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Identifying Outliers in Fuzzy Time Series

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Time series analysis is often associated with the discovery of patterns and prediction of features. Forecasting accuracy can be improved by removing identified outliers in the data set using the Cook's distance and Studentized residual test. In this paper a modified fuzzy time series method is proposed based on transition probability vector membership function. It is experimentally shown that the proposed method minimizes the average forecasting error compared with other known existing methods.

Key words: Membership functions, fuzzy sets, fuzzy logical relations, outliers, Cook's distance, average forecasting error.

Introduction

Time series analysis plays a vital role in most actuarial related problems. Fuzzy time series is a scientific method that can be applied to time series data and in forecasting future events. Commonly actuarial issues are mainly related to the concept of uncertainty, each observation of a fuzzy time series is assumed to be a fuzzy variable along with an associated membership function. The accuracy of fuzzy time series plays a significant role in forecasting. Conventional methods that deal with forecasting problems show their inefficiency when solving problems related to linguistic values.

Several approaches in the literature have been developed to identify outliers in time series analysis. Fox (1972) introduced the concept of outliers in time series analysis and discussed different types of time series outliers. Tsaur (1986) used an iterative fashion to detect multiple outliers.

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A complete survey and discussion regarding outlier detection can be found in Barnett and Lewis, (1984). The Studentized residual analysis method and Cook's distance can be used to detect outliers in time series. According to Barnett and Lewis (1984), the identified outliers can be either accommodated or removed. Chang and Tiao (1988) discussed estimation of time series parameters in the presence of outliers.

Song and Chissom (1993) introduced definitions of fuzzy time series and its modeling by using fuzzy relational equations and approximate reasoning by Zadeh (1965). Song and Chissom (1993) outlined modeling procedures and implemented time-invariant and time-variant models to forecast enrollments at the University of Alabama. Sullivan and Woodall (1994) reviewed the first-order time-variant fuzzy time series model and the first-order time-invariant fuzzy time series model presented. Chen (1996) developed a basic or simplified method for time series forecasting using arithmetic operations rather than complicated max-min composition operations. Sullivan and Woodall (1999) have discussed three methods for estimating Markov transition matrices when observed state probabilities are not all either zeros or ones and a simulation-based comparison of the performance of the estimators. Huarng (2001) worked on finding the effective lengths of intervals to improve forecasting accuracy. Chen (2002) developed a fuzzy time series using arithmetic operations.

Song (2003) has proposed the sample autocorrelation functions of fuzzy time series and used in model selection. The main idea is to select a number of different data sets from each fuzzy set and calculate the sample auto correlation function for each data set. Chung and Hsu (2004) proposed a higher order fuzzy time series applied for Taiwan future exchange. Lee et al., (2004) have presented an improved method to forecast university enrollments based on the fuzzy time series. The method proposed not only defines the supports of the fuzzy numbers that represent the linguistic values of the linguistic variable more appropriately, but also makes the RMSE smaller Sah et al., (2005) presented the method for forecasting given high accuracy and comparing existing methods. Tsaur et al., (2005) have proposed fuzzy relation matrix affecting the forecasting performance and proposed an arithmetic procedure for deriving fuzzy relation matrix method using Fuzzy relation analysis in fuzzy time series. Fuzzy relation is a crucial connector in presenting fuzzy time series model. Also the concept of entropy is applied to measure the degrees of fuzziness when a time invariant matrix is derived. Singh (2007) proposed a method for fuzzy time series forecasting using a simple time variant method. Hao-Tien Liu (2007) has proposed improved time-variant fuzzy time series method. The proposed method takes into consideration of Window base, length of interval, degrees of membership values, and existence of outliers. The improved method provides decision makers with more precise forecasted values.

Fuzzy Time Series

Song and Chissom (1993) proposed a procedure for solving fuzzy time series models described as follows: Let Ube the universe of discourse,

$$U = [V_{\min} - V_1, V_{\max} + V_2],$$

where $U = \{u_1, u_2 ... u_n\}$ is the given historical data, the minimum data is V_{min}, the maximum data is V_{max} and V_1, V_2 are two real numbers. A fuzzy set A_i of U is defined by

$$A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + ... + f_{Ai}(u_n)/u_n$$

where f_A is the membership function of fuzzy set A_i. Let $Y(t), (t = 0, 1, 2...)$ be a subset of R. If $Y(t)$ is the universe of interest defined by the fuzzy set $\mu_i(t), i = 1, \ 2, ...$ then F(t) is called a fuzzy time series of $Y(t)$. If there exists a fuzzy relationship $R\ (t, t-1)$, such that $F(t) - F(t-1) * R(t, t-1)$, where the symbol * is an operator, then $F(t)$ is said to be induced by $F(t-1)$ the relationship can be denoted by $F(t-1) \rightarrow F(t)$. Suppose $F\ (t-1)$ by A_i and $F\ (t)$ by A_j fuzzy logical relationship can be defined by $A_i \rightarrow A_i$ where A_i and A_j are called, respectively, the left hand side and right hand side of the fuzzy logical relationship.

Detection of Outliers

Outlier defines an observation that is numerically distant from the rest of the data, or is any observation in a set of data that is inconsistent with the remainder of the observations in the data set. The outlier is inconsistent in the sense that it is not indicative of possible future behavior of the data sets. Cook's Distance (D_i) defines how much an observation affects a change in a parameter estimate of least square regression analysis:

$$D_i = \frac{e_i^2}{p * MSE}\left(\frac{h_{ii}}{(1-h_{ii})^2}\right).$$

To interpret D_i, compare it to the F-distribution with (p, n–p) degrees of freedom to determine the corresponding percentile; if the percentile value is greater than 50%, then the observation has a major influence on the fitted values and should be examined. Thus, if D_i>F(0.5, p, n–p) then consider influence.

The Studentized residual analysis methods can assist in determining whether outliers exist in historical data. The Studentized test can be employed to examine the outliers as follows: If there are n historical data x_1, x2,…,x_n a square matrixR can be defined as,

$$R = X\left(X^T X\right)^{-1} X^T$$

$$X = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ . & . \\ . & \\ . & . \\ 1 & X_n \end{bmatrix}$$

The Studentized residual can be defined by the Studentized Residual Test:

$$\frac{e_i}{S_j},$$

where

$$S_j = \hat{\sigma}(i)\sqrt{1-r_{ii}}.$$

Here, S_j is the estimated variance of the residual, e_i specifies the residual of the i^{th} datum, $\hat{\sigma}(i)$ is the estimated value of the standard deviation σ without the i^{th} observation, and r_i is the i^{th} diagonal element in matrix R. The data is considered to be an outlier where the absolute residual values having Studentized residuals are greater than 2.0.

Discrete Time Markov Chain

A Markov chain is a discrete random process with the property that the next state depends only on the current state; the past states have no influence on the future.

A Markov chain X is said to be time-homogenous if the conditional probability $P[X_{n+1} = j \mid X_n = i] = P_{ij}$, $i, j \in S$ is independent of *n*, and *S* is the countable state space. The probabilities of P_{ij} are called the transition probabilities for the Markov chain *X*. It is customary to arrange the P_{ij} or $P(i,j) = P_{ij}$ into a square array and to call the resulting matrix $P=(P_{ij})$ the transition probability matrix of the Markov chain *X*; for any i, $j \in S$, $P_{ij} \geq 0$, and $\sum_{j\in E} P_{ij} = 1$ for any $m \in N$, $P[X_{n+m} = j \mid X_n = i] = P_{ij}^m$, $i, j \in S$. Here $P_{ij}^{(m)}$ denotes the probability that the process goes from state *i* to state *j* in m transitions. The transition probabilities P_{ij} can be exhibited as a square matrix

$$P = P_{ij} = \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & \cdots \\ P_{10} & P_{11} & P_{12} & P_{13} & \cdots \\ P_{20} & P_{21} & P_{22} & P_{23} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{i0} & P_{i1} & P_{i2} & P_{i3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

which is called the transition probability matrix of the chain. If the number of states is finite, for example *n*, then there will be n rows and n columns in the matrix P; otherwise the matrix will be infinite.As it is known, $P_{ij} \geq 0$, and $\sum_{j=0}^{\infty} P_{ij} = 1$. for every $i, j = 0,1,2,...$

Modified Method of Forecasting

This article aims to provide better forecasting accuracy using fuzzy time series with forecasts using only historical data. The step by step forecasting procedure is as follows:

1. First identify outliers from the historical data using Cook's distance and the Studentized residual test.

2. After identifying the outlier, compute the appropriate length of interval *l* using the distribution based method by Chen (2002).

3. Compute the number of intervals *m* as follows:

$$m = \frac{(V_{max} + V_2) - (V_{min} - V_1)}{l}$$

   where V_{max} is the maximum value of the historical data, V_2 is the positive integer, V_{min} is the minimum value of the historical data, V_1 is the positive integer and l is the appropriate length of interval.

4. Let U be the universe of discourse, $U = [V_{min} - V_1,\ V_{max} + V_2]$ and partition

into m equal length intervals $\{u_1, u_2, u_3 \ldots u_m\}$.

5. Fuzzify the variations of the historical data and determine the fuzzy logical relationships.

6. If A_i is the fuzzified value of current year n and A_j is the fuzzified value of next year $n+1$, then fuzzy logical relation is denoted by $A_i \rightarrow A_j$.

7. Define Fuzzy sets A_i on universe of discourse U, then determined how many linguistic variables to be fuzzy sets.

8. Define the linguistic terms of A_i represented by the fuzzy sets are as follows:

$$A_1 = \{u_1/0.667, u_2/0.337, u_3/0, \ldots, u_m/0\}$$

$$A_2 = \{u_1/0.25, u_2/0.5, u_3/0.25, \ldots, u_m/0\}$$

$$A_3 = \{u_1/0, u_2/0.25, u_3/0.50, \ldots, 0, u_m/0\}$$

$$\ldots$$

$$A_m = \{u_1/0, \ldots, u_{m-1}/0.333, u_m/.0.667\}$$

9. Fuzzify the historical data are as follows: If the value belongs to u_1, then fuzzified membership into $0.667/A_1+0.333/A_2+0/A_3$ denoted by A_1.If the value belongs to u_i,i=2,3,…,n-1, then the fuzzified membership values into $0.25/A_{i-1} + 0.5/A_i + 0/A_{i+1}$ denoted by A_i. If the value belongs to u_n then the fuzzified membership values $0/A_{n-2}+0.333/A_{n-1}+0.667/A_n$ denoted by A_n.

10. Identify the fuzzy logical relationship of first order fuzzy time series is as follows: $A_{j-1} \rightarrow A_j$.

11. Determine the fuzzy logical relationship $R_i = A_{i-1}^T \times A_i$, i = 1,2,…,n and obtain the transition probability matrix is $P_m = \bigcup_{i=1}^{n} R_i$.

12. Calculate the forecast outputs using transitions state probability membership function as $P'_{t+1} = P'_t \times P_m$, where, P'_{t+1} is the current year historical data is obtained from previous year vector probability membership P'_t and probability matrix P_m .

13. Obtain the average forecasting error using actual and forecasted values:

$$Forcast\ error = \frac{|forcasted\ value - actual\ value|}{(actual\ value)} \times 100\,\%$$

Numerical Example

The proposed approach isdescribed with actual data corresponding to the number of accidentsoccurring in India. The original data set is shown in Table 1.

Table 1: Identifying Outliers Using Cook's Distance and Studentized Residual Test

Year	Number of Accidents	CooksDistance	Student Residual
1985	20700	0.001	-0.168
1986	21550	0.008	-0.415
1987	23400	0.000	0.026
1988	24670	0.000	0.067
1989	27000	0.020	0.837
1990	28260	0.019	0.870
1991	29340	0.014	0.777
1992	26030	**0.115**	**-2.597**
1993	28010	0.067	-1.890
1994	32040	0.000	0.142
1995	34890	0.037	1.292
1996	37120	0.097	2.126
1997	37370	0.048	1.345
1998	38500	0.051	1.290
1999	38640	0.010	0.524
2000	39140	0.000	0.037
2001	40560	0.002	0.182
2002	40750	0.016	-0.528
2003	40670	0.040	-1.504
2004	42990	0.035	-0.667
2005	43920	0.070	-0.882
2006	46090	0.006	-0.085
2007	47920	0.135	0.371

Table.1 shows an unusual residual value (−2.597) in 1992, which has a Studentized absolute value residual greater than 2.0. Studentized residuals measure how many standard deviations each observed value deviates from a model fitted using all of the data except that observation.In this case, there is one Studentized residual greater than 2.0, but none greater than 3.0. The step by step procedure is as follows:

1. First, the appropriate length of interval l is computed using distribution based length procedure to obtain an interval length of $l = 2000$.

2. The calculated number of intervals, $m = \frac{48000 - 20000}{2000} = 14$.

3. Define the universe of discourse or universal set, U = [20000, 48000], and partition U into 14 equal length of intervals, u_i, i=1, 2,…, 14, u_1 =[20000, 22000), u_2 =[22000, 24000), u_3 =[24000, 26000),..., u_{13} =[44000, 46000), and u_{14}= [46000, 48000].

4. It is assumed that the linguistic variable of the historical data can take fuzzy values are as follows: A_1 (very big decrease), A_2 (big decrease), A_{13} (big increase) and A_{14} (very big increase). Then, for the given intervals u_i, i = 1, 2….14, each u_i belongs to a particular A_j, j=1,2,…,14 and is expressed by the real value within the range [0,1]. The complete sets of relationship are shown in Table 2.

$A_1 \rightarrow A_1$	$A_1 \rightarrow A_2$	$A_2 \rightarrow A_3$
$A_3 \rightarrow A_4$	$A_4 \rightarrow A_5$	$A_5 \rightarrow A_5$
$A_5 \rightarrow A_4$	$A_4 \rightarrow A_5$	$A_5 \rightarrow A_7$
$A_7 \rightarrow A_8$	$A_8 \rightarrow A_9$	$A_9 \rightarrow A_9$
$A_9 \rightarrow A_{10}$	$A_{10} \rightarrow A_{10}$	$A_{10} \rightarrow A_{10}$
$A_{10} \rightarrow A_{11}$	$A_{11} \rightarrow A_{11}$	$A_{11} \rightarrow A_{11}$
$A_{11} \rightarrow A_{12}$	$A_{12} \rightarrow A_{12}$	$A_{12} \rightarrow A_{14}$
$A_{14} \rightarrow A_{14}$	$A_{13} \rightarrow A_{14}$	$A_{14} \rightarrow A_{14}$
$A_{13} \rightarrow A_{14}$		

5. The fuzzy relationships are combined into fuzzy logical relations starting from identical left-hand sides. Then R_i, i=1, 2…22 is calculated as a sum of logical relationships in each group. Here, the relation matrix R_i is converted into a transition probability matrix P_m is shown in Figure 1.

6. Table 3 illustrates the defuzzified forecast outputs using transition state probability membership function. The outputs are multiplied with corresponding mid values of the fuzzy interval over the period of years and its overall summation leads the predicted values. For example, year 2004 is forecasted using fuzzified values of 2003 .The midpoints of the intervals u_1,u_2 ,..., u_{14} are multiplied into corresponding defuzzified probability values and its overall summation. The actual and predicted value of number of accidents in India is shown in Figure 2.

7. Finally, the average forecasting error is obtained using actual and forecasted values, when compared with the other existing methods. The result is shown in Table 3.

Conclusion

This article is mainly focused on improving the forecasting accuracy by removing the identified outlier in the data set. This proposed method first predicts the fuzzy time series using transition probability vector membership functions, then, the average forecasting error is calculated based on after removing the outliers in the data. The experimental results show that the average forecasting error is 2.86% for the historical data. After removing the outlier, the method produces 2.60% of average forecasting error. Thereby, the proposed method improves average forecasting accuracy by approximately 9%.The results indicate that the proposed method is more appropriate compared to other existing methods. It is supported by numerical and graphical representations.

Figure 1: Transition Probability Matrix from Relation Matrix

$$P_m=\left[\begin{array}{cccccccccccccc}
0.39 & 0.39 & 0.18 & 0.04 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.22 & 0.28 & 0.28 & 0.18 & 0.04 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.06 & 0.25 & 0.38 & 0.25 & 0.06 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.04 & 0.25 & 0.38 & 0.21 & 0.08 & 0.04 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.18 & 0.36 & 0.25 & 0.14 & 0.07 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.13 & 0.25 & 0.19 & 0.19 & 0.19 & 0.06 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.17 & 0.42 & 0.33 & 0.08 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.05 & 0.25 & 0.40 & 0.25 & 0.05 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.09 & 0.31 & 0.38 & 0.19 & 0.03 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.02 & 0.16 & 0.34 & 0.32 & 0.14 & 0.02 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.05 & 0.20 & 0.34 & 0.25 & 0.10 & 0.06\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.07 & 0.25 & 0.29 & 0.20 & 0.19\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.08 & 0.15 & 0.31 & 0.47\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.33 & 0.67
\end{array}\right]$$

Table 3: Forecasting Number of Accidents from 1985-2007

Year	Actual	Fuzzy Output Vectors														Predicted
		A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	
1985	20700															
1986	21550	0.33	0.35	0.22	0.09	0.01	0	0	0	0	0	0	0	0	0	23202
1987	23400	0.33	0.35	0.22	0.09	0.01	0	0	0	0	0	0	0	0	0	23202
1988	24670	0.20	0.26	0.25	0.19	0.08	0.02	0	0	0	0	0	0	0	0	24488
1989	27000	0.05	0.10	0.21	0.29	0.23	0.08	0.02	0.01	0	0	0	0	0	0	26858
1990	28260	0	0.02	0.08	0.26	0.34	0.18	0.08	0.04	0	0	0	0	0	0	28952
1991	29340	0	0	0.01	0.18	0.33	0.22	0.14	0.09	0.02	0	0	0	0	0	30280
1993	28010	0	0	0.01	0.18	0.33	0.22	0.14	0.09	0.02	0	0	0	0	0	30280
1994	32040	0	0	0.01	0.18	0.33	0.22	0.14	0.09	0.02	0	0	0	0	0	30280
1995	34890	0	0	0	0.03	0.06	0.05	0.14	0.32	0.28	0.10	0.01	0	0	0	34958
1996	37120	0	0	0	0	0	0	0.07	0.25	0.36	0.24	0.07	0.01	0	0	37042
1997	37370	0	0	0	0	0	0	0.01	0.12	0.30	0.34	0.19	0.05	0.01	0	38477
1998	38500	0	0	0	0	0	0	0.01	0.12	0.30	0.34	0.19	0.05	0.01	0	38477
1999	38640	0	0	0	0	0	0	0	0.03	0.17	0.32	0.29	0.14	0.04	0.02	39996
2000	39140	0	0	0	0	0	0	0	0.03	0.17	0.32	0.29	0.14	0.04	0.02	39996
2001	40560	0	0	0	0	0	0	0	0.03	0.17	0.32	0.29	0.14	0.04	0.02	39996
2002	40750	0	0	0	0	0	0	0	0.01	0.06	0.21	0.31	0.23	0.11	0.08	41656
2003	40670	0	0	0	0	0	0	0	0.01	0.06	0.21	0.31	0.23	0.11	0.08	41656
2004	42990	0	0	0	0	0	0	0	0.01	0.06	0.21	0.31	0.23	0.11	0.08	41656
2005	43920	0	0	0	0	0	0	0	0	0.01	0.09	0.23	0.24	0.20	0.23	43441
2006	46090	0	0	0	0	0	0	0	0	0.01	0.09	0.23	0.24	0.20	0.23	43441
2007	47920	0	0	0	0	0	0	0	0	0	0	0.02	0.05	0.32	0.60	46001

Figure 2: Actual and Predicted Values of Accidents in India

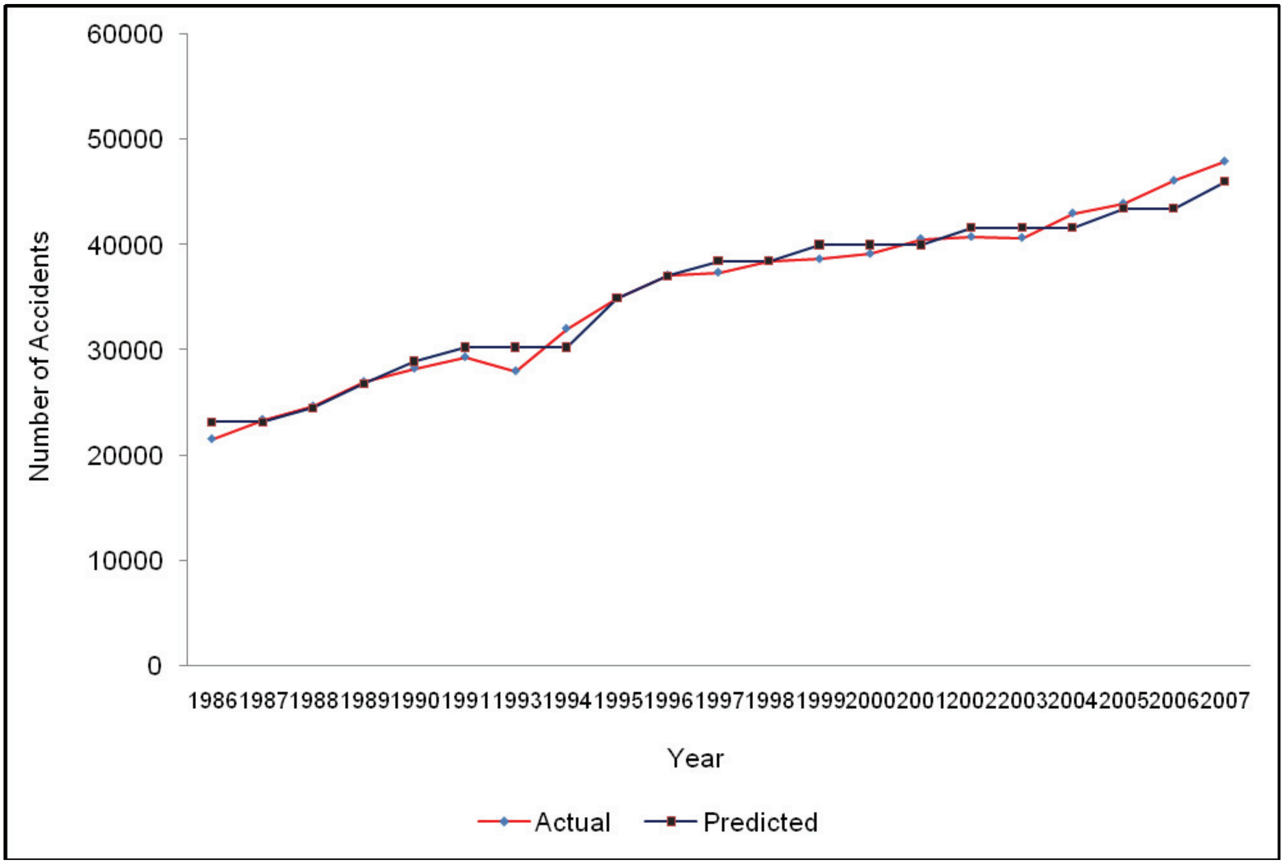

Table 4: Comparison of Average Forecasting Error with Existing Methods

Methods	Chen&Hwang (2000)	Lee, et.al., (2004)	Singh(2007)	Proposed
AFE	3.90%	3.43%	2.89%	2.60 %

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