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Testing the Population Coefficient of Variation

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The coefficient of variation (CV), which is used in many scientific areas, measures the variability of a population relative to its mean and standard deviation. Several methods exist for testing the population CV. This article compares a proposed bootstrap method to existing methods. A simulation study was conducted under both symmetric and skewed distributions to compare the performance of test statistics with respect to empirical size and power. Results indicate that some of the proposed methods are useful and can be recommended to practitioners.

Key words: Coefficient of variation, simulation, size, power of a test, symmetric distribution, skewed distribution.

Introduction

The coefficient of variation (CV), which is the ratio of the standard deviation to the mean, was first introduced by Karl Pearson in 1896. This dimensionless relative measure of dispersion has widespread applications in many disciplines. Researchers have used CV to: measure the risk of a stock (Miller & Karson, 1977), to assess the strength of ceramics (Gong & Li, 1999), to assess homogeneity of bone test samples produced form a particular method (Hamer, et al., 1995), in wildlife studies (Dodd & Murphy, 1995), in dose-response studies (Creticos, et al., 2002) and in uncertainty analyses of fault tree analysis (Ahn, 1995). Nairy and Rao (2003) provided a brief survey of recent applications of CV in business, climatology, engineering and other fields.

The coefficient of variation is presented in virtually all introductory statistics texts, primarily as a descriptive measure; inferential methods regarding population CVs are typically missing in these textbooks. To make an inference regarding a population CV, assumptions regarding the population distribution and knowledge of the distributional properties of the sample CV are needed. Hendricks and Robey (1936) studied the distribution of the sample CV and showed that it can be approximated by a function defined on a positive real line, which depends on the standard normal moment of order $n-1$ about some well-defined point, where n is the sample size. Iglewicz (1967) derived the exact distribution for a sample CV, when the sample is drawn from a normal population. This exact distribution assumed that the chance of obtaining a non-positive sample mean is negligible and, hence, is not useful for inferential purposes.

McKay (1932) gave an approximation of the distribution of a statistic derived from a sample CV based on the Chi-squared distribution. This approximation was determined to be very accurate if CV ≤ 0.33 (Pearson, 1932; Iglewicz, 1967) and reasonably accurate when

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$0.33 \leq \mathrm{CV} \leq 0.67$ (Miller, 1991).The exact distribution of the sample CV is difficult to obtain when the population distribution is not normal. Due to the limited development related to the exact distribution of sample CV for non-normal populations, inferences regarding population CVs did not receive much attention until Sharma and Krishna (1994) developed the asymptotic distribution of the sample inverse coefficient of variation (ICV) without making an assumption about the population distribution; they obtained a confidence interval for the CV by inverting the proposed confidence interval. Curto and Pinto (2009) developed an asymptotic distribution of a sample CV in the case of non-iid (independent and identically distributed) random variables.

Various methods for constructing confidence intervals on CV have recently appeared in the literature (Amiri & Zwanzig, 2010; Carto & Pinto, 2009; Banik & Kibria, 2011). Banik and Kibria (2011) conducted a simulation study to compare the performance of various confidence intervals suggested in the literature. However, despite its widespread use in a wide range of disciplines, tests of hypotheses on CVs do not appear to be of interest to statisticians in general. Although some test statistics have been suggested, there is limited information available regarding the performance of these tests. Moreover, many of these tests are based on normal theory; however, real life data frequently follow right-skewed distributions, particularly when sample sizes are small (Baklizi & Kibria, 2008; Shi & Kibria, 2007; Banik & Kibria, 2009; Almonte & Kibria, 2009).

This article compares the size and the power of some existing tests and their bootstrap versions when data are from both normal and positively skewed distributions. The tests compared were developed based on the sampling distribution of a sample CV due to McKay (1932), Hendricks and Robey (1936), Miller (1991), Sharma and Krishna (1994) and Curto and Pinto (2009).

Test Statistics for Testing Population CV

Let $X_1, X_2, \cdots, X_n$ represent a random sample of size *n* from a normal population with mean μ and SD σ so that $\gamma = \sigma / \mu$ is the population CV. When the distribution is unknown, the parameters μ and σ are estimated from the observed data. The estimated CV is then defined as $\hat{\gamma} = s / \overline{X}$ where $\overline{X}$ and *s* are the sample mean and sample standard deviation respectively. The test hypotheses are:

$$\begin{gathered} H_0 : \gamma = \gamma_0 \\ \text{vs.} \\ H_a : \gamma = \gamma_1 \end{gathered} \tag{1}$$

where $\gamma_1 = \gamma_0 + c$, c is a positive constant and the difference between γ_0 and the true value of the population CV. Because the right skewed distribution is of interest, the upper tailed test was selected. However, the lower tail test may be used by setting $c < 0$. The size of a test can be estimated by setting c = 0. Several test statistics have been suggested for testing the hypotheses in (1).

The *t*-Statistic

Hendricks and Robey (1936) studied the distribution of a sample CV when the sample is drawn from a normal distribution. Koopmans, et al (1964) and Igelewicz (1967) reviewed the relevant literature and proposed the following *t*-statistic for testing γ for a normal distribution:

$$\mathrm{t}_0 = \frac{(\hat{\gamma} - \gamma_0)}{S_{\hat{\gamma}} \sim t_{(n-1)}} \tag{2}$$

where $\hat{\gamma}$ is the sample CV, $S_{\hat{\gamma}} = \hat{\gamma} / \sqrt{2n}$. The hull hypothesis is rejected at the α level of significance if $\mathrm{t}_0 > t_{(n-1),\alpha}$ where $t_{(n-1),\alpha}$ is the upper $(\alpha)^{\text{th}}$ percentile from a t -distribution with (n − 1) degrees of freedom.

McKay's Statistic

McKay (1932) proposed the following test statistic for testing γ:

$$Mc = \left(1 + \gamma_0^{-1}\right)\left(\frac{n\hat{\gamma}^2}{1+\hat{\gamma}^2}\right) \sim \chi^2_{(n-1)} \tag{3}$$

Where $\hat{\gamma}$ is the sample CV. The hull hypothesis is rejected at the α level of significance if $Mc > \chi^2_{(n-1),\ \alpha}$ where $\chi^2_{(n-1),\ \alpha}$ is the upper (α)th percentile from a Chi-square distribution with (n – 1) degrees of freedom.

Miller's Statistic

Miller (1991) provided an asymptotic distribution of the sample CV which can reasonably be assumed to be normal if the parent population is normal. They proposed the following test statistic:

$$\begin{aligned} MiL &= (\hat{\gamma} - \gamma_0) / S_{\hat{\lambda}_M} \sim Z(0,1),\ S_{\hat{\lambda}_M} \\ S_{\hat{\lambda}_M} &= \sqrt{(\hat{\gamma}^4 + 0.5\hat{\gamma}^2)/n} \end{aligned} \tag{3}$$

Sharma and Krishna's Statistic

Sharma and Krishna's (1994) statistic, which is based on the sampling distribution of ICV, is given by

$$SK = n(\hat{\gamma} -_0) \sim (1/Z(0,1)) \tag{4}$$

As noted, this has the advantage of relieving the normality assumption.

Curto and Pinto's Statistic

Curto and Pinto (2009b) proposed a test statistics for non-iid random variables, that is, autocorrelated and heteroskedastic random variables. Their test statistic is given by:

$$CP = (\hat{\gamma} - \gamma_0) / SE(\hat{\gamma}) \tag{5}$$

where

$$SE(\hat{\gamma}) \cong \sqrt{V_{GMM}/n},$$

$$V_{GMM} = \frac{\partial f(\theta)}{\partial \theta} \Sigma \frac{\partial f(\theta)}{\partial \theta'},$$

and

$$\frac{\partial f(\theta)}{\partial \theta'} = \begin{bmatrix} -\sigma/\mu^2 \\ \dfrac{1}{2\sigma\mu} \end{bmatrix}$$

To estimate the asymptotic variance, an estimator for $\partial f(\theta)/\partial\theta'$ may be obtained by substituting into $\hat{\theta}$ and a heteroscedasticity and autocorrelation consistent (HAC) estimator $\hat{\Sigma}$ may be obtained by using Newey and West's (1987) procedure:

$$\hat{\Sigma} = \hat{\Omega}_0 + \sum_{j=1}^{m} \omega(j,m)(\hat{\Omega}_j + \hat{\Omega}_j'),$$

$$\hat{\Omega}_j = \frac{1}{T}\sum_{t=j+1}^{T} \varphi(X_t,\hat{\theta})\varphi(X_t,\hat{\theta})',$$

$$\varphi(X_t,\hat{\theta}) = \begin{bmatrix} X_t - \hat{\mu} \\ (X_t - \hat{\mu})^2 - \hat{\sigma}^2 \end{bmatrix},$$

and

$$\omega(j,m) = 1 - \frac{j}{m+1},$$

where m is the truncated lag that must satisfy the condition m/T.

Proposed Bootstrap Test Statistics for Testing Population CV

Bootstrap, introduced by Efron (1979), is a commonly used computer-based non-parametric tool that does not require assumptions regarding an underlying population and can be applied in a variety of situations. The accuracy of the bootstrap depends on the number of bootstrap samples. If the number of bootstrap samples is large enough, statistics may be more accurate. The number of bootstrap samples is typically between 1,000 and 2,000 because accuracy depends on the size of the samples (Efron & Tibshirani, 1993). This article proposes bootstrap test statistics for testing a population CV. An extensive array of different bootstrap methods are summarized as: Let $X^{(*)} = X_1^{(*)}, X_2^{(*)}, \ldots, X_n^{(*)}$, where the ith sample is denoted $X^{(i)}$ for i = 1, 2 , …, B, and B is the number of bootstrap samples. The bootstrap estimate of CV for the B samples is $CV^*_{(i)}$.

Non-Parametric Bootstrap Statistic

First, compute the CV for all bootstrap samples, then order the sample CVs of each bootstrap sample as:

$$CV^*_{(1)} \le CV^*_{(2)} \ldots \le CV^*_{(B)}$$

The test statistic for testing hypotheses (1) is the t-statistic defined in (2) but the (1−α) sample quantile of the bootstrap samples, $CV^*_{[(1-\alpha)B]}$, is used as the upper critical value for the test.

Parametric Bootstrap t-Statistic

The bootstrap version of the *t*-statistic defined in (2) is given by

$$BT_i^* = \frac{\left(CV_i^* - \overline{\overline{CV}}\right)}{\hat{\sigma}_{CV}}, \quad i = 1, 2, ..., B, \tag{6}$$

where $\hat{\sigma}_{CV} = \frac{1}{B}\sum_{i=1}^{B}\left(CV_i^* - \overline{\overline{CV}}\right)^2$, and $\overline{\overline{CV}} = \frac{1}{B}\sum_{i=1}^{B} CV_i^*$ is the mean of the bootstrap sample CVs. The (1−α)th quantile of the bootstrap t-statistic in (6) is used as the upper critical value for an α level test.

Miller Bootstrap Statistics: Approach 1

This approach suggests replacing $\hat{\gamma}$ in (3) by $\hat{\gamma}^*$, the sample CV of the bootstrap sample, thus, the following test statistic is proposed:

$$BMiL1 = \frac{\hat{\gamma}^* - \gamma_0}{S_{\hat{\lambda}_M}} \sim Z(0,1), \qquad S_{\hat{\lambda}_M} = \sqrt{\frac{\left(\hat{\gamma}^{*4} + 0.5\hat{\gamma}^{*2}\right)}{n}}. \tag{7}$$

Miller Bootstrap Statistics: Approach 2

As noted, the approximate asymptotic normality of the sampling distribution of $\hat{\gamma}$ is based on the assumption that the parent population is normal (Miller, 1991); violation of the normality assumption may lead to undesirable results. The following bootstrap test is thus proposed:

$$BMiL2 = \frac{\hat{\gamma} - \gamma_0}{S_{\hat{\lambda}_M}} \tag{8}$$

where $S_{\hat{\lambda}_M}$ is as defined in (3) The null hypothesis in (1) is rejected if $BMiL2 > Z^*_{\alpha/2}$, where $Z^*_{\alpha/2}$ is the (1 – α)th quantile of

$$Z_i^* = \left(CV_i^* - \overline{\overline{CV}}\right) / \hat{\sigma}_{CV}$$

with

$$\hat{\sigma}_{CV} = \frac{1}{B}\sum_{i=1}^{B}\left(CV_i^* - \overline{\overline{CV}}\right)^2$$

and

$$\overline{\overline{CV}} = \frac{1}{B}\sum_{i=1}^{B} CV_i^*.$$

Curto and Pinto Bootstrap Statistic

The following test statistic for bootstrap version of CP is proposed:

$$BCP = (\hat{\gamma}^* - \gamma_0) / SE(\hat{\gamma}^*) \tag{9}$$

where γ^* is the sample CV of the bootstrap samples and

$$SE(\hat{\gamma}^*) \cong \sqrt{BV_{GMM} / B},$$

$$BV_{GMM} = \frac{\partial f(\theta)}{\partial \theta} \Sigma^* \frac{\partial f(\theta)}{\partial \theta'},$$

$$\hat{\Sigma}^* = \hat{\Omega}_0 + \sum_{j=1}^{m} \omega(j,m)(\hat{\Omega}^*_j + \hat{\Omega}^*_j{}'),$$

$$\hat{\Omega}^*_j = \frac{1}{B}\sum_{t=j+1}^{T} \varphi(X^*_t, \hat{\theta})\varphi(X^*_t, \hat{\theta})',$$

and

$$\varphi(X^*{}_t, \hat{\theta}) = \begin{bmatrix} X^*{}_t - \hat{\mu} \\ (X^*{}_t - \hat{\mu})^2 - \hat{\sigma}^2 \end{bmatrix},$$

where m is the truncated lag that must satisfy the condition m/T. The $(1 - \alpha)^{th}$ quantile of the bootstrap statistic in (9) is used as the upper critical value for an α level test.

Methodology

Monte Carlo simulation experiments were performed to evaluate the performance of the proposed test statistics in terms of size and power. The main objective is to recommend good test statistics for a population CV based on simulation results. Because a theoretical comparison was not possible, a simulation study was used to compare the size and power performances of the test statistics.

Six different configurations of sample sizes: n = 10, 20, 30, 50, 100, 200 were used. Random samples were generated from the *N* (2, 1) and two skewed distributions namely, Gamma (4, 2) and Log-Normal (2, 0.472). This parameter choice resulted in population CVs close to 0.5 for all selected distributions with varying degree of skewness. Note that non-iid data was not used in the simulation. Although the Curto and Pinto statistic was proposed for non-iid, that is, autocorrelated and heteroscedatic random variables, the focus was to compare the size and power of existing test statistics with the proposed bootstrap statistics for testing population CV when data are generated from symmetric and skewed distributions.

For each combination of sample size and population distribution, 10,000 random samples and 1,500 bootstrap replications were generated. The most common 5% level (α) of significance was used. Empirical sizes and powers for each test were calculated as the fraction of the rejections of the null hypothesis out of 10,000 simulation replications by setting *c* = 0.0, 0.04, 0.06, 0.08, 0.10 and 0.12 in $\gamma_1 = \gamma_0 + c$. The size of the test was obtained by setting *c* = 0. Simulation results are presented in Tables 3.1-3.4.

Results

Table 3.1 shows the estimated sizes of the selected test statistics for all three distributions. The row entries represent the proportion of times H_0 was rejected at α = 0.05 under H_0. If a procedure is significantly above the nominal level or significantly above the level of some other procedure, there may be a question about the seriousness of the degree of non-robustness. Rejection rates significantly below the nominal level are not of interest. Such deviations are not problematic for Type I errors and power can be evaluated separately. To gauge the adequacy of robustness in controlling Type I errors, several standards have been used in the past. Cochran (1954) suggested the general guideline of an upper limit of 0.06 for tests run at the 0.05 level. Bradley (1978) considered a liberal criterion of robustness in which he argued that no test should be considered robust if the true Type I error rate exceeds 1.5α; meaning, that an α = 0.05 would require an actual limit of 0.075. Finally Conover, et al. (1981) used a more liberal approach and suggested that a test is non-robust if the Type I error rate exceeds 2α.

From the data shown in Table 3.1 it is clear that none but the *CP* procedure is most conservative. Its size is smaller than the nominal size of 0.05 for all sample size for the normal and Gamma distributions, and for all $n > 10$ for the log-normal distribution. For the Gamma distribution, however, all of the tests suffer from size distortion when $n < 200$

When the underlying distribution is normal all of the procedures, except the *SK* test satisfy Cochran's 0.06 limit (and hence Bradley's 0.075 and Conover, et al.'s 0.1 limit), particularly when $n > 20$. It is noteworthy that the SK procedure does not satisfy Cochran's limit for the normal distribution and no clear superiority of one test is apparent for the log-normal distribution. However, the t-test, SK and NB procedures appear to be clearly non-inferior in controlling Type I errors, especially when $n \geq 100$, as their estimated type I error rate is either very close to or exceeds Conover, et al.'s 0.1 limit. In general, the bootstrap versions of the Mil and CP tests are slightly more conservative than their respective non-bootstrap counterparts for all three distributions and all tests have reasonable size properties when data are generated from a normal distribution as opposed to the two skewed distributions.

The estimated powers of the test statistics for the normal, Gamma and log-normal distributions are presented in Tables 3.2, 3.3 and 3.4 respectively. The first column provides values of *c*, which is the difference between γ_0 and the true value of population CV. The entries in the columns 3-12 represent the proportion of times H_0 was rejected at α = 0.5 under H_1. With few exceptions, Miller's procedure appears to be the most powerful under a normal distribution, while the BMil2 shows some advantages over other tests under a Gamma distribution: no clear pattern of dominance of one test is visible for the log-normal distribution. When $c = 0.04$ all tests considered have very low power (maximum power = 0.48). As expected, power increases with *c*. Most of the tests have reasonable power when $c = 0.12$ and sample size is low, such as n = 30 for the normal and Gamma distributions. A comparison of results presented in Table 3.2 reveals that the Curto and Pinto's test and its bootstrap version, BCP, are both relatively more powerful under a Gamma distribution as well as a log-normal distribution compared to the normal case. In addition, the parametric bootstrap test also shows a clear pattern of improvement in power over the normal case.

Conclusion

This article considered five existing test statistics and five bootstrap versions of three of the tests for testing a population CV under various experimental conditions. Because a theoretical comparison is not possible, a simulation study was conducted to compare the performance of the test statistics. Results indicate that all of the test statistics suffer from size distortion, particularly when data is from either a Gamma or a log-normal distribution and $n \leq 50$. None of the tests is recommended if *c*, the difference between the hypothesized and true value of the population CV is too small, that is, if $c < 0.06$. Although Miller's test appears to be the most powerful under a normal distribution, its bootstrap version, BMil2, shows some advantages over the other tests for Gamma distributions. Sharma and Krishna's test is most powerful for the normal distribution. For a definite statement regarding the performance of the test statistics, additional simulations under variety of experimental conditions are required. It is hoped that the results from this study will be useful to different applied researchers and practitioners who are interested to test a population CV for symmetric and skewed populations.

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Table 3.1: Estimated Type I Error Rates for Various Statistical Tests

Distribution	Tests for CV*	n = 10	20	30	50	100	200
Normal (2, 1)	t-test	0.0300	0.0453	0.0473	0.0580	0.0580	0.0593
	McKay	0.0613	0.0607	0.0480	0.0500	0.0380	0.0480
	MiL	0.0793	0.0747	0.0553	0.0580	0.0427	0.0547
	SK	0.0980	0.0867	0.0640	0.0813	0.0753	0.0700
	CP	0.0107	0.0180	0.0160	0.0200	0.0233	0.0287
	NB	0.0333	0.0378	0.0407	0.0520	0.0527	0.0593
	PB	0.0393	0.0480	0.0473	0.0580	0.0500	0.0427
	BMiL1	0.0667	0.0653	0.0507	0.0520	0.0407	0.0493
	BMiL2	0.0740	0.0667	0.0520	0.0547	0.0373	0.0520
	BCP	0.0080	0.0120	0.0127	0.0200	0.0213	0.0253
Gamma (4, 2)	t-test	0.0060	0.0140	0.0240	0.0427	0.0420	0.0520
	McKay	0.0173	0.0207	0.0253	0.0333	0.0287	0.0320
	MiL	0.0247	0.0280	0.0333	0.0427	0.0327	0.0387
	SK	0.0367	0.0307	0.0420	0.0633	0.0507	0.0580
	CP	0.0260	0.0240	0.0360	0.0367	0.0313	0.0293
	NB	0.0187	0.0247	0.0333	0.0427	0.0420	0.0547
	PB	0.0173	0.0207	0.0347	0.0387	0.0420	0.0520
	BMiL1	0.0080	0.0080	0.0167	0.0280	0.0247	0.0280
	BMiL2	0.0367	0.0280	0.0380	0.0367	0.0327	0.0387
	BCP	0.0400	0.0253	0.0427	0.0307	0.0307	0.0293
Log-Normal (2, 0.4720)	t-test	0.0160	0.0287	0.0480	0.0607	0.0953	0.1033
	McKay	0.0313	0.0367	0.0480	0.0487	0.0720	0.0707
	MiL	0.0413	0.0433	0.0560	0.0607	0.0793	0.0767
	SK	0.0487	0.0487	0.0613	0.0827	0.1120	0.1127
	CP	0.0527	0.0440	0.0380	0.0280	0.0293	0.0213
	NB	0.0200	0.0367	0.0593	0.0707	0.1007	0.1240
	PB	0.0467	0.0407	0.0733	0.0753	0.0973	0.0973
	BMiL1	0.0160	0.0193	0.0267	0.0347	0.0573	0.0593
	BMiL2	0.0560	0.0487	0.0733	0.0693	0.0813	0.0720
	BCP	0.0793	0.0533	0.0533	0.0347	0.0313	0.0173

*Notes: t, t statistic; McKay, McKay; MiL, Miller; SK, Sharma and Krishna; CP, Curto and Pinto; NB, Non-parametric bootstrap; PB, Parametric bootstrap; BMiL1, Miller bootstrap 1; BMiL2, Miller bootstrap 2; BCP, Bootstrap Curto and Pinto

Table 3.2: Estimated Power of Various Tests for the Normal (2, 1) Distribution

c	n	Tests for CV*									
		t	McKay	MiL	SK	CP	NB	PB	BMiL1	BMiL2	BCP
0.04	10	0.0427	0.1093	0.1340	0.1307	0.0233	0.0693	0.0413	0.1087	0.1067	0.0100
	20	0.1047	0.1467	0.1647	0.1487	0.0513	0.0713	0.1187	0.1433	0.1587	0.0407
	30	0.1327	0.1447	0.1593	0.1487	0.0680	0.1420	0.1367	0.1427	0.1540	0.0620
	50	0.2087	0.2020	0.2193	0.2313	0.1187	0.2313	0.2267	0.2007	0.2267	0.1307
	100	0.2807	0.2400	0.2987	0.2833	0.1727	0.3053	0.2567	0.2447	0.2373	0.1500
	200	0.4773	0.4187	0.4840	0.4587	0.3380	0.4453	0.4327	0.4180	0.3787	0.2813
0.06	10	0.0620	0.1413	0.1653	0.1467	0.0273	0.1113	0.0493	0.1393	0.1333	0.0107
	20	0.1527	0.2020	0.2273	0.1827	0.0773	0.1473	0.1773	0.1953	0.2313	0.0787
	30	0.2013	0.2327	0.2553	0.2107	0.1107	0.1913	0.2107	0.2240	0.2453	0.1053
	50	0.2793	0.2787	0.2973	0.2913	0.1827	0.2393	0.2693	0.2713	0.2820	0.1607
	100	0.5047	0.4720	0.5187	0.4833	0.3787	0.4733	0.4733	0.4660	0.4533	0.3427
	200	0.7367	0.6813	0.7400	0.6873	0.6193	0.7133	0.7647	0.6800	0.7280	0.6587
0.08	10	0.0847	0.2007	0.2287	0.1920	0.0480	0.1793	0.1173	0.1847	0.2187	0.0393
	20	0.1947	0.2547	0.2800	0.2067	0.1220	0.2247	0.2240	0.2373	0.2820	0.1287
	30	0.2827	0.3280	0.3493	0.2667	0.1760	0.2827	0.2720	0.3147	0.3300	0.1500
	50	0.4373	0.4440	0.4667	0.4240	0.3107	0.4020	0.4040	0.4273	0.4320	0.2627
	100	0.7000	0.6753	0.7207	0.6527	0.5727	0.7067	0.6900	0.6680	0.6760	0.5513
	200	0.9240	0.9053	0.9327	0.8913	0.8767	0.9140	0.9240	0.9040	0.9100	0.8727
0.10	10	0.1207	0.2647	0.2880	0.2313	0.0720	0.1593	0.2193	0.2487	0.3100	0.0960
	20	0.3040	0.3807	0.4033	0.2980	0.1753	0.3173	0.3140	0.3607	0.3900	0.1613
	30	0.4293	0.4827	0.5013	0.3900	0.3033	0.4367	0.4547	0.4627	0.5027	0.3053
	50	0.6087	0.6293	0.6440	0.5587	0.4607	0.6167	0.5553	0.6100	0.6033	0.3940
	100	0.8587	0.8507	0.8560	0.8047	0.7720	0.8540	0.8633	0.8433	0.8593	0.7793
	200	0.9880	0.9867	0.9873	0.9787	0.9773	0.9900	0.9907	0.9847	0.9873	0.9793
0.12	10	0.1580	0.3253	0.3500	0.2507	0.0973	0.1887	0.1700	0.2900	0.3180	0.0673
	20	0.4000	0.4887	0.5007	0.3680	0.2707	0.4333	0.4227	0.4607	0.4960	0.2580
	30	0.5467	0.5973	0.6173	0.4640	0.4133	0.5140	0.5213	0.5793	0.5880	0.3533
	50	0.7453	0.7627	0.7733	0.6853	0.6260	0.7240	0.7233	0.7493	0.7520	0.5740
	100	0.9507	0.9480	0.9513	0.9180	0.9173	0.9327	0.9420	0.9420	0.9420	0.9040
	200	1.0000	1.0000	1.0000	0.9987	0.9993	1.0000	1.0000	1.0000	1.0000	0.9993

*Notes: t, t statistic; McKay, McKay; MiL, Miller; SK, Sharma and Krishna; CP, Curto and Pinto; NB, Non-parametric bootstrap; PB, Parametric bootstrap; BMiL1, Miller bootstrap 1; BMiL2, Miller bootstrap 2; BCP, Bootstrap Curto and Pinto

Table 3.3: Estimated Power of Various Tests for the Gamma (4, 2) Distribution

c	n	Tests for CV*									
		t	McKay	MiL	SK	CP	NB	PB	BMiL1	BMiL2	BCP
0.04	10	0.0107	0.0493	0.0673	0.0660	0.0720	0.0280	0.0247	0.0200	0.0653	0.0680
	20	0.0560	0.0787	0.0993	0.0840	0.0873	0.0787	0.0780	0.0553	0.1093	0.0987
	30	0.0920	0.0993	0.1220	0.1033	0.1080	0.1160	0.1167	0.0787	0.1367	0.1213
	50	0.1353	0.1313	0.1447	0.1553	0.1413	0.1573	0.1327	0.1093	0.1340	0.1267
	100	0.2553	0.2127	0.2333	0.2553	0.2207	0.2493	0.2627	0.1987	0.2420	0.2253
	200	0.4427	0.3760	0.4000	0.4200	0.3933	0.4407	0.4187	0.3700	0.3667	0.3673
0.06	10	0.0240	0.0907	0.1073	0.0933	0.0927	0.0393	0.1253	0.0467	0.1767	0.1607
	20	0.0740	0.1047	0.1280	0.0893	0.1227	0.0853	0.1900	0.0787	0.1320	0.1287
	30	0.1640	0.1887	0.2033	0.1713	0.1893	0.1600	0.2080	0.1540	0.2287	0.2193
	50	0.2807	0.2780	0.2993	0.2907	0.2673	0.2667	0.3040	0.2493	0.3113	0.2840
	100	0.4560	0.4167	0.4400	0.4360	0.4253	0.4753	0.4473	0.3933	0.4220	0.4060
	200	0.7473	0.6920	0.7067	0.6967	0.7160	0.7387	0.7820	0.6773	0.7460	0.7573
0.08	10	0.0300	0.1273	0.1540	0.1167	0.1267	0.0740	0.0740	0.0713	0.1673	0.1413
	20	0.1513	0.2087	0.2333	0.1673	0.2053	0.2300	0.1513	0.1720	0.2093	0.1793
	30	0.2347	0.2633	0.2887	0.2173	0.2693	0.2227	0.2760	0.2293	0.3240	0.3000
	50	0.4213	0.4293	0.4460	0.4000	0.4107	0.4240	0.4507	0.3840	0.4620	0.4253
	100	0.7180	0.7033	0.7120	0.6860	0.7053	0.7340	0.7047	0.6880	0.6987	0.6720
	200	0.9327	0.9140	0.9167	0.8987	0.9193	0.9373	0.9247	0.9053	0.9073	0.9073
0.10	10	0.0560	0.1780	0.2013	0.1433	0.1607	0.0900	0.1093	0.1187	0.2147	0.1720
	20	0.2320	0.3227	0.3473	0.2267	0.3000	0.3160	0.2853	0.2693	0.3660	0.3127
	30	0.3753	0.4427	0.4627	0.3300	0.4220	0.5067	0.4300	0.3860	0.4780	0.4467
	50	0.6153	0.6327	0.6553	0.5633	0.6100	0.6713	0.6527	0.5900	0.6747	0.6293
	100	0.8860	0.8760	0.8853	0.8393	0.8713	0.8740	0.8933	0.8573	0.8880	0.8813
	200	0.9867	0.9827	0.9847	0.9800	0.9833	0.9820	0.9880	0.9827	0.9860	0.9873
0.12	10	0.0780	0.2693	0.2993	0.1947	0.2287	0.2273	0.1807	0.1913	0.3160	0.2500
	20	0.3347	0.4280	0.4527	0.3067	0.3947	0.3547	0.5113	0.3767	0.5713	0.5327
	30	0.5347	0.6160	0.6400	0.4527	0.5767	0.5627	0.5627	0.5580	0.6353	0.5747
	50	0.7467	0.7680	0.7873	0.6820	0.7493	0.7500	0.7620	0.7393	0.7927	0.7567
	100	0.9620	0.9607	0.9627	0.9293	0.9613	0.9573	0.9513	0.9560	0.9527	0.9453
	200	1.0000	0.9993	0.9993	0.9987	0.9993	0.9993	0.9993	0.9993	0.9993	0.9993

*Notes: t, t statistic; McKay, McKay; MiL, Miller; SK, Sharma and Krishna; CP, Curto and Pinto; NB, Non-parametric bootstrap; PB, Parametric bootstrap; BMiL1, Miller bootstrap 1; BMiL2, Miller bootstrap 2; BCP, Bootstrap Curto and Pinto

Table 3.4: Estimated Power of Various Tests for the Log-Normal (2, 0.4724) Distribution

c	n	Tests for CV*									
		t	McKay	MiL	SK	CP	NB	PB	BMiL1	BMiL2	BCP
0.04	10	0.0220	0.0593	0.0740	0.0733	0.0733	0.0287	0.0333	0.0293	0.0707	0.0707
	20	0.0573	0.0867	0.0973	0.0873	0.0780	0.0653	0.0993	0.0473	0.1140	0.1027
	30	0.1100	0.1240	0.1400	0.1293	0.1013	0.1273	0.1540	0.0873	0.1613	0.1367
	50	0.1713	0.1660	0.1787	0.1853	0.1107	0.1707	0.1813	0.1253	0.1813	0.1180
	100	0.2647	0.2387	0.2480	0.2667	0.1600	0.2747	0.2720	0.2147	0.2513	0.1633
	200	0.4220	0.3680	0.3833	0.3987	0.2560	0.4287	0.4333	0.3427	0.3920	0.2707
0.06	10	0.0287	0.0747	0.0887	0.0780	0.0980	0.1067	0.1027	0.0347	0.1447	0.1607
	20	0.1140	0.1487	0.1620	0.1367	0.1380	0.1653	0.1393	0.1067	0.1667	0.1447
	30	0.1527	0.1687	0.1867	0.1540	0.1447	0.1447	0.1840	0.1300	0.2040	0.1567
	50	0.2347	0.2340	0.2513	0.2400	0.1813	0.2927	0.2300	0.1967	0.2353	0.1660
	100	0.4360	0.4060	0.4227	0.4147	0.3247	0.4860	0.4293	0.3840	0.4100	0.3120
	200	0.6607	0.6120	0.6253	0.6147	0.5120	0.6720	0.6147	0.5893	0.5787	0.4313
0.08	10	0.0513	0.1200	0.1360	0.1133	0.1347	0.0553	0.1107	0.0740	0.1640	0.1700
	20	0.1440	0.2013	0.2167	0.1560	0.1893	0.2013	0.1653	0.1407	0.2173	0.1893
	30	0.2273	0.2653	0.2853	0.2140	0.2393	0.2453	0.2667	0.2127	0.2960	0.2573
	50	0.3753	0.3813	0.4013	0.3600	0.3147	0.3560	0.4000	0.3313	0.4080	0.3333
	100	0.6253	0.6027	0.6153	0.5800	0.5047	0.6427	0.6560	0.5680	0.6347	0.5467
	200	0.8640	0.8327	0.8433	0.8173	0.7893	0.8607	0.8480	0.8207	0.8300	0.7633
0.10	10	0.0640	0.1533	0.1760	0.1273	0.1640	0.0953	0.1133	0.0967	0.1940	0.1773
	20	0.2147	0.2940	0.3087	0.2113	0.2773	0.2420	0.2400	0.2213	0.3080	0.2727
	30	0.3280	0.3813	0.3980	0.2920	0.3480	0.3500	0.3673	0.3193	0.4053	0.3587
	50	0.5053	0.5247	0.5400	0.4560	0.4487	0.4940	0.5167	0.4660	0.5380	0.4447
	100	0.7980	0.7860	0.7967	0.7420	0.7387	0.8100	0.8027	0.7627	0.7967	0.7400
	200	0.9673	0.9560	0.9600	0.9387	0.9413	0.9560	0.9573	0.9540	0.9527	0.9300
0.12	10	0.0693	0.1880	0.2093	0.1333	0.1960	0.1647	0.2560	0.1247	0.3213	0.3180
	20	0.2873	0.3787	0.3987	0.2533	0.3440	0.3473	0.4353	0.3100	0.4847	0.4533
	30	0.4213	0.4900	0.5053	0.3433	0.4400	0.4560	0.4307	0.4167	0.4973	0.4280
	50	0.6787	0.7067	0.7233	0.6100	0.6533	0.7353	0.6500	0.6580	0.6873	0.6113
	100	0.9100	0.9093	0.9133	0.8707	0.8807	0.9140	0.9253	0.8980	0.9227	0.8953
	200	0.9940	0.9940	0.9940	0.9900	0.9913	0.9940	0.9947	0.9940	0.9947	0.9940

*Notes: t, t statistic; McKay, McKay; MiL, Miller; SK, Sharma and Krishna; CP, Curto and Pinto; NB, Non-parametric bootstrap; PB, Parametric bootstrap; BMiL1, Miller bootstrap 1; BMiL2, Miller bootstrap 2; BCP, Bootstrap Curto and Pinto

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