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An Extension of the Seasonal KPSS Test

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The limit theory of the seasonal KPSS test is established under the null hypothesis using seasonal dummy variables. Taking these variables into account can result in improved finite sample performance of the test. The seasonal KPSS test can be interpreted as a test of deterministic seasonality and it may be used in addition to seasonal unit root tests to analyze the dynamic properties of time series. The seasonal indicator variables provide the test with an explicit model-based regression that in itself constitutes a support for its limit theory.

Key words: KPSS test, deterministic seasonality, Brownian motion, C32 time series models.

Introduction

The use of seasonally unadjusted data has become increasingly popular in empirical studies; there are several possible reasons for this. One key reason is the argument that seasonal adjustment distorts inference in dynamic models, for example, seasonal unit roots can be seriously affected when working with seasonally adjusted data. In this respect, Ghysels and Perron (1993) showed that seasonal adjustment filters affect finite sample distributions of unit root test statistics under the null hypothesis. Further, the seasonal component is an unobserved part of a time series, thus it must be taken into consideration because its elimination can lead to detrimental information loss. It was found in several cases that the seasonal component and other systematic components, such as trend and cycle, are in fact non-separable. From a statistical viewpoint this could be attributed to the fact that economic propagation mechanisms transmitting seasonal fluctuations from exogenous to endogenous variables are systematically related to business cycle fluctuations. Beaulieu, MacKie-Mason and Miron (1992) and Miron (1996) showed this in their studies of international economic aggregates such as output, labor input, interest rates, and prices. Canova and Ghysels (1994) also found that seasonality tends to differ across business cycle stages of recessions and expansions referring to an empirical study of U. S. macroeconomic time series. Consequently, a forced seasonal adjustment may lead to inaccurate predictions, which in turn may result in wrong decisions.

The literature presents several different models of seasonality. As highlighted by Canova and Hansen (1995), the first approach is to model seasonality as a deterministic component. This approach is generally adopted by macroeconomists, as shown by Barsky and Miron (1989). The second approach is to consider seasonality as a deterministic process along with its stationary stochastic pattern as illustrated by Canova (1992). The third approach is to consider seasonality as a stochastic component by allowing for seasonal unit roots.

A famous testing framework proposed by Hylleberg, et al. (1990) used the null hypothesis of seasonal non-stationarity induced by the presence of seasonal unit root(s) to make the distinction between unit roots at different seasonal frequencies. The subsequent rejection of their null hypothesis implies a strong result

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that the series exhibits a stationary seasonal pattern but their test was found to suffer from the problem of low power with moderate sample sizes. Because testing for seasonal unit roots is an important step in time series analysis these tests are often used as a pre-test for seasonal co-integration (Johansen & Schaumberg, 1999), several authors have contributed to the development of these types of tests (Canova & Hansen, 1995; Caner, 1998). In the testing approach the rejection of the null hypothesis would show evidence that the data are non-stationary. Another reason why this testing method is interesting to practitioners could be explained by the necessity to take into account the cost of spurious inference when testing the dummy variables model as argued in Franses, Hylleberg and Lee (1995).

Canova and Hansen (1995) and Taylor (2003) generalized the KPSS testing framework to seasonal data to test the stationarity hypothesis against (seasonal) unit roots. After specifying a general regression for their tests, the authors examined specific cases related to testing stationarity against (seasonal) unit root or some unit roots among a well-defined set. Lyhagen (2006) proposed another version of the KPSS test in the seasonal context which results in a frequency-based test and tested the hypothesis of level stationarity against a single seasonal unit root. In this study, seasonal indicator variables are included in the seasonal KPSS suggested by Lyhagen (2006). This approach may several have advantages over existing methods: It provides a model-based regression to the test, which is different from Lyhagen's method, where the limit theory based on an explicit form of the model was not established. The novelty of results from this study is the development of an asymptotic theory of the test in the presence of seasonal dummies which leads to a natural extension of the SKPSS to include deterministic seasonality.

Preliminaries on the Seasonal KPSS Test

Let y_t be a time series observed quarterly. This frequency was chosen because it provides a simple and clear analysis, however it should be noted that results of this study are valid for all seasonal frequencies (e.g., monthly or daily data) by simply defining seasonal unit roots according to their corresponding seasonal frequencies. Because the goal of this research is to test for the presence of negative unit root, it would be suitable to use the appropriate filter in order to isolate the effects of other unit roots in the series. Therefore, the test will be applied to the transformed series:

$$y_t^{(1)} = (1 - L + L^2 - L^3)y_t,$$

where L is the lag operator.

Next, test the unit root of -1 in the series

$$\begin{aligned} y_t^{(1)} &= x_t^{'}\beta + r_t + u_t, \\ t &= 1, \ldots, T, \end{aligned} \tag{1}$$

where $T = 4N$, $\beta^{'} x_t = \sum_{i=1}^{4} a_i D_{it}$, the shorthand notation $D_{it} = \delta(i, t - 4[(t-1)/4])$, [.] denotes the largest integer function and $\delta(i, j)$ is the Kronecker's δ function. The term u_t is zero mean weakly dependent process with autocovariogram $\gamma_h = E(u_t u_{t+h})$ and a strictly positive long run variance ω_u^2.

The component r_t is drawn from the following process:

$$r_t = -r_{t-1} + v_t, \tag{2}$$

where v_t is zero mean weakly process with variance σ_v^2 and long run variance $\omega_v^2 > 0$. The transformation required to carry out the seasonal KPSS test for complex unit roots $\pm i$ is given by the variable, $y_t^{(2)} = (1 - L^2)y_t$. The test of such complex unit roots is based on the regression,

$$y_t^{(2)} = x_t^{'}\lambda + c_t + e_t, \tag{3}$$

where e_t is zero mean weakly dependent process with long run variance $\omega_e^2 > 0$ and $\lambda' x_t = \sum_{i=1}^{4} b_i D_{it}$. The component c_t is given by

$$c_t = -c_{t-2} + \varepsilon_t, \tag{4}$$

where ε_t is another zero mean weakly dependent process with variance σ_ε^2 and strictly positive long run variance ω_ε^2.

Adding the deterministic terms in (1) and (3) is very important because it enables the seasonal KPSS test to be extended to include deterministic seasonality. The testing procedure follows in two steps: First, test for the existence of unit root −1, and second, test for the complex roots where the null hypothesis will be specified thereafter.

The seasonal KPSS test is a Lagrange Multiplier-based test, hence, the null hypothesis of a root equal to −1 is $H_0 : \sigma_v^2 = 0$. Under this null hypothesis

$$y_t^{(1)} = x_t' \beta + u_t, \tag{5}$$

where the series is trend stationary after seasonal mean correction. Under the alternative hypothesis $H_1 : \sigma_v^2 > 0$, $y_t^{(1)}$ has a unit root corresponding to Nyquist frequency.

Let $\tilde{u}_t$ be the residual series obtained from least squares regression applied to equation (5), $t = 1, 2, ..., T$. Following Breitung and Franses (1998, Eq. (18), p. 209), Busetti and Harvey (2003, Eq. (8), p. 422) and Taylor (2003, Eq. (38), p. 605), replace the long-run variance ω_u^2 by an estimate of (2π times) the spectrum at the observed frequency in order to deal with unconditional heteroscedasticity and serial correlation. This nonparametric estimation of the long-run variance is a useful solution to the nuisance parameter problem (Taylor, 2003). Thus, the Nyquist frequency is

$$\tilde{\omega}_u^2(l) = \\ T^{-1} \sum_{t=1}^{T} \tilde{u}_t^2 + 2T^{-1} \sum_{k=1}^{l} w(k,l) (\sum_{t=k+1}^{T} \tilde{u}_t \tilde{u}_{t-k}) \cos(\pi k), \tag{6}$$

where the weight function $w(k,l) = 1 - \frac{k}{l+1}$ and l is a lag truncation parameter such that $l \to \infty$ as $T \to \infty$ and $l = o(n^{1/2})$. From equation (6), a Bartlett kernel following Newey and West (1987) can now be chosen. It should be noted that Andrews (1991) and Kwaitkowski, et al. (1992) showed that that such a truncation lag can produce good results in practice. Similarly, the null hypothesis of the test regarding complex unit roots is given by $H_0 : \sigma_\varepsilon^2 = 0$; under this null hypothesis

$$y_t^{(2)} = x_t' \lambda + e_t \tag{7}$$

Using the residuals $\tilde{e}_t$ obtained from the least squares regression of equation (7), the Bartlett kernel estimator ω_e^2 is computed as:

$$\tilde{\omega}_e^2(l) = \\ T^{-1} \sum_{t=1}^{T} \tilde{e}_t^2 + 2T^{-1} \sum_{k=1}^{l} w(k,l) (\sum_{t=k+1}^{T} \tilde{e}_t \tilde{e}_{t-k}) \cos(\frac{\pi}{2} k), \tag{8}$$

with the partial sums defined as $\tilde{S}_t = \sum_{j=1}^{t} e^{i\pi j} \tilde{u}_j$ and $\tilde{P}_t = \sum_{j=1}^{t} e^{i\frac{\pi}{2} j} \tilde{e}_t$.

It follows that the test statistics for unit root of −1 is given by:

$$\eta^{(-1)} = \frac{1}{T^2} \frac{\sum_{t=1}^{T} \tilde{S}_t \overline{\tilde{S}}_t}{\tilde{\omega}_u^2(l)}. \tag{9}$$

This statistic may be written for the complex unit roots, as

$$\eta^{(\pm i)} = \frac{1}{T^2} \frac{\sum_{t=1}^{T} \widetilde{P}_t \overline{\widetilde{P}}_t}{\widetilde{\omega}_e^2(l)}, \tag{10}$$

where $\overline{\widetilde{S}}_t$ and $\overline{\widetilde{P}}_t$ are the conjugate numbers of $\widetilde{S}_t$ and $\widetilde{P}_t$, respectively.

Asymptotic Results

Next, the asymptotic distribution of $\eta^{(1)}$ and $\eta^{(\pm i)}$ is shown.

Theorem

a) Under $H_0 : \sigma_v^2 = 0$, $\eta^{(-1)} \rightarrow_d \int_0^1 V(r)^2 dr$ where $V(r)$ is a standard Brownian bridge, and $\rightarrow_d$ denotes weak convergence in probability and $r \in [0,1]$.

b) Under $H_0 : \sigma_\varepsilon^2 = 0$, $\eta^{(\pm i)} \rightarrow_d \frac{1}{2}\int_0^1 [V_R^c(\tau)^2 + V_I^c(\tau)^2] d\tau$ where $V_R^c(\tau)$ and $V_I^c(\tau)$ are two independent standard Brownian bridges and $\tau \in [0,1]$.

Proof

Starting with the first part of the theorem and referring to Jin and Phillips (2002) whose results showed that seasonal dummies maintain the asymptotic properties of the KPSS test unchanged. Also, given the mirror image of negative unit roots,

$$\frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j u_j \rightarrow_d B(r)$$

where $B(r)$ is a Brownian motion. The standardized partial sum process can be written as follows:

$$\begin{aligned}
\frac{\tilde{S}_{[Tr]}}{\sqrt{T}} &= \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} e^{i(\pi j)} \tilde{u}_j \\
&= \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j \tilde{u}_j \\
&= \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j [u_j - x_j'(\tilde{\beta} - \beta)] \\
&= \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j u_j - \\
&\quad \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j x_j' \left(X'X \right)^{-1} (X'u),
\end{aligned}$$

where $r \in [0,1]$. Thus the following is obtained,

$$\frac{\tilde{S}_{[Tr]}}{\sqrt{T}} = \frac{1}{\sqrt{T}} \sum_{j=1}^{[Tr]} (-1)^j u_j - \left(\frac{\sum_{j=1}^{[Tr]} (-1)^j x_j'}{T} \right) \left(\frac{X'X}{T} \right)^{-1} \left(\frac{X'u}{\sqrt{T}} \right). \tag{11}$$

In addition,

$$\frac{1}{T} \sum_{j=1}^{[Tr]} (-1)^j x_j' \rightarrow \left(-\frac{r}{4}, \frac{r}{4}, -\frac{r}{4}, \frac{r}{4} \right),$$

$$T^{-1} X' X \rightarrow (1/4) I_4,$$

and

$$\frac{1}{\sqrt{T}}X'u = \frac{1}{2}\frac{1}{\sqrt{\frac{T}{4}}}\begin{pmatrix}\sum_{j=1}^{\frac{T}{4}} u_{4j-3} \\ \sum_{j=1}^{\frac{T}{4}} u_{4j-2} \\ \sum_{j=1}^{\frac{T}{4}} u_{4j-1} \\ \sum_{j=1}^{\frac{T}{4}} u_{4j}\end{pmatrix} \to \frac{1}{2}\begin{pmatrix}B_{u.1}\\ B_{u.2}\\ B_{u.3}\\ B_{u.4}\end{pmatrix}$$

$$\equiv \frac{1}{2}N\left(0,\begin{pmatrix}\omega_0 & \omega_1 & \omega_2 & \omega_3\\ \omega_1 & \omega_0 & \omega_1 & \omega_2\\ \omega_2 & \omega_1 & \omega_0 & \omega_1\\ \omega_3 & \omega_2 & \omega_1 & \omega_0\end{pmatrix}\right), \quad (12)$$

where $B_{u.i}(1) =_d N(0,\omega_0)$, $i = 1,...,4$, $\omega_0 = \gamma_0 + 2\sum_{h=1}^{\infty}\gamma_{4h}$, $\omega_1 = \sum_{h=1}^{\infty}\gamma_{2h-1} = \omega_3$, and $\omega_2 = 2\sum_{h=1}^{\infty}\gamma_{4h-2}$. It follows that

$$\left(\frac{\sum_{j=1}^{[Tr]}(-1)^j x_j'}{T}\right)\left(\frac{X'X}{T}\right)^{-1}\left(\frac{X'u}{\sqrt{T}}\right) \to$$

$$_d\left(-\frac{r}{4},\frac{r}{4},-\frac{r}{4},\frac{r}{4}\right)\begin{pmatrix}4&0&0&0\\0&4&0&0\\0&0&4&0\\0&0&0&4\end{pmatrix}\frac{1}{2}\begin{pmatrix}B_{u.1}(1)\\B_{u.2}(1)\\B_{u.3}(1)\\B_{u.4}(1)\end{pmatrix}$$

$$\to_d \frac{1}{2}r\,[-B_{u.1}(1)+B_{u.2}(1)-B_{u.3}(1)+B_{u.4}(1)]$$

$$\to_d \frac{1}{2}r\,[-B_1(1)+B_2(1)-B_3(1)+B_4(1)]=rB\,(1)$$

and

$$\frac{1}{\sqrt{T}}\tilde{S}_{[Tr]} \to_d B(r) - rB(1),$$

which results in the following:

$$T^{-2}\sum_{t=1}^{T}\tilde{S}_t^2 \to_d \omega_u^2\int_0^1 V(r)^2 dr, \quad (13)$$

where $V(r)$ is a standard Brownian bridge process. Further, because $\tilde{\omega}_u^2(l)$ is a consistent estimate of ω_u^2, it can be shown that $\eta^{(-1)} \to_d \int_0^1 V(r)^2 dr.$

Next it is necessary to prove the second part of the theorem. Because complex-valued roots come in conjugate pairs, it is only necessary to consider the complex root i associated with frequency $\frac{\pi}{2}$. In this case the standardized partial sum process can be written as follows:

$$\frac{\tilde{P}_{[T\tau]}}{\sqrt{T}} = \frac{1}{\sqrt{T}}\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})}\tilde{e}_j$$
$$= \frac{1}{\sqrt{T}}\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})}\tilde{e}_j$$
$$= \frac{1}{\sqrt{T}}\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})}[e_j - x_j'(\tilde{\lambda}-\lambda)]$$

where

$$\frac{\tilde{S}_{[Tr]}}{\sqrt{T}} =$$

$$\frac{1}{\sqrt{T}}\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})}e_j - \left(\frac{\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})}x_j'}{T}\right)\left(\frac{X'X}{T}\right)^{-1}\left(\frac{X'e}{\sqrt{T}}\right)$$

Chan and Wei (1988) showed that

$$\frac{1}{\sqrt{T}}\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} e_j \to_d B^*(\tau),$$

where $B^*(\tau) = B_R^*(\tau) + iB_I^*(\tau)$ and $B_R^*(\tau)$ and $B_I^*(\tau)$ are two independent real Brownian motions and

$$\frac{1}{T}\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j^{'} \to \left(0,-\frac{r}{4},0,\frac{r}{4}\right) + i\left(\frac{r}{4},0,-\frac{r}{4},0\right).$$

Therefore,

$$\left(\frac{\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j^{'}}{T}\right)\left(\frac{X'X}{T}\right)^{-1}\left(\frac{X'e}{\sqrt{T}}\right) \to$$

$$\left[\begin{array}{l} {}_d 4\times\left(0,-\frac{r}{4},0,\frac{r}{4}\right) I_4 \times \frac{1}{2}\begin{pmatrix} B_{e.1} \\ B_{e.2} \\ B_{e.3} \\ B_{e.4}\end{pmatrix} \\ +i[4\times\left(\frac{r}{4},0,-\frac{r}{4},0\right) I_4 \times \frac{1}{2}\begin{pmatrix} B_{e.1} \\ B_{e.2} \\ B_{e.3} \\ B_{e.4}\end{pmatrix}] \end{array}\right],$$

where $B_{e.i}$ are defined similarly to $B_{u.i}$ in equation (12), $i = 1,...,4$. It follows that

$$\left(\frac{\sum_{j=1}^{[T\tau]} e^{i(j\frac{\pi}{2})} x_j^{'}}{T}\right)\left(\frac{X'X}{T}\right)^{-1}\left(\frac{X'e}{\sqrt{T}}\right) \to$$

$$\left(\begin{array}{l} {}_d \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{2}\tau(-B^{(2)}(1)+B^{(4)}(1)) \\ +i\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{2}\tau(B^{(1)}(1)-B^{(3)}(1)) \end{array}\right)$$

$$= \frac{1}{\sqrt{2}}\tau\ (B_R^*(1) + i\ B_I^*(1)) = \frac{1}{\sqrt{2}}\tau\ B^*(1),$$

where $B^{(i)}(\tau)$, $i = 1,..,4$, $B_R^*(\tau)$ and $B_I^*(\tau)$ are all real Brownian motions and the last two processes are independent.

It is evident that $\frac{\widetilde{P}_{[T\tau]}}{\sqrt{T}} \to_d \sqrt{\frac{\omega_e^2}{2}}\, V^c(\tau)$, where $V^c(\tau)$ is a complex Brownian bridge that can be written as $V^c(\tau) = V_R^c(\tau) + iV_I^c(\tau)$ $V_R^c(\tau)$ and $V_I^c(\tau)$ are two independent real standard Brownian bridges. As a result,

$$\frac{1}{T^2}\sum_{t=1}^{T} \tilde{P}_t \bar{\tilde{P}}_t \to_d \frac{\omega_e^2}{2}\int_0^1 [(V_R^c)^2(\tau) + (V_I^c)^2(\tau)]d\tau \tag{14}$$

because $\widetilde{\omega}_e^2(l)$ is a convergent estimate of ω_e^2 , it may be concluded that

$$\eta^{(\pm i)} \ \to_d \ \frac{1}{2}\int_0^1 [(V_R^c)^2(\tau) + (V_I^c)^2(\tau)]d\tau, \tag{15}$$

as claimed.

It should be noted that asymptotically $\eta^{(-1)}$ has the Cramer-von Mises distribution (CvM) under the null hypothesis, although the limit theory of $\eta^{(\pm i)}$ was shown as a function of a generalized CvM with two degrees, specifically, $\eta^{(\pm i)} \to_d \frac{1}{2} CvM(2)$. The critical values of the seasonal KPSS test with seasonal dummies can be computed from Nyblom (1989) or from Canova and Hansen (1995). These critical values are shown in Table 1.

Table 1: Critical Values of the Seasonal KPSS Test

	1%	5%	10%
Root -1	0.743	0.461	0.347
Roots $\pm i$	0.537	0.374	0.3035

Monte Carlo Analysis

To evaluate the size performance of the seasonal KPSS statistic, Monte Carlo simulation experiments were conducted using seasonal roots of a quarterly process. The data generating process (DGP) for the negative unit root is

$$y_t = x_t^{'}\beta + r_t, \quad t = 1, ..., T, \tag{16a}$$

where $x_t^{'}\beta$ is defined as in (1) and the autoregressive process r_t is given by:

$$r_t = \alpha \; r_{t-1} + v_t, \tag{16b}$$

The error terms v_t are normally distributed with zero mean and unit variance. The DGP for complex unit roots is given by:

$$y_t = x_t^{'}\lambda + c_t, \quad t = 1, ..., T, \tag{17a}$$

where $x_t^{'}\lambda$ is defined in (3) and the process c_t is given by:

$$c_t = \alpha \; c_{t-2} + \varepsilon_t, \tag{17b}$$

and ε_t are normally distributed with zero mean and unit variance.

Alternative values of $\alpha \in \{-1, -0.8, -0.2, 0, 0.2, 0.8\}$ were chosen and only the 5% nominal size was considered. The bandwidth values chosen in these experiments are given by: $l0 = 0$, $l4 = \text{integer}\left[4(T/100)^{1/4}\right]$ and $l12 = \text{integer}\left[12(T/100)^{1/4}\right]$. Twenty-thousand (20,000) replications were conducted and all the simulation experiments were carried out with Matlab programs.

Results in Table 2 show that the test size increases as values of α decrease. Also note that larger data samples do not significantly affect the test size. These simulations raised another point, which was observed by Lyhagen (2006) for similar testing but without deterministic components: they show that, as opposed to the original KPSS testing framework, $l4$ and $l12$ do not have better size performance than $l0$. In fact, in the seasonal KPSS framework, the test size deterioration induces substantial power. Results of the simulation experiments performed in this study (see Table 1) suggest an overall good power performance of the seasonal KPSS test, particularly against near seasonal unit root alternatives.

Conclusion

The joint use of unit root and stationarity tests is recommended in empirical studies. Such a joint use can lead to a more in-depth analysis of the dynamic properties of the time series. This article established asymptotic theory of the seasonal KPSS test in the presence of seasonal dummies and extended SKPSS to include deterministic seasonality. Given that seasonal unit root tests have low power in moderate samples, the test represents an adequate solution as illustrated by the simulation results. Lyhagen (2006) also showed good power properties of the test when there is no deterministic term in the model, however, it would be interesting to study both power and size performance of the test when factors affecting the time series such as measurement errors and additive outliers are present. Khedhiri and El Montasser (2010) used

Monte Carlo methods to show that the seasonal KPSS test is robust to the magnitude and the number of additive outliers. Furthermore, the statistical results obtained demonstrate overall good performance on the finite-sample properties of the test.

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Table 2**:** Rejection Frequencies for the Seasonal KPSS Test with Seasonal Dummies for Seasonal Quarterly Unit Roots

$\eta^{(-1)}$				$\eta^{(\pm i)}$			
α	T	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
-1	80	0.99	0.8192	0.5314	0.9969	0.9708	0.7755
	200	0.9997	0.9469	0.7268	1	0.9970	0.9264
-0.9	80	0.9229	0.4780	0.1409	0.9736	0.8436	0.3735
	200	0.9700	0.4960	0.1727	0.9973	0.9014	0.4613
-0.2	80	0.1359	0.0558	0.0279	0.1598	0.0851	0.0423
	200	0.1262	0.0575	0.0425	0.1645	0.0861	0.0515
0	80	0.0543	0.0418	0.0240	0.0505	0.0420	0.0325
	200	0.0526	0.0449	0.0389	0.0508	0.0467	0.0413
0.2	80	0.0183	0.0308	0.0205	0.0112	0.0223	0.0246
	200	0.0144	0.0360	0.0351	0.0086	0.0251	0.0326
0.9	80	0.00	0.0066	0.0038	0.00	0.0055	0.0050
	200	0.00	0.0002	0.0066	0.00	0.0001	0.0012

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