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Improved Estimators in Finite Population Surveys: Theory and Applications

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Improved estimators are proposed for estimating the population mean $\overline{Y}$ of the study variable y using auxiliary variable x in simple random sampling. Explicit expression for the bias and MSE of the proposed family are derived to the first order of approximation. The proposed estimators are compared with other estimators and theoretical findings are illustrated by two numerical examples.

Key words: Ratio estimator, product estimator, regression estimator, bias, mean square error.

Introduction

Consider a simple random sample of size n drawn without replacement from a finite population $U = (U_1, U_2, ..., U_N)$ of N units, let $\overline{y}$ and $\overline{x}$ be the sample mean estimates of $\overline{Y}$ and $\overline{X}$, respectively, the population means of the study variable y and auxiliary variable x. To date in the literature of survey sampling, the efficiencies of estimators of unknown population means of a study variable y have been increased by the use of known information on an auxiliary variable x which is highly correlated with study variable y, the well-known ratio estimator is

$$t_R = \overline{y}\frac{\overline{X}}{\overline{x}}. \qquad (1)$$

When the correlation between auxiliary variable x and study variable y is highly negative, then the conventional product estimator for $\overline{Y}$ is defined as

$$t_P = \overline{y}\frac{\overline{x}}{\overline{X}}. \qquad (2)$$

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Theoretically, it has been established that, in general, the linear regression estimator is more efficient than the ratio and product estimators except in the case where the regression line passes through the neighborhood of the origin, thus the classical regression estimator is

$$t_g = \overline{y} + b_{yx}\left(\overline{X} - \overline{x}\right). \qquad (3)$$

Furthermore, to find more precise estimates several authors have used prior values of certain population parameter(s). Searls (1964) used known coefficient of variation (CV) of study variable at estimation stage. Sisodiya and Dwivedi (1981) extended the Searls (1964) work by using the known CV of the auxiliary variable for estimating population mean of study variable y in a ratio method of estimation. Thus, use of prior value of coefficient of kurtosis in estimating the population variance of study variable y was first conducted by Singh, et al. (1973). It was later, used by Sen (1978), Upadhyaya and Singh (1984) and Searls and Interpanich (1990) in the estimation of population mean of a study variable. Further, Singh and Tailor (2003) proposed a modified ratio estimator by using a known value of a correlation coefficient. This study suggests a new family of estimators to estimate a population mean $\overline{Y}$ of a study variable y by using estimators from Khoshnevisan, et al.

(2007); the optimum cases of the suggested family of estimators are also obtained.

The Suggested Family of Estimators
        A family of estimators proposed by Khoshnevisan, et al. (2007) for estimating the population mean is

$$t = \bar{y}\left\{\frac{a\bar{X}+c}{\alpha(a\bar{x}+c)+(1-\alpha)(a\bar{X}+c)}\right\}^{g}$$

(4)

where $a(\neq 0)$, c are either real numbers or functions of known parameters of an auxiliary variable x such as Standard Deviation (S_x), Coefficient of Variation (C_x), Skewness ($\beta_1(x)$), Kurtosis ($\beta_2(x)$) and Correlation Coefficient (ρ_{yx}) of the population, and α and g are suitably chosen scalars.

The expressions of bias and MSE of the Khoshnevisan's estimator are respectively given as

$$B(t) = \left(\frac{1-f}{n}\right)\bar{Y}\left\{\frac{g(g+1)}{2}\alpha^2 v^2 C_x^2 - \alpha v g C_{yx}\right\}$$

(5)

and

$$MSE(t) = \left(\frac{1-f}{n}\right)\left\{S_y^2 + \alpha^2 v^2 g^2 R^2 S_x^2 - 2\alpha v g R S_{yx}\right\}$$

(6)

which is minimum, when $\alpha = \dfrac{K}{vg}$, then

$$\min.MSE(t) = \left(\frac{1-f}{n}\right)\left(1-\rho_{yx}^2\right)S_y^2 = MSE(t_g).$$

(7)

        Following Khoshnevisan, et al. (2007), a class of estimators for estimating the population mean $\bar{Y}$ of the study variable y can be defined by combining regression estimator with t, the

Khoshnevisan, et al. (2007) estimator for full information case is

$$T =$$

$$\left\{\bar{y}+b_{yx}\left(\bar{X}-\bar{x}\right)\right\}\left\{\frac{a\bar{X}+c}{\alpha(a\bar{x}+c)+(1-\alpha)(a\bar{X}+c)}\right\}^{g},$$

(8)

where $\left\{\bar{y}+b_{yx}\left(\bar{X}-\bar{x}\right)\right\}$ is the classical regression estimator, b_{yx} is the sample estimate of β_{yx}, $\bar{y}=(1/n)\sum_{i=1}^{n}y_i$, $\bar{x}=(1/n)\sum_{i=1}^{n}x_i$, $\bar{X}=(1/N)\sum_{i=1}^{N}X_i$, $\bar{Y}=(1/N)\sum_{i=1}^{N}Y_i$, $b_{yx}=\left(s_{yx}/s_x^2\right)$, $\beta_{yx}=\left(S_{yx}/S_x^2\right)$, $s_{xy}=(1/(n-1))\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})$, $s_x^2=(1/(n-1))\sum_{i=1}^{n}(x_i-\bar{x})^2$, $S_{xy}=(1/(N-1))\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})$, and $S_x^2=(1/(N-1))\sum_{i=1}^{N}(x_i-\bar{X})^2$.

        To obtain the bias and MSE of the class of estimators T,

$$\bar{y} = \bar{Y}(1+\varepsilon_0),$$
$$\bar{x} = \bar{X}(1+\varepsilon_1),$$
$$s_{xy} = S_{xy}(1+\varepsilon_2),$$

and

$$s_x^2 = S_x^2(1+\varepsilon_3),$$

such that $E(\varepsilon_i)=0$, $\forall i = 0\,to\,3$,

$$E(\varepsilon_0^2)=\left(\frac{1-f}{n}\right)\frac{S_y^2}{\bar{Y}^2}, \qquad E(\varepsilon_1^2)=\left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{X}^2},$$

$$E(\varepsilon_0\varepsilon_1)=\left(\frac{1-f}{n}\right)\frac{S_{yx}}{\bar{Y}\bar{X}}, \qquad C_x=\frac{S_x}{\bar{X}}, C_y=\frac{S_y}{\bar{X}},$$

$$C_{yx}=\frac{S_{xy}}{\bar{Y}\bar{X}}, \qquad\qquad \rho_{yx}=\frac{\mu_{11}}{\mu_{20}^{1/2}\mu_{02}^{1/2}},$$

$$E(\varepsilon_1\varepsilon_2)=\left(\frac{1-f}{n}\right)\frac{\lambda_{12}S_x}{\bar{X}\rho_{yx}},$$

$$E(\varepsilon_1\varepsilon_3) = \left(\frac{1-f}{n}\right)\frac{\lambda_{03}S_x}{\overline{X}}, \quad \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}, \text{ and}$$

$$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \overline{Y})^r(x_i - \overline{X})^s, \quad (r,s) \text{ being}$$

non-negative integers.

Expanding T in terms of $\varepsilon's$ results in

$$T =$$
$$\overline{Y}\left\{1+\varepsilon_0 - A\varepsilon_1(1+\varepsilon_2)(1+\varepsilon_3)^{-1}\right\}(1+\alpha v\varepsilon_1)^{-g},$$
(9)

where $A = (\beta_{yx}/R)$, $R = (\overline{Y}/\overline{X})$, $v = \frac{a\overline{X}}{a\overline{X}+c}$.

It is assumed that $|\varepsilon_3| < 1$ and $|\alpha v\varepsilon_1| < 1$ so that $(1+\varepsilon_3)^{-1}$ and $(1+\alpha v\varepsilon_1)^{-g}$ are expandable in terms of $\varepsilon's$. Further, expanding the right hand side of (9), in terms of $\varepsilon's$ and neglecting terms of $\varepsilon's$ having power greater than two, results in

$$(T-\overline{Y}) = \overline{Y}\left\{\begin{array}{c}\varepsilon_0 - A\varepsilon_1 - g\alpha v\varepsilon_1 - g\alpha v\varepsilon_0\varepsilon_1 \\ -A\varepsilon_1\varepsilon_2 + A\varepsilon_1\varepsilon_3 + g\alpha v\varepsilon_1^2 \\ +\frac{g(g+1)}{2}\alpha^2 v^2\varepsilon_1^2\end{array}\right\}.$$
(10)

Taking expectations of both sides on (10), give the bias of T to the first degree of approximation, as

$$B(T) =$$
$$\left[\frac{1}{\overline{X}}\left(\frac{1-f}{n}\right)\left\{\begin{array}{c}\frac{g(g+1)}{2}\alpha^2 v^2 RS_x^2 \\ +g\alpha v(A-\beta_{yx})RS_x^2\end{array}\right\} - \left(\frac{\lambda_{12}}{\rho_{yx}}-\lambda_{03}\right)S_x\right]$$
(11)

Squaring both sides of (10) and neglecting terms of $\varepsilon's$ having power greater than two, results in

$$(T-\overline{Y})^2 = \overline{Y}^2\left\{\varepsilon_0 - (A+g\alpha v)\varepsilon_1\right\}^2$$
$$= \overline{Y}^2\left\{\begin{array}{c}\varepsilon_0^2 + A^2\varepsilon_1^2 + g^2\alpha^2 v^2\varepsilon_1^2 - 2A\varepsilon_0\varepsilon_1 \\ -2g\alpha v\varepsilon_0\varepsilon_1 + 2Ag\alpha v\varepsilon_1^2\end{array}\right\}$$
(12)

Taking expectations of both sides on (12), gives the MSE of T to the first degree of approximation as

$$MSE(T) =$$
$$\left(\frac{1-f}{n}\right)\left[S_y^2 + R(A+g\alpha v)\left\{R(A+g\alpha v)-2\beta_{yx}\right\}S_x^2\right]$$
(13)

which is minimum when, for example, $\alpha = 0 = \alpha_0$. Thus, the resulting minimum MSE of T, is given by

$$min.MSE(T) = \left(\frac{1-f}{n}\right)(1-\rho_{yx}^2)S_y^2$$
$$= MSE(t_g)$$
$$= min.MSE(t)$$
(14)

The ratio-cum-regression estimators (see Table 1) are in the same family of (8) and the bias and mean squared error (MSE) in (13) for these estimators can be expressed as

$$B(T_i) = \begin{cases}\frac{1}{\overline{X}}\left(\frac{1-f}{n}\right)\left\{\begin{array}{c}(2-\beta_{yx})RS_x^2 \\ -\left(\frac{\lambda_{12}}{\rho_{yx}}-\lambda_{03}\right)S_x\end{array}\right\}; \\ i = 1 \\ \frac{1}{\overline{X}}\left(\frac{1-f}{n}\right)\left\{\begin{array}{c}\left(\frac{v_{(i-1)}}{2}\right)RS_x^2 - \left(\frac{\lambda_{12}}{\rho_{yx}}-\lambda_{03}\right)S_x \\ +1-\beta_{yx}\end{array}\right\}, \\ i = 3,5,7,\dots,17\end{cases}$$
(15)

and

$$MSE\left(T_i\right)=\begin{cases}\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-R^2\right)S_x^2\right\};\\[2mm]i=1\\[2mm]\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-v_{\frac{(i-1)}{2}}R^2\right)S_x^2\right\},\\[2mm]i=3,5,7,...,17.\end{cases}$$

(16)

For the product-cum-regression estimators given in Table 1, the bias and MSE of the estimators are

$$B\left(T_j\right)=$$

$$\begin{cases}\dfrac{1}{\overline{X}}\left(\dfrac{1-f}{n}\right)\left\{\left(\beta_{yx}-1\right)RS_x^2-\left(\dfrac{\lambda_{12}}{\rho_{yx}}-\lambda_{03}\right)S_x\right\};\\[2mm]j=2\\[2mm]\dfrac{1}{\overline{X}}\left(\dfrac{1-f}{n}\right)\left\{\left(\beta_{yx}-1\right)v_{(j/2)-1}RS_x^2-\left(\dfrac{\lambda_{12}}{\rho_{yx}}-\lambda_{03}\right)S_x\right\},\\[2mm]j=4,6,8,...,18\end{cases}$$

(17)

$$MSE\left(T_j\right)=\begin{cases}\left(\dfrac{1-f}{n}\right)\left\{S_y^2+\left(\beta_{yx}^2+R^2\right)S_x^2\right\};\\[2mm]j=2\\[2mm]\left(\dfrac{1-f}{n}\right)\left\{S_y^2+\left(\beta_{yx}^2+v_{(j/2)-1}R^2\right)S_x^2\right\},\\[2mm]j=4,6,8,...,18\end{cases}$$

(18)

where $v_1=\dfrac{\overline{X}}{\overline{X}+C_x}$, $v_2=\dfrac{\beta_2(x)\overline{X}}{\beta_2(x)\overline{X}+C_x}$,

$v_3=\dfrac{C_x\overline{X}}{C_x\overline{X}+\beta_2(x)}$, $v_4=\dfrac{\overline{X}}{\overline{X}+S_x}$,

$$v_5=\dfrac{\beta_1(x)\overline{X}}{\beta_1(x)\overline{X}+S_x},\ v_6=\dfrac{\beta_2(x)\overline{X}}{\beta_2(x)\overline{X}+S_x},$$

$$v_7=\dfrac{\overline{X}}{\overline{X}+\rho_{yx}},\ \text{and}\ v_8=\dfrac{\overline{X}}{\overline{X}+\beta_2(x)}.$$

Many more estimators can be generated from the proposed estimator in (8) by substituting the different values of α, g, a and c.

Efficiency Comparisons

The expressions of MSE of various estimators to the first degree of approximation are

$$Var\left(\overline{y}\right)=\left(\dfrac{1-f}{n}\right)S_y^2;$$

(19)

$$MSE\left(t_R\right)=\left(\dfrac{1-f}{n}\right)\left(S_y^2+R^2S_x^2-2RS_{yx}\right)$$

(20)

$$MSE\left(T_1\right)=\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-R^2\right)S_x^2\right\}$$

(21)

$$MSE\left(T_3\right)=\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-v_1R^2\right)S_x^2\right\};$$

(22)

$$MSE\left(T_5\right)=\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-v_2R^2\right)S_x^2\right\};$$

(23)

$$MSE\left(T_7\right)=\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-v_3R^2\right)S_x^2\right\};$$

(24)

$$MSE\left(T_9\right)=\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-v_4R^2\right)S_x^2\right\};$$

(25)

$$MSE\left(T_{11}\right)=\left(\dfrac{1-f}{n}\right)\left\{S_y^2-\left(\beta_{yx}^2-v_5R^2\right)S_x^2\right\};$$

(26)

Table 1: Some Members of the Family of Estimators of T

Ratio-Cum-Regression Estimators $(g=1)$	Product-Cum-Regression Estimators $(g=-1)$	α	a	c
$T_1 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\dfrac{\bar{X}}{\bar{x}}$	$T_2 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\dfrac{\bar{x}}{\bar{X}}$	1	1	0
$T_3 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{X} + C_x}{\bar{x} + C_x}\right)$	$T_4 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{x} + C_x}{\bar{X} + C_x}\right)$	1	1	C_x
$T_5 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x} + C_x}\right)$	$T_6 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\beta_2(x)\bar{x} + C_x}{\beta_2(x)\bar{X} + C_x}\right)$	1	$\beta_2(x)$	C_x
$T_7 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{C_x\bar{X} + \beta_2(x)}{C_x\bar{x} + \beta_2(x)}\right)$	$T_8 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{C_x\bar{x} + \beta_2(x)}{C_x\bar{X} + \beta_2(x)}\right)$	1	C_x	$\beta_2(x)$
$T_9 = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{X} + S_x}{\bar{x} + S_x}\right)$	$T_{10} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{x} + S_x}{\bar{X} + S_x}\right)$	1	1	S_x
$T_{11} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\beta_1(x)\bar{X} + S_x}{\beta_1(x)\bar{x} + S_x}\right)$	$T_{12} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\beta_1(x)\bar{x} + S_x}{\beta_1(x)\bar{X} + S_x}\right)$	1	$\beta_1(x)$	S_x
$T_{13} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\beta_2(x)\bar{X} + S_x}{\beta_2(x)\bar{x} + S_x}\right)$	$T_{14} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\beta_2(x)\bar{x} + S_x}{\beta_2(x)\bar{X} + S_x}\right)$	1	$\beta_2(x)$	S_x
$T_{15} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{X} + \rho_{yx}}{\bar{x} + \rho_{yx}}\right)$	$T_{16} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{x} + \rho_{yx}}{\bar{X} + \rho_{yx}}\right)$	1	1	ρ_{yx}
$T_{17} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)}\right)$	$T_{18} = \left\{\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\right\}\left(\dfrac{\bar{x} + \beta_2(x)}{\bar{X} + \beta_2(x)}\right)$	1	1	$\beta_2(x)$

$$MSE(T_{13}) = \left(\frac{1-f}{n}\right)\{S_y^2 - (\beta_{yx}^2 - v_6 R^2)S_x^2\};$$

(27)

$$MSE(T_{15}) = \left(\frac{1-f}{n}\right)\{S_y^2 - (\beta_{yx}^2 - v_7 R^2)S_x^2\};$$

(28)

$$MSE(T_{17}) = \left(\frac{1-f}{n}\right)\{S_y^2 - (\beta_{yx}^2 - v_8 R^2)S_x^2\}.$$

(29)

The efficiency comparison of the proposed estimator T at its optimum with the usual unbiased estimator $\bar{y}$, ratio estimator t_r and $T_i; i = 1,3,5,7,...,17$, the results are:

$$Var(\bar{y}) - \min.MSE(T) = \rho_{yx}^2 S_y^2 > 0$$

$$MSE(t_R) - \min.MSE(T) = \{R^2 + \beta_{yx}(\beta_{yx} - 2R)\}S_x^2 > 0$$

$$MSE(T_1) - \min.MSE(T) = R^2 S_x^2 > 0$$

$$MSE(T_3) - \min.MSE(T) = v_1 R^2 S_x^2 > 0$$

$$MSE(T_5) - \min.MSE(T) = v_2 R^2 S_x^2 > 0$$

$$MSE(T_7) - \min.MSE(T) = v_3 R^2 S_x^2 > 0$$

$$MSE(T_9) - \min.MSE(T) = v_4 R^2 S_x^2 > 0$$

$$MSE(T_{11}) - \min.MSE(T) = v_5 R^2 S_x^2 > 0$$

$$MSE(T_{13}) - \min.MSE(T) = v_6 R^2 S_x^2 > 0$$

$$MSE(T_{15}) - \min.MSE(T) = v_7 R^2 S_x^2 > 0$$

$$MSE(T_{17}) - \min.MSE(T) = v_8 R^2 S_x^2 > 0$$

$$MSE(t_R) - Var(\bar{y}) < 0; \text{if } R < 2\beta_{yx}$$

$$MSE(T_1) - Var(\bar{y}) < 0; \text{if } R^2 < \beta_{yx}^2$$

$$MSE(T_i) - Var(\bar{y}) < 0; \text{if } v_j R^2 < \beta_{yx}^2;$$
$$i = 3,5,7,9,11,13,15,17 \text{ and } j = 1 \text{ to } 8.$$

If these defined conditions are true, then the proposed estimator is more efficient than the usual unbiased estimator $\bar{y}$ and unbiased ratio estimator t_R.

Empirical Study

Population I (Koyuncu & Kadilar, 2009)

Data concerning primary and secondary schools for 923 districts of Turkey in 2007 (Source: Ministry of Education, Republic of Turkey), taking the number of teachers as study variable and the number of students as auxiliary variable in both primary and secondary schools. A sample of size $n = 180$ was selected from the dataset and the correlations observed between auxiliary and study variables are positive. Therefore, ratio estimators were used for the estimation of the population mean. The summary statistics about the population are:
$N = 923$, $n = 180$, $S_y = 749.9395$, $S_x = 21331.1315$, $\bar{Y} = 436.4345$, $\bar{X} = 11440.4984$, $\rho_{yx} = 0.9543$, $\beta_2(x) = 18.7208$ and $\beta_1(x) = 3.9365$.

Population II (Singh, 2001)

Consider the data used by Anderson (1958). The parameters of the population consist of 25 families as follows: y: head length of second son; x: head breadth of first son; $N = 25$, $n = 7$, $C_y = 0.0546$, $C_x = 0.0488$, $\bar{Y} = 183.84$, $\bar{X} = 151.12$, $\rho_{yx} = 0.6932$, $\beta_2(x) = 2.6519$, and $\beta_1(x) = 0.0002$.

Here, the percent relative efficiencies (PRE) of different suggested estimators were computed with respect to the usual unbiased estimator $\bar{y}$ for both populations. The outcomes are shown in Table 2.

Table 2: Percent Relative Efficiencies of Different Suggested Ratio Type
Estimators with Respect to the Usual Unbiased Estimator $\bar{y}$

Population I		Population II	
Ratio Type estimators	PRE($., \bar{y}$)	Ratio Type estimators	PRE($., \bar{y}$)
$\bar{y}$	100.00	$\bar{y}$	100.00
t_r	939.7177	t_r	178.6661
T_1	78.94465	T_1	75.85505
T_3	78.95661	T_3	75.86713
T_5	78.94529	T_5	75.85961
T_7	79.00904	T_7	87.91403
T_9	199.8646	T_9	78.05581
T_{11}	112.5773	T_{11}	191.3018
T_{13}	86.20194	T_{13}	76.6948
T_{15}	78.95077	T_{15}	6.02636
T_{17}	79.06471	T_{17}	76.50771
$T_{(opt.)}$	1119.6777	$T_{(opt.)}$	192.5025

Conclusion

It is envisaged from Table 2 that the proposed estimator at its optimum is more efficient than among all discussed estimators. Also, the estimators T_1, T_3, T_5, T_7, T_{13}, T_{15} and T_{17} are less efficient than the usual unbiased estimator because the condition $v_j R^2 < \beta_{yx}^2$; $i = 3,5,7,9,11,13,15,17$ and $j = 1\ to\ 8$ is not satisfied for both the populations.

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