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Estimation and Testing in Type I Generalized Half Logistic Distribution

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A generalization of the half logistic distribution is developed through exponentiation of its cumulative distribution function and termed the Type I Generalized Half Logistic Distribution (GHLD). GHLD's distributional characteristics and parameter estimation using maximum likelihood and modified maximum likelihood methods are presented with comparisons. Comparison of Type I GHLD and the exponential distribution is conducted via likelihood ratio criterion.

Key words: Generalized half logistic distribution, ML estimation, modified ML estimation, testing, likelihood ratio criterion.

Introduction

In life testing and reliability studies a combination of monotone and constant failure rates over various segments of the range of lifetime of a random variable is known as a bathtub or a non-monotone failure rate. In the biological and engineering sciences there are situations of non-monotone failure rates available to model such data; a comprehensive narration of the models is given in Rajarshi & Rajarshi (1988). Mudholkar, et al. (1995) presented an extension of the Weibull family that contains unimodel distributions with bathtub failure rates and also allows for a broader class of monotone hazard rates; they named their extended version the Exponentiated Weibull Family. Gupta and Kundu (1999) also proposed a new model called generalized exponential distribution. If θ is a positive real number and F(x) is the cumulative distribution function (cdf) of a continuous positive random variable, then $[F(x)]^{\theta}$ and the corresponding probability distribution may be termed exponentiated or generalized versions of F(x).This generalization is adapted to the half logistic distribution and the resulting model is considered in this study.

A half logistic model obtained as the distribution of an absolute standard logistic variate is a probability model of recent origin (Balakrishnan, 1985). Its standard probability density function (pdf), cdf and hazard functions are given by:

$$f(x) = \frac{2e^{-x}}{(1+e^{-x})^2}, x \geq 0, \tag{1.1}$$

$$F(x) = \left[\frac{1-e^{-x}}{1+e^{-x}}\right], x \geq 0, \tag{1.2}$$

$$h(x) = \frac{1}{1+e^{-x}}, x \geq 0. \tag{1.3}$$

The pdf, cdf and hazard functions of the generalized half logistic distribution (GHLD-I) are given by

$$f(x) = \frac{2\theta e^{-x}(1-e^{-x})^{\theta-1}}{(1+e^{-x})^{\theta+1}}, x > 0, \theta > 0, \tag{1.4}$$

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$$F(x) = \left[\frac{1-e^{-x}}{1+e^{-x}}\right]^{\theta}, \quad (1.5)$$
$$x > 0,\ \theta > 0,$$

$$h(x) = \frac{2\theta e^{-x}(1-e^{-x})^{\theta-1}}{(1+e^{-x})[(1+e^{-x})^{\theta}-(1-e^{-x})^{\theta}]},$$
$$x > 0,\ \theta > 0 \quad (1.6)$$

Balakrishnan and Sandhu (1995) have suggested a new probability model with a standard pdf and cdf given by the following equations

$$f(x) = \frac{2(1-kx)^{(1/k)-1}}{[1+(1-kx)^{1/k}]^2},\ 0 \le x \le \frac{1}{k}, k \ge 0, \quad (1.7)$$

$$F(x) = \frac{1-(1-kx)^{1/k}}{1+(1-kx)^{1/k}},\ 0 \le x \le \frac{1}{k}, k \ge 0. \quad (1.8)$$

The limits of (1.7) and (1.8) as $k \to \infty$ are respectively (1.1) and (1.2), the pdf and cdf of HLD; for this reason Balakrishnan and Sandhu (1995) called the distribution (1.7) and (1.8) a generalized HLD.

Olapade (2007, 2008) considered two distributions and discussed their distributional properties and order statistics in samples from these distributions; he termed these two distributions type-I and type-III GHLD respectively. The generalized HLD of Olapade (2007, 2008) are obtained via truncation of the type-I and type-III generalized logistic distributions of Balakrishnan and Leung (1988). Thus, the proposed type-I GHLD is conceptually different from the GHLDs of Balakrishnan and Sandhu (1995) and Olapade (2007, 2008).

Estimation

The pdf and distribution function of GHLD-I with scale parameter σ and shape parameter θ are:

$$f(x) = \frac{2\theta(1-e^{-x/\sigma})^{\theta-1}e^{-x/\sigma}}{\sigma(1+e^{-x/\sigma})^{\theta+1}}, \quad (2.1)$$
$$0 < x < \infty,\ \sigma > 0,\ \theta > 0,$$

and

$$F(x) = \left[\frac{1-e^{-x/\sigma}}{1+e^{-x/\sigma}}\right]^{\theta} \quad (2.2)$$
$$0 < x < \infty,\ \sigma > 0,\ \theta > 0.$$

Let $x_1 < x_2 < \ldots < x_n$ be an ordered sample of size *n* from GHLD-I (because the theory of order statistics is required in the estimation, an ordered sample is itself first considered). The log likelihood equations to estimate the parameters θ and σ are given by

$$\frac{\partial \log L}{\partial \sigma} = 0,$$

and

$$\frac{\partial \log L}{\partial \theta} = 0,$$

where L is the likelihood function of the considered sample:

$$\log L = \left[\begin{array}{l} n\log\frac{2\theta}{\sigma} + (\theta-1)\sum_{i=1}^{n}\log(1-e^{-x_i/\sigma}) \\ -(\theta+1)\sum_{i=1}^{n}\log(1+e^{-x_i/\sigma}) - \sum_{i=1}^{n}\frac{x_i}{\sigma} \end{array}\right]$$

$$\frac{\partial \log L}{\partial \sigma} = 0 \Rightarrow \left[\begin{array}{l} n - \sum_{i=1}^{n} z_i + (\theta-1)\sum_{i=1}^{n}\frac{z_i e^{-z_i}}{1-e^{-z_i}} \\ -(\theta+1)\sum_{i=1}^{n}\frac{z_i e^{-z_i}}{1+e^{-z_i}} \end{array}\right] = 0 \quad (2.3)$$

where $z_i = \frac{x_i}{\sigma}$

$$\frac{\partial \log L}{\partial \theta} = 0 \Rightarrow \hat{\theta} = \frac{n}{\sum_{i=1}^{n}\log(\frac{1+e^{-z_i}}{1-e^{-z_i}})}. \quad (2.4)$$

These two equations must be solved iteratively for θ and σ given a sample. The asymptotic variances and covariance of MLEs of θ and σ can be obtained by inverting the information matrix whose elements are the mathematical expectations of the following expressions.

$$-\left(\frac{\partial^2 \log L}{\partial \sigma^2}\right)=$$

$$\sum_{i=1}^{n}\frac{2z_i}{\sigma^2}+(\theta-1)\sum_{i=1}^{n}\frac{z_i e^{-2z_i}+z_i^2 e^{-2z_i}-z_i e^{-z_i}}{(1-e^{-z_i})^2}$$

$$+(\theta+1)\sum_{i=1}^{n}\frac{\frac{z_i}{\sigma}e^{-z_i}+\frac{2z_i^2}{\sigma}e^{-x_i/\sigma}-\frac{2z_i}{\sigma^2}e^{-2z_i}}{(1+e^{-z_i})^2}-\frac{n}{\sigma^2} \quad (2.5)$$

$$-\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)=\frac{n}{\theta^2}, \quad (2.6)$$

and

$$-\left(\frac{\partial^2 \log L}{\partial \sigma \partial \theta}\right)=\frac{1}{\sigma}\left[\sum_{i=1}^{n}\frac{z_i e^{-z_i}}{1-e^{-z_i}}+\sum_{i=1}^{n}\frac{z_i e^{-z_i}}{1+e^{-z_i}}\right]. \quad (2.7)$$

Expressions (2.5), (2.6) and (2.7) evaluated at estimates of θ, σresult in an estimated dispersion matrix. This process is iterative in nature; to obtain analytical estimators the estimating equations can be reasonably approximated by some admissible expressions. The solutions of resulting approximating equations are termed approximate MLEs or modified MLEs. Such modifications have been proposed and studied by many researchers in various models, including: Tiku (1967); Mehrotra & Nanda (1974); Pearson & Rootzen (1977); Tiku& Suresh (1992); Rosaiah, et al. (1993a); Rosaiah, et al. (1993b); Kantam & Srinivasa Rao (1993); Kantam & Srinivasa Rao (2002); Kantam & Sri Ram (2003. These works generally estimate the scale parameter of the model and the shape parameter (if any), is either assumed to be known or estimated by another method and the resulting estimator of the shape parameter is used to for modified ML estimation of the scale parameter. The former is the most usual situation and the latter is occasional. This study attempts to estimateσ when θ is known. In order to obtain an analytical expression for σ, the expressions in Equation (2.3) are approximated by some linear function in the respective population quantiles:

$$G(z_i)=\frac{Z_i e^{-z_i}}{1-e^{-z_i}}\cong \gamma_i+\delta_i z_i, \quad (2.8)$$

$$K(z_i)=\frac{Z_i e^{-z_i}}{1+e^{-z_i}}\cong \alpha_i+\beta_i z_i, \quad (2.9)$$

where $\alpha_i,\beta_i,\gamma_i$ and δ_i are to be found. After using the approximations in (2.3) the solution for σ is:

$$\hat{\sigma}=\frac{\sum_{i=1}^{n}x_i-(\theta-1)\sum_{i=1}^{n}\delta_i x_i-(\theta+1)\sum_{i=1}^{n}\beta_i x_i}{n+(\theta-1)\sum_{i=1}^{n}\gamma_i+(\theta+1)\sum_{i=1}^{n}\alpha_i}. \quad (2.10)$$

This estimator is named the MMLE of σ, which is a linear estimator in x_i's. Now consider a method to obtain$\alpha_i,\beta_i,\gamma_I$ and δ_i.

Let $p_i=\frac{i}{n+1}$; i = 1, 2, …, n, and let t_i,t_i^* be the solutions of the following equations, for example:

$$F(t_i)=p_j-\sqrt{\frac{p_i q_i}{n}}=p_i', \quad (2.11)$$

$$F(t_i^*)=p_i+\sqrt{\frac{p_i q_i}{n}}=p_i'', \quad (2.12)$$

with $q_i=1-p_i$ and where F(.) is cdf of GHLD-I.

The intercepts γ_i, α_i and slopes δ_I and β_i, of linear approximations (2.8) and (2.9) are respectively given by

$$\delta_i = \frac{G(t_i^*) - G(t_i)}{t_i^* - t_i};$$
$$\beta_i = \frac{K(t_i^*) - K(t_i)}{t_i^* - t_i}, \quad 2.13)$$

$$\gamma = G(t_i^*) - \delta_i t_i^*, \quad (2.14)$$

and

$$\alpha_i = K(t_i^*) - \beta_i t_i^*, \quad (2.15)$$

Using the cdf of GHLD-I, the expressions for t_i, t_i^* are given by

$$t_i = log\left[\frac{1+(p_i^{'})^{1/\theta}}{1-(p_i^{'})^{1/\theta}}\right];\ t_i^* = log\left[\frac{1+(p_i^{''})^{1/\theta}}{1-(p_i^{''})^{1/\theta}}\right].$$

The values of $\gamma_i, \alpha_i, \beta_i$ and δ_i for various θ and *n* are shown in Tables 2.1a, 2.1b and 2.1c.

In this modified method, the basic principle is that certain expressions in the log likelihood equation are linearized in the neighborhood of the population quantile, which also depends on the size of the sample: The larger the sample, the closer the approximation, that is, the exactness of the approximation becomes finer for large values of *n*. Hence the approximate log likelihood equation and the exact log likelihood equation tend to each other as $n \to \infty$. Thus, the exact and modified MLEs are asymptotically identical (Tiku, et al., 1986). The same may not be true in small samples. Due to non-availability of analytical sampling variances, the modified ML method was compared with the exact ML method via Monte Carlo simulation. The empirical bias, variance, mean square error (MSE) of the exact and modified MLEs for samples of sizes 5(5) 20 and θ = 2(1) 4 are given in Table 2.2. The following observations are made based on results in Table 2.2: The MMLE over estimates the bias compared to the exact MLE for some sizes of samples and the exact MLE is slightly less biased than the MMLE. The empirical variance, MSE of MLE, MMLE corrected to third decimal place are nearly equal in most combinations of *n* and θ, indicating that exact MLE and MMLE are almost equally efficient in small samples.

GHLD-I vs. Exponential Models

The comparison between the GHLD-I and Exponential model is made with the help of the likelihood ratio (LR) criterion. GHLD-I is specified as the null population (P_o) and the Exponential model as alternative population (P_1). The test hypotheses proposed are:

H_0:a given sample belongs to GHLD-I (P_0)
vs.
H_1: the sample belongs to the population Exponential model (P_1).

Let L_1, L_0 respectively represent the likelihood function of a sample with population $P_1$and P_0. The LR criterion L_1/L_0 percentiles are obtained by simulation: 10,000 random samples sized *n*=5, 10, 15, 20 are generated from the null population $P_0$and its parameters are estimated using each sample. The value of the likelihood function of the null population is computed at the generated sample observations and the corresponding parameter estimates; this value is denoted by L_0.

Using the same sample generated from P_0, the parameters and likelihood function value of the alternative population are calculated and called L_1. The values of L_1/L_0 over 10,000 runs are sorted and selected percentiles are identified for a given *n* and θ (see Table 3.1). The entries under the column headings 0.95 in Table 3.1 may be considered the 5% level of significance critical values for discriminating between the GHLD-I and Exponential models. The powers of the test statistic L_1/L_0 are also evaluated through simulation by calculating L_1/L_0 with samples generated from exponential population (P_1) and estimating, the parameters calculating the values of the likelihood functions L_1, L_0 with sample from P_1. The proportion of L_1/L_0 values falling above 95th percentile of L_1/L_0 become power of the LR test criterion are shown in Table 3.2, which reveals the following: (1) The GHLD-I and Exponential model are indistinguishable with 0 power or negligible power of 0.02 for *n* = 5; (2) as *n* increases, the power increases specifically at θ = 4 and *n* = 20, resulting in significant discrimination between two populations with respect to the LR criterion.

Table 2.1a: Intercept and Slope of the Approximations
$G(Z_i) = \gamma_i + \delta_i z_i$ and $K(Z_i) = \alpha_i + \beta_i z_i$ when $\theta = 2$

n	i	γ_i	δ_i	α_i	β_i
5	1	0.0000	0.3660	0.0000	0.2113
	2	0.8945	-0.2953	0.2309	0.0090
	3	0.7922	-0.2269	0.3543	-0.0660
	4	0.6516	-0.1566	0.4121	-0.0888
	5	0.2576	0.0000	0.2103	0.0000
10	1	0.0000	0.6726	0.0000	0.2867
	2	0.9503	-0.3592	0.1281	0.1210
	3	0.9093	-0.3180	0.2123	0.0393
	4	0.8640	-0.2817	0.2853	-0.0169
	5	0.8127	-0.2474	0.3465	-0.0559
	6	0.7532	-0.2134	0.3933	-0.0809
	7	0.6821	-0.1784	0.4215	-0.0932
	8	0.5937	-0.1409	0.4236	-0.0925
	9	0.4745	-0.0980	0.3839	-0.0768
	10	0.1579	0.0000	0.1428	0.0000
15	1	0.0000	0.9142	0.0000	0.3232
	2	0.9675	-0.3859	0.0883	0.1815
	3	0.9417	-0.3538	0.1489	0.1074
	4	0.9146	-0.3263	0.2045	0.0522
	5	0.8855	-0.3011	0.2556	0.0091
	6	0.8540	-0.2773	0.3019	-0.0248
	7	0.8196	-0.2543	0.3428	-0.0513
	8	0.7816	-0.2315	0.3775	-0.0714
	9	0.7393	-0.2086	0.4049	-0.0854
	10	0.6915	-0.1852	0.4234	-0.0937
	11	0.6365	-0.1608	0.4309	-0.0963
	12	0.5716	-0.1350	0.4239	-0.0929
	13	0.4919	-0.1067	0.3966	-0.0825
	14	0.3868	-0.0739	0.3368	-0.0633
	15	0.1173	0.0000	0.1098	0.0000
20	1	0.0000	1.1201	0.0000	0.3456
	2	0.9758	-0.4015	0.0673	0.2202
	3	0.9571	-0.3743	0.1143	0.1528
	4	0.9377	-0.3513	0.1585	0.1011
	5	0.9174	-0.3306	0.2002	0.0593
	6	0.8960	-0.3113	0.2394	0.0246
	7	0.8732	-0.2929	0.2761	-0.0044
	8	0.8490	-0.2752	0.3100	-0.0286
	9	0.8230	-0.2578	0.3407	-0.0488
	10	0.7951	-0.2406	0.3681	-0.0651
	11	0.7650	-0.2234	0.3915	-0.0780
	12	0.7321	-0.2061	0.4104	-0.0876
	13	0.6961	-0.1885	0.4242	-0.0939
	14	0.6561	-0.1704	0.4319	-0.0969
	15	0.6112	-0.1516	0.4321	-0.0967
	16	0.5598	-0.1318	0.4232	-0.0928
	17	0.4998	-0.1106	0.4022	-0.0850
	18	0.4268	-0.0873	0.3643	-0.0726
	19	0.3318	-0.0602	0.2994	-0.0537
	20	0.0946	0.0000	0.0900	0.0000

Table 2.1b: Intercept and Slope of the Approximations
$G(Z_i) = \gamma_i + \delta_i z_i$ and $K(Z_i) = \alpha_i + \beta_i z_i$ when $\theta = 3$

n	i	γ_i	δ_i	α_i	β_i
5	1	0.0000	0.2211	0.0000	0.1533
	2	0.8191	-0.2449	0.3321	-0.0531
	3	0.7007	-0.1829	0.4098	-0.0891
	4	0.5559	-0.1230	0.4096	-0.0857
	5	0.1950	0.0000	0.1704	0.0000
10	1	0.0000	0.3825	0.0000	0.2167
	2	0.8959	-0.3060	0.2355	0.0193
	3	0.8393	-0.2652	0.3183	-0.0380
	4	0.7834	-0.2313	0.3743	-0.0705
	5	0.7247	-0.2003	0.4105	-0.0883
	6	0.6605	-0.1705	0.4283	-0.0956
	7	0.5877	-0.1407	0.4264	-0.0939
	8	0.5014	-0.1096	0.4003	-0.0838
	9	0.3908	-0.0749	0.3393	-0.0639
	10	0.1175	0.0000	0.1099	0.0000
15	1	0.0000	0.5000	0.0000	0.2500
	2	0.9232	-0.3332	0.1871	0.0659
	3	0.8838	-0.2997	0.2584	0.0069
	4	0.8467	-0.2727	0.3119	-0.0310
	5	0.8098	-0.2489	0.3537	-0.0572
	6	0.7721	-0.2270	0.3860	-0.0753
	7	0.7328	-0.2062	0.4098	-0.0873
	8	0.6911	-0.1861	0.4255	-0.0944
	9	0.6464	-0.1663	0.4327	-0.0972
	10	0.5975	-0.1463	0.4309	-0.0961
	11	0.5431	-0.1259	0.4187	-0.0911
	12	0.4809	-0.1046	0.3940	-0.0823
	13	0.4071	-0.0817	0.3526	-0.0692
	14	0.3132	-0.0558	0.2856	-0.0504
	15	0.0866	0.0000	0.0828	0.0000
20	1	0.0000	0.5948	0.0000	0.2716
	2	0.9377	-0.3498	0.1577	0.0986
	3	0.9068	-0.3203	0.2200	0.0406
	4	0.8782	-0.2969	0.2685	0.0015
	5	0.8506	-0.2766	0.3083	-0.0271
	6	0.8230	-0.2582	0.3414	-0.0487
	7	0.7950	-0.2411	0.3690	-0.0652
	8	0.7664	-0.2249	0.3915	-0.0776
	9	0.7369	-0.2093	0.4093	-0.0867
	10	0.7060	-0.1940	0.4225	-0.0929
	11	0.6736	-0.1790	0.4310	-0.0966
	12	0.6392	-0.1641	0.4346	-0.0979
	13	0.6024	-0.1491	0.4330	-0.0970
	14	0.5625	-0.1338	0.4257	-0.0940
	15	0.5188	-0.1182	0.4118	-0.0889
	16	0.4701	-0.1020	0.3901	-0.0814
	17	0.4145	-0.0848	0.3586	-0.0716
	18	0.3488	-0.0662	0.3138	-0.0587
	19	0.2659	-0.0451	0.2483	-0.0419
	20	0.0694	0.0000	0.0671	0.0000

Table 2.1c: Intercept and Slope of the Approximations
$G(Z_i) = \gamma_i + \delta_i z_i$ and $K(Z_i) = \alpha_i + \beta_i z_i$ when $\theta = 4$

n	i	γ_i	δ_i	α_i	β_i
5	1	0.0000	0.1580	0.0000	0.1200
	2	0.7573	-0.2116	0.3848	-0.0787
	3	0.6333	-0.1552	0.4235	-0.0931
	4	0.4907	-0.1026	0.3903	-0.0787
	5	0.1591	0.0000	0.1437	0.0000
10	1	0.0000	0.2657	0.0000	0.1735
	2	0.8453	-0.2687	0.3104	-0.0335
	3	0.7805	-0.2299	0.3771	-0.0717
	4	0.7198	-0.1986	0.4137	-0.0896
	5	0.6585	-0.1705	0.4300	-0.0962
	6	0.5936	-0.1440	0.4290	-0.0952
	7	0.5222	-0.1178	0.4105	-0.0878
	8	0.4397	-0.0909	0.3714	-0.0746
	9	0.3371	-0.0615	0.3032	-0.0546
	10	0.0948	0.0000	0.0901	0.0000
15	1	0.0000	0.3408	0.0000	0.2026
	2	0.8788	-0.2949	0.2655	0.0002
	3	0.8313	-0.2623	0.3305	-0.0432
	4	0.7890	-0.2367	0.3731	-0.0681
	5	0.7484	-0.2146	0.4021	-0.0834
	6	0.7081	-0.1946	0.4210	-0.0924
	7	0.6671	-0.1758	0.4314	-0.0968
	8	0.6246	-0.1577	0.4341	-0.0976
	9	0.5799	-0.1401	0.4292	-0.0954
	10	0.5320	-0.1226	0.4163	-0.0904
	11	0.4795	-0.1049	0.3946	-0.0829
	12	0.4208	-0.0866	0.3623	-0.0726
	13	0.3524	-0.0672	0.3164	-0.0594
	14	0.2673	-0.0454	0.2495	-0.0421
	15	0.0694	0.0000	0.0671	0.0000
20	1	0.0000	0.4000	0.0000	0.2222
	2	0.8975	-0.3113	0.2357	0.0255
	3	0.8588	-0.2821	0.2969	-0.0196
	4	0.8250	-0.2595	0.3392	-0.0473
	5	0.7934	-0.2403	0.3706	-0.0660
	6	0.7628	-0.2232	0.3943	-0.0789
	7	0.7326	-0.2074	0.4119	-0.0878
	8	0.7023	-0.1926	0.4243	-0.0937
	9	0.6714	-0.1784	0.4320	-0.0970
	10	0.6398	-0.1648	0.4353	-0.0982
	11	0.6071	-0.1514	0.4343	-0.0976
	12	0.5729	-0.1382	0.4289	-0.0954
	13	0.5368	-0.1250	0.4190	-0.0917
	14	0.4982	-0.1117	0.4042	-0.0864
	15	0.4566	-0.0982	0.3839	-0.0797
	16	0.4107	-0.0843	0.3571	-0.0715
	17	0.2994	-0.0541	0.2766	-0.0495
	18	0.2254	-0.0366	0.2141	-0.0346
	19	0.0555	0.0000	0.0541	0.0000
	20	0.0000	0.1580	0.0000	0.1200

Table 3.1: Percentiles of L_1/L_0
(P_0: GHLD-I; P_1: Exponential)

θ	n p	0.00135	0.001	0.025	0.05	0.9500	9750	0.99865
2	5	0.0295	0.0493	0.0743	0.1126	3.5	4.6	8.8
	10	0.0072	0.0303	0.0663	0.1241	11.0	15.3	58.6
	15	0.0068	0.0337	0.0744	0.1657	32.2	46.0	153.6
	20	0.0061	0.0308	0.0885	0.2034	88.1	133.9	575.4
3	5	0.0166	0.0308	0.0515	0.0860	10.5	15.9	44.7
	10	0.0037	0.0222	0.0613	0.1408	104.0	170.6	1217.2
	15	0.0045	0.0367	0.1059	0.3192	899.8	1595.0	10736.5
	20	0.0055	0.0518	0.2048	0.6646	7345.6	14907.5	110093.4
4	5	0.0134	0.0236	0.0417	0.0759	28.7547	49.0	207.9
	10	0.0026	0.0207	0.0714	0.1927	879.7	1698.5	20257.6
	15	0.0043	0.0551	0.2066	0.7839	21578.0	47370.6	587772.50
	20	0.0079	0.1255	0.6597	2.9169	496183.4	1314785.0	19609204.91

Table 3.2: Powers of LR Test Criterion at
α = 0.05: GHLD vs Exponential

θ	*n*	Power
2	5 10 15 20	0.0207 0.7786 0.8919 0.9422
3	5 10 15 20	0 0.9135 0.9772 0.9938
4	5 10 15 20	0 0.9552 0.9940 0.9941

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