was established by del Barrio Castro (2007)

$$\frac{1}{\sigma \sqrt{N}} Y_{[rN]} \rightarrow_d B_i(r),$$

$$B_i(r) = C_i \Gamma^*(1) B(r),$$

$$C_i = \Theta_0^i + \Theta_1^i,$$

$$i = 1, 2, \dots, 5, \quad (8)$$

where the symbol \rightarrow_d denotes the convergence of probability measures, $B_i(r)$ is a 4×1 vector Brownian motion process with variance matrix $\Omega_i = \sigma^2 C_i \Gamma^*(1) \Gamma^*(1)' C_i'$ and $B(r)$ is a vector Brownian motion with variance matrix $\sigma^2 I_4$. The subscript i corresponds to the alternative (A.i), $i = 1, \dots, 5$.

Note that the rank of C_i , $i = 1, \dots, 5$, is the number of (seasonal) unit roots implied by the process (A.i), $i = 1, \dots, 5$. In order to determine the number of cointegration relations between the quarters corresponding to every process (A.i), $i = 1, \dots, 5$, it is necessary to subtract from the periodicity of the quarterly data, that is 4, the rank of the matrix C_i , $i = 1, \dots, 5$. Equation (8) may be rewritten more precisely by identifying the stochastic processes $B_i(r)$, $i = 1, 2, \dots, 5$, on the grounds that there is always cointegration among the quarters of the time series (see del Barrio Castro, 2007, p.915). Limit Theory of the Kunst Test Under Nonstationary Alternatives

The following lemma can be directly deduced from the preceding result of del Castro Barrio (2007) and lemma A.1 of Osborn and Rodrigues (2002).

Lemma

Supposing that the DGP of y_t in (1) is given by the alternatives (A.1)-(A.5) and also that the vector $(e_{1n}, \dots, e_{4n}), \forall n$, satisfies assumption 1 of Phillips (1986, p.313), then under the null of the Kunst test is $T \rightarrow \infty$

(a)

$$N^{-2} \sum_{n=1}^N Y_n Y_n' \rightarrow_d \sigma^2 \int_0^1 M_i B(r) B(r)' M_i' dr,$$

$$i = 1, 2, \dots, 5.$$

(b)

$$N^{-1} \sum_{n=1}^N Y_{n-1} \varepsilon_n' \rightarrow_d \sigma^2 M_i \int_0^1 B(r) dB(r)' M_i',$$

$$i = 1, 2, \dots, 5.$$

(c)

$$T^{-2} \sum_{t=1}^T y_{t-k}^2 \rightarrow_d \frac{\sigma^2}{16} \int_0^1 B(r)' M_i' M_i B(r) dr,$$

$$k = 1, \dots, 4, \quad i = 1, 2, \dots, 5.$$

(d)

$$T^{-2} \sum_{t=1}^T y_{t-k} y_{t-j} \rightarrow_d \frac{\sigma^2}{16} \int_0^1 B(r)' M_i' H_k' H_j M_i B(r) dr,$$

$$k \neq j, \quad i = 1, 2, \dots, 5.$$

(e)

$$T^{-1} \sum_{t=1}^T y_{t-k} \varepsilon_t \rightarrow_d \frac{\sigma^2}{4} \int_0^1 B(r)' M_i' H_k' M_i dB(r),$$

$$k = 1, \dots, 4, \quad i = 1, 2, \dots, 5,$$

where

$$\varepsilon_n = (\varepsilon_{1n}, \varepsilon_{2n}, \varepsilon_{3n}, \varepsilon_{4n})'$$

and

$$M_i = C_i \Gamma^*. \quad (1)$$

The matrix $H_k, k = 1, 2, 3, 4$, is a particular permutation matrix order 4 which produces the following elementary operations: let a matrix K have 4 lines, the operation $H_1 K$ moves the last row of K to the top row of $H_1 K$ and the other rows move down one place. More generally, $H_i K$ shifts the final i^{th} rows to the top of the matrix while the remaining rows correspondingly move down; note that $H_4 = I_4$ (see Golub & Van Loan, 1996, p. 109-112, for details). The OLS estimator $\hat{\alpha}$ of the vector $\alpha = (\alpha_1, \alpha_2, \alpha_3, \delta)'$ defined in Equation (1) satisfies under the null of Kunst test the asymptotic results represented by the following theorem.

Theorem

If that the DGP of y_t in (1) is given by one of the alternatives (A.1) - (A.5), then:

(a)

$$\frac{T}{4} (\hat{\alpha} - \alpha) \rightarrow_d F^{-1} f,$$

(matrices for F and f are shown in Figure 1);

(b) The Student statistic $t_{\hat{\alpha}_i}$ corresponding to the i^{th} component of vector $\hat{\alpha}$ satisfies the following result:

$$t_{\hat{\alpha}_i} \rightarrow_d \frac{(F^{-1} f)_i}{((F^{-1})_{ii})^{1/2}};$$

and

(c) The F-type statistic of Kunst test verifies:

$$F_{\hat{\alpha}_1, \dots, \hat{\alpha}_3, \delta}^* \rightarrow_d f' F^{-1} f.$$

Figure 1: F and f Matrices for $\frac{T}{4}(\hat{\alpha} - \alpha) \rightarrow_d F^{-1}f$

$$F = \begin{bmatrix} \int_0^1 B(r)' M_i' H_1' H_1 M_i B(r) dr & \int_0^1 B(r)' M_i' H_1' H_2 M_i B(r) dr & \int_0^1 B(r)' M_i' H_1' H_3 M_i B(r) dr & \int_0^1 B(r)' M_i' H_1' M_i B(r) dr \\ \int_0^1 B(r)' M_i' H_2' H_1 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2' H_2 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2' H_3 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2' M_i B(r) dr \\ \dots & \dots & \dots & \dots \\ \int_0^1 B(r)' M_i' H_3' M_i B(r) dr & \int_0^1 B(r)' M_i' H_3' M_i B(r) dr & \int_0^1 B(r)' M_i' H_3' M_i B(r) dr & \int_0^1 B(r)' M_i' M_i B(r) dr \end{bmatrix}$$

$$f = \begin{bmatrix} \int_0^1 B(r)' M_i' H_1' M_i dB(r) \\ \int_0^1 B(r)' M_i' H_2' M_i dB(r) \\ \dots \\ \int_0^1 B(r)' M_i' H_3' M_i dB(r) \\ \int_0^1 B(r)' M_i' M_i dB(r) \end{bmatrix} \forall i = 1, 2, \dots, 5.$$

Proof

Before showing the proof of result (a) of the theorem, consider the properties of the matrix F : the elements of the main diagonal of F are all equal and the elements of F along each diagonal line parallel to the principal diagonal are equal, thus, F is a Toeplitz matrix. Toeplitz matrices belong to the larger class of persymmetric matrices. A square matrix B of order n is persymmetric if it is symmetric about the northeast-southwest diagonal, that is, $b_{ij} = b_{n-j+1, n-i+1}$ for all i and j . Moreover, from the properties of the matrices H_k , $k = 1, 2, 3, 4$, it can be shown that the matrix F is also symmetric. Equation (1) can be written in matrix form:

$$Y = X\alpha + \varepsilon, \quad (9)$$

where

$$X = \begin{bmatrix} y_0 & y_{-1} & y_{-2} & y_{-3} \\ y_1 & y_0 & y_{-1} & y_{-2} \\ \dots & \dots & \dots & \dots \\ y_{T-1} & y_{T-2} & y_{T-3} & y_{T-4} \end{bmatrix}$$

and

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \dots \\ \varepsilon_T \end{bmatrix}.$$

In addition, $\hat{\alpha} - \alpha = (X'X)^{-1}X'\varepsilon$,

where

$$X'X = \begin{bmatrix} \sum_1^T y_{t-1}^2 & \sum_1^T y_{t-1}y_{t-2} & \sum_1^T y_{t-1}y_{t-3} & \sum_1^T y_{t-1}y_{t-4} \\ \sum_1^T y_{t-2}y_{t-1} & \sum_1^T y_{t-2}^2 & \sum_1^T y_{t-2}y_{t-3} & \sum_1^T y_{t-2}y_{t-4} \\ \sum_1^T y_{t-3}y_{t-1} & \sum_1^T y_{t-3}y_{t-2} & \sum_1^T y_{t-3}^2 & \sum_1^T y_{t-3}y_{t-4} \\ \sum_1^T y_{t-4}y_{t-1} & \sum_1^T y_{t-4}y_{t-2} & \sum_1^T y_{t-4}y_{t-3} & \sum_1^T y_{t-4}^2 \end{bmatrix},$$

and

$$X'\varepsilon = \begin{bmatrix} \sum_1^T y_{t-1}\varepsilon_t \\ \dots \\ \sum_1^T y_{t-4}\varepsilon_t \end{bmatrix}.$$

Also: $\frac{T}{4}(\hat{\alpha} - \alpha) = \frac{1}{4}(\frac{X'X}{T^2})^{-1} \frac{X'\epsilon}{T}$ and, due to parts (c), (d) and (e) of the preceding lemma and the fact that $H_4 = I_4$, the result of the theorem holds.

The asymptotic distributions of Student statistics corresponding to the parameters of Equation 1 can be deduced from result (a) of the theorem. To prove result (c), the F-type statistic of Kunst can be written as follows: $F_{\hat{\alpha}_1, \dots, \hat{\alpha}_3, \hat{\delta}}^* = \hat{\alpha}'[(S^2)^{-1}X'X]\hat{\alpha}$, where S^2 is the OLS estimator of the residual variance in Equation (1). The (Toeplitz) circulant matrix F and its inverse are symmetric, consequently result (c) holds and the theorem is proved.

Empirical quantiles of the Kunst test for the processes (A.1)-(A.5) were generated and associated with nominal levels 90%, 95% and 99%. The sample size considered is 4,000 (1,000 years) with 20,000 replications; it has been shown that these empirical quantiles tend to be infinite (these results are not presented, but are available upon request). Consequently, it is possible to predict that in 100% of cases the null hypothesis will be rejected for the processes (A.1)-(A.5) for nominal levels of 5% and 1%. Table 1 shows the rejection frequencies for a sample size of 100 (25 years) and 20,000 replications. All simulations were conducted using the software Matlab.

In addition, augmented regression (1) corresponding to the Kunst test was carried out by lagged values of the independent variable, thus, this regression becomes

$$\Delta_4 y_t = \alpha_1 y_{t-1} + \dots + \alpha_3 y_{t-3} + \delta y_{t-4} + \sum_{i=1}^p \Delta_4 y_{t-i} + \epsilon_t, \quad t = 1, \dots, T. \quad (10)$$

Table 2 reports the power of the augmented Kunst test against the nonstationary alternatives (A.1) - (A.5). Results in Table 2 show that perfect power is maintained across all the alternatives (A.1) - (A.5) even if the number of lagged terms of the dependent variable increases. At this level, a slight exception to this general finding was detected for the alternative (A.5) and for $p = 4$ or $p = 6$. Particularly, and for this alternative, the exception is much clearer for $p = 6$ and the nominal level 1%. In fact, the test power decreases and reaches a value of approximately 66%.

Conclusion

A large amount of literature on testing for seasonal unit roots has appeared during the last two decades. However, the majority of econometricians treating this topic have bent over backwards to give the limit theory of the tests for unit roots at the zero, Nyquist and

Table 1: Empirical Rejection Frequencies of Kunst Test under Nonstationary Alternatives

Kunst Test	Processes					
	(A.0)	(A.1)	(A.2)	(A.3)	(A.4)	(A.5)
$F_{\hat{\alpha}_1, \dots, \hat{\alpha}_3, \hat{\delta}}^*$ Nominal Size 5%	0.095	1	1	1	1	1
$F_{\hat{\alpha}_1, \dots, \hat{\alpha}_3, \hat{\delta}}^*$ Nominal Size 1%	0.0158	1	1	1	1	1

Number of replications: 20,000; Sample size $4N = 100$ observations

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Table 2: Empirical Rejection Frequencies of the Kunst Augmented Test under Nonstationary Alternatives

		Processes					
		(A.0)	(A.1)	(A.2)	(A.3)	(A.4)	(A.5)
Nominal Size 5%	p = 2	0.0592	1	1	1	1	1
	p = 4	0.0549	1	1	1	1	0.9920
	p = 6	0.0522	1	1	0.9980	0.9976	0.9038
Nominal Size 1%	p = 2	0.0140	1	1	1	1	0.9998
	p = 4	0.0141	1	1	0.9992	0.9991	0.9271
	p = 6	0.0121	0.9990	0.9990	0.97770	0.97460	0.6632

harmonic seasonal frequencies by considering either an additional determinist component or a modified hypothesis set concerning the error terms which appear in the regression models associated with such tests. Seldom have works concerning this topic studied the power of seasonal unit roots against nonstationary alternatives. Ghysels, et al. (1994) studied this problem and, in a simulation study, they hypothesized that the DHF test may not separate unit roots at each frequency. Having enriched this analysis by a large sample investigation, Taylor (2003) found that the DHF statistics did not diverge to minus infinity when the DGP of the series is a conventional random walk. del Barrio Castro (2006, 2007) considered an extended set of nonstationary alternatives and studied their asymptotic effects on the DHF and HEGY statistics.

This article extended the problem treated by Taylor (2003) and del Castro Barrio (2007) to the overall F-type tests for seasonal integration. It has been realized that the most renowned tests, that is, those of HEGY (1990) and Kunst (1997), are asymptotically related. For this reason, this research focused on asymptotic effects of the nonstationary alternatives, (A.1) - (A.5) on Kunst F-type test. To reach this goal, I had the circulant-matrix-based approach introduced by Osborn and Rodrigues (2002) was chosen. Moreover, in a simulation study, it was found that the Kunst F-

type statistic maintained high power when the totality of unit roots implied by the filter $(1-L^4)$ were not present. In addition, these high-power properties are preserved when the regression model of the test was augmented with lagged dependent variables. The approach adopted in this article can be applied to cases other than the quarterly one. To this aim, it is enough to write the adequate matrices C_i defined from Equation (5) following the data observation frequency.

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