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Logistic regression models for transition probabilities of higher order Markov models are developed for the sequence of chain dependent repeated observations. To identify the significance of these models and their parameters a test procedure for a likelihood ratio criterion is developed. A method of model selection is suggested on the basis of AIC and BIC procedures. The proposed models and test procedures are applied to analyze the occurrences of daily rainfall data for selected stations in Bangladesh. Based on results from these models, the transition probabilities of first order Markov model for temperature and humidity provided the most suitable option to model forecasts for daily rainfall occurrences for five selected stations in Bangladesh.

Key words: Logistic regression, transition probabilities, Markov chain, ML estimation, LR test, AIC, BIC, daily rainfall occurrences data.

Introduction

A Markov chain model is constructed for describing transition probabilities for time or chain dependent process under change or random process. A logistic regression model is used as probabilistic model for analyzing covariate dependent binary data. The logistic regression model may define covariate dependent transition probabilities of a Markov chain. Muenz and Rubinstein (1985) made an attempt to develop covariate dependent first order transition probabilities for Markov chain models. In their model two-health states, distress and no distress, recorded as binary responses 1 and 0 respectively were incorporated; they showed that healthy patients feel less distress than others at the time of biopsy as time proceeds.

To identify the pattern of daily rainfall occurrences Gabriel and Neumann (1962) developed a Markov chain model for Tel Aviv data. They showed that dry and wet spells follow a geometric distribution. For the same data, Green (1964) fitted the probability models better than that of Gabriel and Neumann's models assuming that dry and wet spells follow an exponential distribution. Parthasarathy and Dhar (1974) identified the negative trend for south Asian daily rainfall occurrences using regional rainfall over India for the period 1901 to 1960. Similar studies analyzing daily rainfall data have been conducted by Islam (1980), Stern (1980a), Stern (1980b), Stern, et al. (1982), Stern and Coe (1984), Sinha and Paul (1992), Sinha and Islam (1994), Shimizu and Kayano (1994), Sinha, et al. (2006), Sinha, et al. (2009) and others. However, these did not develop covariate dependent probability models for analyzing the patterns of daily rainfall occurrences. To identify the patterns and forecasting models for the occurrence or non-occurrence of rainfall, different types of covariate dependent transition

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probabilities of Markov chain models need to be developed for logistic regression.

Transition Probabilities of Markov Chain for Logistic Regression Models

To develop logistic regression models for higher order transition probabilities (t. p.) of Markov chains, consider the chain dependent repeated observations $x_1, x_2, ..., x_n$ at time t (t = 1, 2, ..., T), $X_n(t)$. Here the assumption that observations occurring depend on different covariates, $Z_n(t)$ is made. The first order transition count, $n_{jk}(t)$ denotes the number of individuals in state j at time t-1 and in state k at t. If the second order transition count, $n_{ijk}(t)$ denotes the number of individuals in state i at time t-2, in state j at t-1 and in state k at t, then the first and second order transition probabilities of the Markov chain are denoted by $p_{jk}(t)$ and $p_{ijk}(t)$ respectively, for all i, j, k = 1, 2, ..., m and t = 1, 2, ..., T.

For stationarity, these probabilities are denoted by p_{jk} and p_{ijk}, respectively. Similarly, higher order stationary or non-stationary transition probabilities of Markov chain $p_{ij...krl}$ or $p_{ij....krl}(t)$, respectively, may be defined for transition count $n_{ij...krl}$ or $n_{ij...krl}(t)$. The term $p_{ij...krl}$ or $p_{ij....krl}(t)$ indicates the transition probability of state l at time t, given the state r at time t-1, ..., the state j at time t-s+1 and state i at time t-s, where t = s, s+1, ..., T, and for all i, j, ..., k, r, l = 0, 1. The ML estimate (Anderson & Goodman, 1957; Muenz & Rubinstein, 1985; Sinha, et al., 2006; Sinha, et al., 2009) of higher order stationary or non-stationary transition probabilities for the transition probability matrices are

$$\hat{p}_{ij....kl} = \frac{n_{ij...kl}}{n_{ij...k.}} = \frac{n_{ij....kl}(t)}{n_{ij...k}(t-1)}$$

where

$$n_{ij....k.} = \sum_{l=1}^{m} n_{ij...kl} \,.$$

To develop the covariate dependent two-state transition probabilities of the Markov chain, consider the parameters p_{01} and p_{11} for first order, and p_{001}, p_{101}, p_{011} and p_{111} for a second order Markov chain. Here p_{01}, p_{11}, p_{001}, p_{101}, p_{011} and p_{111} specify the transition probabilities of $0 \to 1$, $1 \to 1$, $0 \to 0 \to 1$, $1 \to 0 \to 1$, $0 \to 1 \to 1$ and $1 \to 1 \to 1^{th}$ transitions respectively. Similarly, 2^r parameters may be defined for r^{th} order two-state transition probabilities of the Markov chain. To formulate such a Markov chain, the following assumptions are made: (i) each observation of chain dependent process depends on different covariates; (ii) observations of the chain dependent process follow a logistic form; (iii) the counts $n_j(0)$ and $n_{jk}(1)$ are non-random; and (iv) each row of transition in the probability matrix is independent.

To estimate the covariate dependent transition probabilities for the Markov chain, consider logistic regression models for first, second and higher order transition probabilities (Muenze & Rubinstein, 1985) which are defined as

$$P_{ij...kr1} = \frac{\exp(z_{ij...kr1}(q,t))}{1+\exp(z_{ij....kr1}(q,t))}, \quad (2.1)$$

where

$$Z_{ij..kr1}(q,t) = \alpha_{ij..kr1} + \sum_{h=1}^{n} \beta_{h(ij..kr1)} Z_{h(ij...kr1)q(t-r)},$$

for all i, j, ..., k, r = 0, 1, h = 1, 2, ..., n , q = 1, 2, ..., Q, t = 1, 2, ..., where T and r are the order of the Markov chain. Here, $Z_{h(ij..kr1)q(t-r)}$ is the h^{th} covariate for $i \to j \to ..... \to k \to r \to l(=1)^{th}$ state for $(t\text{-}r)^{th}$ day of q^{th} year, $\beta_{h(ij....k1)}$ is the parameter of h^{th} covariate for $i \to j \to ..... \to k \to r \to l(=1)^{th}$ state and $\alpha_{ij...r1}$ is the intercept term. Further for saturated model the term $Z_{ij....kr1}(q,t)$ for (2.1) can be defined as

$$\begin{aligned} Z_{ij..kr1}(q,t) = {} & \\ & \alpha_{ij..kr1} + \sum_{h=1}^{n} \beta_{h(ij..kr1)} Z_{h(ij...kr1)q(t-r)} \\ & + \sum_{(h<g)=1}^{n} \beta_{hg(ij..kr1)} Z_{h(ij...kr1)q(t-r)} Z_{g(ij...kr1)q(t-r)} \\ & + \text{higher order interaction effect} \end{aligned} \quad (2.2)$$

where $\beta_{h(ij \ldots kr1)}$ is the main effect for $Z_{h(ij..kr1)q(t-r)}$ th covariate and $\beta_{hg(ij \ldots kr1)}$ is the interaction effect for the $Z_{h(ij..kr1)q(t-r)}$ and $Z_{g(ij..kr1)q(t-r)}$ th covariates.

Estimation of Parameters for Covariate Dependent Transition Probabilities of Markov Chain Model

To identify the effect of different covariates for the changes of transition probabilities of Markov chain model the parameters of the models 2.1 and 2.2 are to be estimated. To estimate the parameters for transition probabilities of Markov chain model, Anderson and Goodman (1957), Muenz and Rubinstein (1985), Sinha, et al. (2006) and Sinha, et al. (2009) suggested the ML estimation method. Thus the method of MLE is used to estimate the parameters of model 2.1. The log likelihood function (Formula 2.3) is shown in Figure 1. To obtain the estimated value of parameters by ML estimation method under Newton-Raphson iteration procedure, the information matrix and information vector are denoted by I and U respectively, where I^{-1} is the variance covariance matrix with respect to parameters. Similarly, the parameters of model (2.2) may be estimated.

Test of Hypothesis

To test the significance of the parameters and models for logistic regression models for transition probabilities of a Markov chain, Wald (1943) suggested test statistic W as consistent and asymptotically equivalent to the likelihood ratio test under the null hypothesis. This test statistic provides a significant result for the iterative nature of maximum likelihood estimate than that of likelihood ratio test. However, Rao (1965), Hauck and Donner (1977) and Jennings (1986) found that the test statistic W is less powerful compared to likelihood ratio test. Furthermore, for a large sample Hosmer and Lemeshow (1989) recommended the likelihood ratio test as opposed to Wald's test, because often it fails to reject the co-efficient when it is significant. Due to these, the likelihood ratio test procedure is employed to test the significance of parameters and models for 2.1 and 2.2.

Likelihood Ratio Test

To identify the significance of the covariate dependent transition probabilities of Markov chain models and their parameters; consider hypotheses 1 and 2 for model 2.1, and 3 and 4 for model 2.2.

Hypotheses 1, Model 2.1:

$$H_0: \beta_{1(ij...r1)} = \beta_{2(ij...r1)} = ... = \beta_{p(ij...r1)} = \beta_{(p+1)(ij...r1)} = ... = \beta_{h(ij...r1)} = 0$$

vs.

$$H_1: \beta_{1(ij...r1)} = \beta_{2(ij...r1)} = ... = \beta_{p(ij...r1)} = \beta_{(p+1)(ij...r1)} = ... = \beta_{h(ij...r1)} \neq 0$$

Figure 1: Log Likelihood Function for the ML Estimation Method (Formula 2.3)

$$\log L = \sum_{q=1}^{Q} \sum_{t=1}^{T} n_{ij..r1}(q,t)\left(\alpha_{ij..r1} + \sum_{h=1}^{n} \beta_{h(ij..r1)} Z_{h(ij...r1)q(t-r)}\right) - \sum_{q=1}^{Q} \sum_{t=1}^{T} (n_{ij..r0}(q,t) + n_{ij..r1}(q,t)) \log\left(1 + \exp\left(\alpha_{ij..r1} + \sum_{h=1}^{n} \beta_{h(ij...r1)} Z_{h(ij..r1)q(t-r)}\right)\right), \quad (2.3)$$

$n_{ij...r0}(q,t)$ and $n_{ij...r1}(q,t)$ are the transition counts for the $i, j, .., r, l(=0)^{th}$ state and $i, j, .., r, l(=1)^{th}$ state respectively for the t^{th} day of the q^{th} year, where q = 1, 2, ...,Q and t = 1,2, ..., T

Hypotheses 2, Model 2.1:

$$H_0: \beta_{p(ij...r1)} = 0 \text{ and } H_0: \alpha_{ij...r1} = 0$$

vs.

$$H_1: \beta_{p(ij...r1)} \neq 0 \text{ and } H_1: \alpha_{ij...r1} \neq 0$$

Hypotheses 3, Model 2.2:

$$H_0: \beta_{1(ij...r1)} = \beta_{2(ij...r1)} = ... = \beta_{h(ij...r1)} = \beta_{\substack{hg(ij\;...\;r1) \\ h<g}} = ... = 0$$

vs.

$$H_1: \beta_{1(ij...r1)} = \beta_{2(ij...r1)} = ... = \beta_{h(ij...r1)} = \beta_{\substack{hg(ij\;...\;r1) \\ h<g}} = ... \neq 0$$

Hypotheses 4, Model 2.2:

$$H_0: \beta_{\substack{pk(ij...r1) \\ p<k}} = 0$$

vs.

$$H_1: \beta_{\substack{pk(ij\;...\;r1) \\ p<k}} \neq 0 \;,$$

where h = 1, 2, ..., p, p+1, ..., n, g = 1, 2, ..., k, k+1, ..., n, $\beta_{h(ij\;...\;r1)}$ is the parameter of the h^{th} covariates for i, j, ..., r, l(=1) transition and $\beta_{hg(ij\;...\;r1)}$ is the interaction effect between h^{th} and g^{th} covariates. The likelihood ratio test statistic ($-2\log\lambda_{ij\;...\;r1}$) is asymptotically distributed as $\chi^2_{ij....r1}$ (Kendall & Stuart, 1973) with h, 1, (h+(h(h-1)/2) + number of higher order interaction effect) and 1 degree of freedom for the null hypotheses 1, 2, 3 and 4 respectively, where $\lambda_{ij\;...\;r1}$ is the likelihood ratio for i, j, .., r, l(=1)th (for all i, j, ..., r, s = 0, 1) transitions of the Markov chain. For the overall transition probabilities of the Markov chain this test statistic is defined as

$$\chi^2 = \sum_{ij\;...\;r1} \chi^2_{ij\;..\;r1} \qquad (2.4)$$

for all i, j, ..., r = 0, 1 with 2^rh, 2^r, (h+(h(h-1)/2) + number of higher order interaction effect) and 2^r degrees of freedom for null hypotheses 1, 2, 3 and 4 respectively, where r is the order of the Markov chain.

Methods of Model Selection

To identify the best model among the significant models, several authors including McCullagh & Nelder (1983) and Agresti (1984) suggested various model selection procedures and they also identified some limitations and drawbacks. For example, these selection procedures sometimes provide almost equal emphasis for several possible models; often procedures do not provide the best model among the models sufficiently for a true alternative hypothesis. For overcoming these problems, Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) procedures are employed for the selection of appropriate covariate dependent transition probabilities for the Markov model (Sakamoto, 1991; Shimizu, 1993).

Akaike (1972b) developed AIC by the utilization of a likelihood ratio criterion under the extension form of maximum likelihood. Akaike (1970) defined AIC on the basis of final prediction error (FPE) as the mean square prediction error of a predictor to identify the autoregressive model. Schwarz (1978) developed BIC as a more consistent and optimal procedure than the AIC. Sakamoto (1991) used a minimum of AIC (MAICE) and a minimum of BIC (MBICE) to identify the optimal explanatory variable for the model. For covariate dependent transition probabilities of Markov chain models, to develop a model selection procedure, the MAICE and MBICE are employed by utilizing the likelihood function and the ML estimate of parameters. For a large n, Bayes estimators are asymptotically equivalent to ML estimators (Kendall & Stuart, 1973) and the procedures are defined as

$$\text{AIC(i)} < \text{AIC(i+1)} < \ldots < \text{AIC(i+s)}, \qquad (3.1)$$

and

$$\text{BIC(i)} < \text{BIC(i+1)} < \ldots < \text{BIC(i+s)}, \qquad (3.2)$$

where i = 1, 2, …, ∝ , s = 0, 1, 2, …, ∝, (i+s) indicate the number of models, AIC = -2(maximum log likelihood) + 2(number of estimable parameters in the model) and BIC = -2(maximum log likelihood) + 2(number of estimable parameters in the model)×log n. The terms AIC(i) and BIC(i) indicate the best model among models AIC(i+1), …, AIC(i+s) and BIC(i+1), …, BIC(i+s) respectively.

Data

To identify the utility of the proposed models, the daily rainfall occurrence data during the rainy season for the period 1964-1990 for five selected stations, namely Chittagong, Mymensingh, Rajshahi, Faridpur and Satkhira of Bangladesh, were utilized. These data are collected by the Department of Meteorology, Government of People's Republic of Bangladesh. The period between the months of April and October is considered as the rainy season. The major agricultural crops (Aus and Aman rice) under the traditional system of this country, Bangladesh, are produced during this period and depend greatly on the occurrences of rainfall due to the shortage of sufficient irrigation facilities.

Logistic Regression Models for Transition Probabilities of Markov Chain for the Occurrence of Rainfall

A comprehensive idea regarding the probability of rainfall is essential in view of economic implications for crop production. The probabilities for the occurrences of rainfall are used in agricultural planning purposes, such as land-use, choice of crops and cropping system. Several researchers (Virmani, 1975; 1982; Dale, et al., 1981; Davy, et al., 1976) analyzed the occurrences of rainfall to identify the determinant of rainfall occurrences. They found that the occurrences of rainfall depend mainly on different climatic factors, such as temperature and humidity. Further, Shimizu (1993) developed a bivariate mixed lognormal distribution for assessing the probability of rainfall by using the Automated Meteorological Data Acquisition System (AMeDAS) data set of Japan.

In order to develop covariate dependent transition probabilities of a Markov chain model for assessing and analyzing the occurrences of rainfall for the five selected areas of Bangladesh, consider probability models 2.1 and 2.2. The climatic variables temperature and humidity (Virmani, 1975, 1982) are employed to perform these models. For these variables, the term $Z_{ij....kr1}(q,t)$ for model 2.1 may be defined as:

$$Z_{1(ij...kr1)}(q,t) = \alpha_{1(ij..kr1)} + \beta_{1(ij..kr1)} X_{1(ij..kr1)q(t-r)}, \quad (5.1)$$

$$Z_{2(ij...kr1)}(q,t) = \alpha_{2(ij..kr1)} + \beta_{2(ij..kr1)} X_{2(ij..kr1)q(t-r)}, \quad (5.2)$$

$$Z_{3(ij...kr1)}(q,t) = \alpha_{3(ij..kr1)} + \beta_{1(ij..kr1)} X_{1(ij..kr1)q(t-r)} + \beta_{2(ij..kr1)} X_{2(ij..kr1)q(t-r)}, \quad (5.3)$$

$$Z_{4(ij...kr1)}(q,t) = \alpha_{4(ij..kr1)} + \beta_{1(ij..kr1)} X_{1(ij..kr1)q(t-r)} + \beta_{2(ij..kr1)} X_{2(ij..kr1)q(t-r)} + \beta_{12(ij..kr1)} X_{1(ij..kr1)q(t-r)} X_{2(ij..kr1)q(t-r)}, \quad (5.4)$$

for all i, j, .., k, r = 0, 1 , q = 1, 2, .., Q, t = 1, 2, …, T and r is the order of Markov model. Here i, j, …, k, r represent the transitions of the Markov model and q and t indicate the number of year and the number of days in the year respectively. The term (q, t) represents the t^{th} day of the q^{th} year. The variables $X_{1(ij..kr1)q(t-r)}$ and $X_{2(ij..kr1)q(t-r)}$ indicate the maximum temperature and average humidity respectively of the $(t-r)^{th}$ day for the q^{th} year for $(i, j, .., k, r, 1)^{th}$ transitions. The terms $\beta_{1(ij..kr1)}$ and $\beta_{2(ij..kr1)}$ indicate the effects of temperature and humidity respectively, $\beta_{12(ij..kr1)}$ indicates the interaction effect between temperature and humidity on the occurrences of rainfall and the terms 1 and 0 indicate wet and dry days respectively.

To test the significance of probability models 5.1-5.4 and their estimated parameters, the likelihood ratio test statistic is utilized. For performing this test statistic, the following four null hypotheses are considered.

a. For models (5.1) and (5.2):

$$H_0: \beta_{1(j1)} = 0 \text{ and}$$
$$H_0: \beta_{1(ij1)} = 0$$
$$\text{vs.}$$
$$H_1: \beta_{1(j1)} \neq 0 \text{ and}$$
$$H_1: \beta_{1(ij1)} \neq 0.$$

b. For model (5.3):

$$H_0: \beta_{1(j1)} = \beta_{2(j1)} = 0 \text{ and}$$
$$H_0: \beta_{1(ij1)} = \beta_{2(ij1)} = 0$$
$$\text{vs.}$$
$$H_1: \beta_{1(j1)} = \beta_{2(j1)} \neq 0 \text{ and}$$
$$H_1: \beta_{1(ij1)} = \beta_{2(ij1)} \neq 0.$$

c. For model (5.4):

$$H_0: \beta_{1(j1)} = \beta_{2(j1)} = \beta_{12(j1)} = 0 \text{ and}$$
$$H_0: \beta_{1(ij1)} = \beta_{2(ij1)} = \beta_{12(ij1)} = 0$$
$$\text{vs.}$$
$$H_1: \beta_{1(j1)} = \beta_{2(j1)} = \beta_{12(j1)} \neq 0 \text{ and}$$
$$H_1: \beta_{1(ij1)} = \beta_{2(ij1)} = \beta_{12(ij1)} \neq 0.$$

d. For models (5.1-5.4):

$$H_0: \alpha_{j1} = 0 \text{ and}$$
$$H_0: \alpha_{ij1} = 0$$
$$\text{vs.}$$
$$H_1: \alpha_{j1} \neq 0 \text{ and}$$
$$H_1: \alpha_{ij1} \neq 0.$$

e. For models (5.3) and (5.4) respectively:

$$H_0: \beta_{m(j1)} = 0 \text{ and}$$
$$H_0: \beta_{m(ij1)} = 0$$
$$\text{vs.}$$
$$H_1: \beta_{m(j1)} \neq 0 \text{ and}$$
$$H_1: \beta_{m(ij1)} \neq 0,$$

where m = 1, 2.

f. For model (5.3):

$$H_0: \beta_{12(j1)} = 0 \text{ and}$$
$$H_0: \beta_{12(ij1)} = 0$$
$$\text{vs.}$$
$$H_1: \beta_{12(j1)} \neq 0 \text{ and}$$
$$H_1: \beta_{12(ij1)} \neq 0.$$

To test the significance of transition probabilities for the occurrences of rainfall for first and second order Markov models 5.1-5.4, the values of χ^2 under the LR criterion for null hypotheses (a), (b) and (c) are identified. Further, to test the significance of parameters for transition probabilities of first and second order Markov models (5.1-5.4), the values of χ^2 under LR criterion for null hypotheses (d), (e) and (f) are also identified (the values of χ^2 for these null hypotheses are not shown herein, however). But based on these χ^2-values, the significance of parameters and models are identified. To test the null hypothesis by the χ^2 statistic, it is always observed that the value of χ^2 increases as sample size increases. For overcoming this problem, although it is small, consider a p-value up to 0.001 as the cut-off point.

Results

Significance of Estimated Parameters and Models

To estimate the parameters of models 5.1-5.4, consider the ML estimation method under the Newton-Raphson iteration procedure. To identify the order of the transition probabilities of a Markov chain for daily rainfall occurrences Sinha (1997) and Sinha, et al. (2009) showed that the Chittagong and Faridpur stations follow first order and the Mymensingh, Rajshahi and Satkhira stations follow second order transition probabilities of a Markov chain. To estimate the parameters of these models, consider t = 214, Q = 27 and r = 1 for the Chittagong and Faridpur stations and r = 1 and 2 for the remaining three stations. For transition probabilities of daily rainfall occurrences for first order Markov models 5.1-5.4 for the five selected stations and second order Markov models 5.1-5.4 for the Mymensingh, Rajshahi and Satkhira stations of Bangladesh, the estimated values of parameters and their significance are shown in Table 1.

For the first order Markov models, the effect of temperature for model 5.1 and the effect of humidity for model 5.2 on the occurrences of transition probabilities (t.p.) of rainfall are found to be positive for the five selected stations (see Table 1). The exception to this result occurs for transition type Wet/Wet for model 5.1 for all selected stations and for model 5.2 for the Chittagong and Rajshahi stations.

The effect of humidity and temperature for model 5.3, and the effect of humidity and the interaction term between temperature and humidity for model 5.4 are also positive on the occurrences of t.p. of rainfall for all the selected stations. The exceptions to this result occur for transition type Wet/Wet for the Rajshahi, Faridpur and Satkhira stations and for all transitions of Chittagong station for temperature for model 5.3. Such an exceptional result is also observed for transition type Wet/Wet for the Chittagong station for humidity and for transition types Wet/Dry for the Chittagong station and Wet/Wet for the Satkhira station for the interaction term for model 5.4.

The positive effect of temperature and humidity and their interaction effect for the occurrences of rainfall transitions indicate that the probability of the occurrence of rainfall increases with increases of these variables for two consecutive days. The result for model 5.4 implies that the effect of temperature and humidity and their interaction effect on the occurrences of rainfall are inversely related.

The effect of temperature and of humidity on the occurrences of rainfall for different types of transition probabilities of first order Markov models 5.1-5.4 are significant (p-value < 0.001) for the five selected stations. To assess the probability of rainfall occurrences for first order Markov models, the results of χ_{ij}^2 indicate that all transitions for model 5.1 are significant at the Chittagong, Rajshahi and Satkhira stations, for model 5.2 are significant at the Chittagong and Mymensingh stations, and for models 5.3 and 5.4 are significant at all selected stations. For the overall transition probability of rainfall occurrences, the χ^2 value indicates that the first order Markov models 5.1-5.4 are significant for all selected stations.

For second order Markov models, results show in Table 1 indicate that the effect of temperature for model 5.1 and the effect of humidity for model 5.2 are positive on the occurrences of transition probabilities of rainfall for the Mymensingh, Rajshahi and Satkhira stations. This result is an exception for transition type Wet/Dry/Wet for the Mymensingh and Rajshahi stations and Wet/Wet/Wet for the Satkhira station for model 5.1. Further, the effect of temperature and humidity for model 5.3, and the effect of humidity and interaction term (temperature and humidity) for model 5.4 are observed to be positive on the occurrences of t.p. of rainfall for these three stations. However, for the Rajshahi station is an exception for transition types Wet/Dry/Wet and Wet/Wet/Dry for temperature for model 5.3. This exceptional result is also found for model 5.4 for transition type Wet/Dry/Dry at the Satkhira station for the interaction term and for transition types Wet/Dry/Wet, Wet/Wet/Dry and Wet/Wet/Wet at Rajshahi, and Wet/Wet/Wet at Satkhira for humidity. This positive effect of temperature, humidity and their interaction effect for the occurrences of rainfall transitions imply that the probability of rainfall increases with increases of these variables for three consecutive days.

For different types of second order transition probabilities of Markov models, Table 1 shows that the effect of temperature for model 5.1, the effect of humidity for model 5.2, the effect of temperature and humidity for model 5.3, and the effect of temperature and humidity and their interaction effect for model 5.4 are nonsignificant (p-value < 0.001) on the occurrences of rainfall for the maximum number of transitions for the Mymensingh, Rajshahi and Satkhira stations.

Further, to assess the probability of rainfall occurrences for second order Markov models, the results of χ^2_{ijk} indicate that all transitions for models 5.3-5.4 are significant for these stations. The exceptions to this result occur for transition types Wet/Dry/Wet and Wet/Wet/Dry for the Mymensingh station and Wet/Dry/Wet and Wet/Wet/Wet for the Rajshahi station for model 5.3, and transition type Wet/Dry/Wet for the Mymensingh station for model 5.4. However, for overall transition probability of rainfall occurrences, the values of

Table 1: Estimated Parameters for Logistic Regression for Transition Probabilities of First and Second Order Markov Models 5.1-5.4 and their Significance for Five Selected Areas of Bangladesh

Station Name	Transition Type	Model 5.2		Model 5.2		Model 5.3			Model 5.4			
		$\alpha_{1(j1)}$	$\beta_{1(j1)}$	$\alpha_{2(j1)}$	$\beta_{2(j1)}$	$\alpha_{3(j1)}$	$\beta_{1(j1)}$	$\beta_{2(j1)}$	$\alpha_{4(ij1)}$	$\beta_{1(ij1)}$	$\beta_{2(ij1)}$	$\beta_{12(ij1)}$
Chittagong	0 → 1	-1.66321*	0.02460*	-15.80485*	0.18630*	-13.87005*	-0.05096*	0.18216*	-34.61039*	0.61502	0.44442*	-0.00842
		(0.20331)	(0.00649)	(0.93470)	(0.01155)	(1.2856)	(0.01173)	(0.01161)	(8.95026)	(0.28473)	(0.11991)	(0.00360)
	1 → 1	8.55298*	-0.24899*	-17.11358*	-0.21235*	-10.07113	- 0.07826*	0.15733*	25.11332	-1.22477*	- 0.24801	0.01323*
		(0.74237)	(0.02383)	(0.83400)	(0.00995)	(1.07783)	(0.01855)	(0.00924)	(7.91970)	(0.25361)	(0.08778)	(0.00282)
Mymeningh	0 → 1	-1.80030*	0.02794*	-4.91382*	0.05003*	-5.91503*	0.02303*	0.05448*	- 4.21334*	-0.04123	0.03434*	0.00077*
		(0.10932)	(0.00360)	(0.33400)	(0.00421)	(0.41529)	(0.00405)	(0.00466)	(0.47261)	(0.01928)	(0.00586)	(0.00024)
	1 → 1	0.85874*	-0.00711	-7.71641*	0.09883*	-7.30683*	0.00455*	0.09249*	0.08709*	-0.26779*	0.00564	0.00321*
		(0.15992)	(0.00540)	(0.57352)	(0.00678)	(0.61389)	(0.00551)	(0.00674)	(1.15585)	(0.04208)	(0.01344)	(0.00049)
	0→0→1	-2.27199*	0.03130*	-3.00491*	0.02145*	-3.48158*	0.02376*	0.01915*	- 3.15954*	-0.01040	0.01471	0.00045
		(0.13929)	(0.00455)	(0.24970)	(0.00321)	(0.30151)	(0.00473)	(0.00365)	(0.31601)	(0.01664)	(0.00435)	(0.00022)
	1→0→1	-0.11962	-0.00206	-2.31835	0.02603	-2.43567	0.00243	0.02661	-1.09384	-0.02267	0.01102*	0.00028
		(0.23879)	(0.00797)	(0.79070)	(0.00957)	(0.88411)	(0.00818)	(0.00977)	(2.88316)	(0.09513)	(0.03384)	(0.00112)
	0→1→1	0.34499	0.00071	-2.04344*	0.02968	-2.11570	0.00195	0.02987	-0.57187	-0.02340	0.01103	0.00031
		(0.22895)	(0.00753)	(0.65770)	(0.00805)	(0.72134)	(0.00774)	(0.00813)	(0.96734)	(0.03652)	(0.01183)	(0.00045)
	1→1→1	0.70411*	0.00401	-1.30477	0.02447	-1.56163	0.00643	0.02534	-0.51836	-0.04736	0.01326	0.00063
		(0.19879)	(0.00680)	(0.67893)	(0.00782)	(0.73602)	(0.00682)	(0.00792)	(1.28677)	(0.04874)	(0.01487)	(0.00057)
Rajshahi	0 → 1	-2.61113*	0.05089*	-9.73000*	0.11039*	-10.82544*	0.03616*	0.11045*	-5.35333*	-0.19978*	0.03782*	0.00307*
		(0.12574)	(0.00392)	(0.47914)	(0.00591)	(0.54733)	(0.00539)	(0.01958)	(0.59776)	(0.02632)	(0.00838)	(0.00035)
	1 → 1	1.43630*	-0.03344*	-12.59192*	-0.15132	-9.44802*	-0.03370	0.12696*	0.07798	-0.50250*	0.00351	0.00586*
		(0.26954)	(0.00860)	(0.78959)	(0.00917)	(0.82519)	(0.00971)	(0.00274)	(0.60304)	(0.03493)	(0.00770)	(0.00013)
	0→0→1	-2.91907*	0.05491*	-5.11202*	0.05136*	-5.80392*	0.04121*	0.04480*	- 4.10119*	-0.04681	0.01762	0.00130*
		(0.14711)	(0.00457)	(0.33424)	(0.00436)	(0.39331)	(0.00521)	(0.00478)	(0.41056)	(0.01853)	(0.00647)	(0.00027)
	1→0→1	-0.13410	-0.01329	-2.97782*	0.02952	-2.50378	-0.01697	0.030175	-0.36481	-0.14755	-0.01027	0.00176
		(0.34177)	(0.01076)	(0.88895)	(0.01073)	(0.91911)	(0.01123)	(0.01055)	(1.27667)	(0.05058)	(0.01720)	(0.00067)
	0→1→1	0.01713	0.00408	-3.04452*	0.03864	-2.96257*	-0.00344	0.03896*	0.24531	-0.15739*	-0.00762	0.00210*
		(0.29250)	(0.00120)	(0.75410)	(0.00908)	(0.78487)	(0.00964)	(0.00909)	(0.93122)	(0.03921)	(0.01285)	(0.00016)
	1→1→1	0.31405	0.00934	-0.90133	0.01694	-1.04218	0.00606	0.01647	1.15412	-0.12338*	-0.01350	0.00163*
		(0.29781)	(0.00974)	(0.64385)	(0.00726)	(0.69783)	(0.01003)	(0.00743)	(0.81518)	(0.04037)	(0.00971)	(0.00048)
Faridpur	0 → 1	-1.49605*	0.02027*	-7.23936*	0.08018	-7.58175*	0.00712	0.08165*	-1.94471*	-0.34008*	0.00022	0.00465*
		(0.13717)	(0.00433)	(0.45168)	(0.00554)	(0.55765)	(0.0064)	(0.00578)	(0.20908)	(0.02180)	(0.00492)	(0.00031)
	1 → 1	1.21035*	-0.02112	-12.9266*	0.15741*	-0.74976*	-0.15087*	0.07049*	0.07996	-0.45381*	0.00639	0.00529*
		(0.23697)	(0.00771)	(0.76910)	(0.00897)	(0.28198)	(0.01329)	(0.00490)	(0.26813)	(0.02845)	(0.00515)	(0.00038)
Satkhira	0 → 1	-2.99935*	0.05913*	-7.26723*	0.08084	-7.63173*	0.01052	0.08114*	-6.16671*	-0.04065	0.06227*	0.00066
		(0.15545)	(0.00471)	(0.40541)	(0.00510)	(0.48543)	(0.00698)	(0.00164)	(0.71258)	(0.02520)	(0.01010)	(0.00035)
	1 → 1	4.06126*	-0.10761*	-11.13848*	0.14027*	-9.66700*	-0.01306*	0.12768*	-13.75171*	0.11648	0.17769*	-0.00159
		(0.56772)	(0.01774)	(0.66308)	(0.00789)	(0.83070)	(0.01140)	(0.00790)	(4.44781)	(0.13820)	(0.05397)	(0.00534)
	0→0→1	-3.26893*	0.05882*	-4.47600*	0.04262	-4.64660*	0.01328	0.03926*	- 4.92094*	0.02978	0.04603*	-0.00031
		(0.17891)	(0.00540)	(0.27989)	(0.00369)	(0.30980)	(0.00677)	(0.00419)	(0.42540)	(0.01857)	(0.00689)	(0.00028)
	1→0→1	-1.22253	0.02539	-3.32119*	0.03614	-5.25520*	0.04439	0.04233*	-2.45716	-0.04177	0.00693	0.00109
		(0.51918)	(0.01589)	(0.80828)	(0.00994)	(1.21875)	(0.02007)	(0.01051)	(4.97238)	(0.15117)	(0.06278)	(0.00193)
	0→1→1	0.13733	0.00460	-2.13771*	0.03007	-2.46701	0.00832	0.03080*	-0.53421	-0.07461	0.00277	0.00116
		(0.43185)	(0.00135)	(0.71428)	(0.00881)	(0.91527)	(0.01394)	(0.00901)	(1.12239)	(0.04103)	(0.01571)	(0.00058)
	1→1→1	1.44444*	-0.01870	-3.85287*	0.05447	-4.38068*	0.01034	0.05688*	4.64829	-0.25704	-0.04568	0.00304
		(0.46885)	(0.00476)	(0.82413)	(0.00952)	(1.09066)	(0.01396)	(0.01007)	(5.81100)	(0.18482)	(0.06665)	(0.00213)

Notes: The figure in parentheses indicates the standard deviation of estimated parameters. The transitions 0→1 and 1→1 indicates the transition of the type dry to wet and wet to wet respectively. The transitions 0→0→1, 1→0→1, 0→1→1 and 1→1→1 indicate the transition of the type dry to dry to wet, wet to dry to wet, dry to wet to wet and wet to wet to wet respectively. $^*p < 0.001$.

Table 2: Values of AIC and BIC for First and Second Order Transition Probabilities of Markov Models 5.1-5.4 for Five Selected Stations of Bangladesh

Station Name	Test Criteria	Order of Markov Models	Transition Types	Model 5.1	Model 5.2	Model 5.3	Model 5.4
Chittagong	AIC	First Order	0→1	3496.83	3085.17	3067.95	3064.85
			1→1	3286.95	2979.59	2936.83	2916.97
	BIC	First Order	0→1	3498.67	3087.01	3070.71	3068.53
			1→1	3289.56	2982.20	2940.74	2922.19
Mymensingh	AIC	First Order	0→1	3700.92	3511.57	3477.76	3464.93
			1→1	3205.04	2998.41	2984.48	2957.16
		Second Order	0→0→1	2326.60	2321.67	2295.10	2291.80
			1→0→1	1178.27	1172.80	1173.51	1175.33
			0→1→1	1157.12	1161.34	1154.29	1146.28
			1→1→1	2027.04	2017.16	2018.28	2018.41
	BIC	First Order	0→1	3702.78	3513.43	3480.55	3468.65
			1→1	3210.70	3000.85	2988.14	2962.04
		Second Order	0→0→1	2327.94	2323.01	2297.11	2294.48
			1→0→1	1179.45	1173.98	1175.28	1177.69
			0→1→1	1158.52	1162.74	1156.45	1149.08
			1→1→1	2029.16	2019.28	2021.46	2022.65
Rajshahi	AIC	First Order	0→1	3728.45	3229.80	3176.66	3114.17
			1→1	2839.46	2529.84	2528.50	2384.42
		Second Order	0→0→1	2539.36	2498.56	2419.86	2399.60
			1→0→1	1114.18	1107.21	1106.97	1102.27
			0→1→1	1173.59	1152.07	1153.94	1140.69
			1→1→1	1661.40	1656.43	1658.07	1643.82
	BIC	First Order	0→1	3730.31	3231.66	3179.45	3117.89
			1→1	2841.66	2531.96	2531.80	2388.82
		Second Order	0→0→1	2540.82	2500.02	2422.05	2402.52
			1→0→1	1115.16	1108.19	1108.44	1104.23
			0→1→1	1174.89	1153.37	1155.89	1143.29
			1→1→1	1663.22	1658.25	1660.80	1647.46
Faridpur	AIC	First Order	0→1	3838.85	3493.74	3494.47	3322.31
			1→1	3365.90	3147.79	3115.20	2944.74
	BIC	First Order	0→1	3840.79	3495.68	3497.38	3326.19
			1→1	3368.34	3150.23	3118.86	2949.62
Satkhira	AIC	First Order	0→1	3449.47	3044.67	3044.27	3044.16
			1→1	2829.18	2525.72	2508.95	2510.00
		Second Order	0→0→1	2312.46	2214.23	2212.15	2213.98
			1→0→1	1027.00	1016.03	1011.64	1013.33
			0→1→1	1051.62	1039.12	1038.84	1037.48
			1→1→1	1796.37	1764.78	1766.25	1768.22
	BIC	First Order	0→1	3451.23	3046.43	3046.91	3047.68
			1→1	2831.52	2528.06	2512.46	2514.67
		Second Order	0→0→1	2313.78	2215.55	2214.13	2216.62
			1→0→1	1027.96	1016.99	1013.08	1015.25
			0→1→1	1052.90	1040.40	1040.76	1040.04
			1→1→1	1798.41	1766.82	1766.29	1772.30

Notes: For Table 2.7 the transitions 0→1 and 1→1 indicates the transition of the type dry to wet and wet to wet respectively. The transitions 0→0→1, 1→0→1, 0→1→1 and 1→1→1 indicate the transition of the type dry to dry to wet, wet to dry to wet, dry to wet to wet and wet to wet to wet respectively.

χ^2 indicate that the second order Markov models 5.1-5.4 are significant for all the selected stations.

Probability Model for Forecasting Rainfall

To select a forecasting model among the models for the occurrences of rainfall, AIC and BIC criteria were utilized. The values of AIC and BIC for covariate dependent transition probabilities of Markov models 5.1-5.4 for the occurrences of rainfall are shown in Table 2.

Table 2 indicates that the values of AIC and BIC are minimum for different types of transition probabilities of rainfall occurrences for first order Markov model 5.4 for all the selected stations. However, the effect of temperature and humidity for this model are not sufficiently effective Table 2.1 in explaining all the transition probabilities of rainfall occurrences for all stations. Therefore, results lack strong grounds to select this model as an appropriate forecasting model for daily rainfall occurrences.

To identify this model, consider next minimum value to the values of model 5.4 for these criteria. Table 2 shows that the values of AIC and BIC for all transition probabilities of first order Markov model 5.3 are the next minimum values to the values of model 5.4; therefore, the transition probabilities of first order Markov model 5.3 may be considered an appropriate forecasting model for daily rainfall occurrences for all selected stations. Although the effect of temperature for transition Wet/Wet for the Rajshahi station and Wet/Dry for the Faridpur and Satkhira stations is non-effective, overall transitions this effect are significant.

For second order transition probabilities of the Markov model, Table 2 shows that the values of AIC and BIC for models 5.3 and 5.4 for Mymensingh and Satkhira stations are approximately equal and these values are observed minimum compared to values of models 5.1 and 5.2. For the Rajshahi station these values are observed minimum for model 5.4 compared to models 5.1, 5.2 and 5.3. However, the effect of temperature and humidity for model 5.4 is not significant (Table 1) for maximum number of transitions. Therefore, this model is not selected as an appropriate forecasting model for daily rainfall occurrences for the Rajshahi station. To select this model, consider next minimum values of these criteria rather than values of model 5.4. Table 2 indicates that the values of AIC and BIC for model 5.3 provide next minimum values compared to the values of model 5.4. Therefore, the transition probabilities of second order Markov model 5.3 may be selected for forecasting the occurrences of rainfall for the Mymensingh, Rajshahi and Satkhira stations. However, Table 1 shows that the effect of temperature and humidity for the transition probabilities of rainfall for first order Markov model 5.3 are significantly effective and for second order Markov model 5.3, but these effects are not sufficiently effective. Based on this logical view, it may be concluded that the transition probabilities of first order Markov model 5.3 make it an adequate choice for forecasting the occurrences of rainfall than that of second order Markov model 5.3 for all the selected stations of Bangladesh.

Conclusion

Logistic regression models for higher order transition probabilities of Markov chains for the sequence of chain dependent repeated observations have been developed. An assumption is made that the sequence of repeated observations can be explained by certain covariates. These models are developed as an extension of the model proposed by Muenz and Rubinstein (1985). To identify the significance of covariate dependence in transition probabilities for higher order Markov models and also to identify the significance of parameters of these models, a test procedure under likelihood ratio criterion has been developed. Further, a method of model selection procedure is suggested in this study employing AIC and BIC procedures (Sakamoto; 1991).

The proposed models and test procedures have been used to analyze the occurrences of daily rainfall data for selected stations in Bangladesh. To apply these models, two climatic variables - temperature and humidity - were considered. These applications reveal that the proposed models and test procedures can be useful to identify the forecasting models for daily rainfall occurrences. From the results of these models and test procedures, the effects of temperature and humidity on the occurrences of rainfall can be summarized for first order the Markov model 5.3 that provides statistically significant results.

From the analysis of models 5.1-5.4, positive results were observed for the effect of temperature for model 5.1 and the effect of humidity for model 5.2 on the occurrences of rainfall for maximum number of first and second order rainfall transitions of Markov models for all the selected stations. The effects of temperature and humidity for first and second order Markov models 5.3 on the occurrences of rainfall show similar results.

The first and second order Markov models 5.4 also provide positive effects for the humidity and interaction term (temperature and humidity) on the occurrences of rainfall for maximum number of rainfall transitions for all selected stations. These positive effects indicate that the probability of rainfall is positively associated with temperature and humidity. The effect of temperature and the effect of humidity on the occurrences of rainfall for first order Markov models 5.1 and 5.2 respectively, and the effect of these covariates for model 5.3 are observed to be significant for the maximum number of transitions for all selected stations. It is also demonstrated that the method of model selection procedure provides sufficient evidence that the first order Markov model 5.3 is the most suitable among the models investigated as the forecasting model for daily rainfall occurrences for the five selected stations of Bangladesh.

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