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Generalized Modified Ratio Estimator for Estimation of Finite Population Mean

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A generalized modified ratio estimator is proposed for estimating the population mean using the known population parameters. It is shown that the simple random sampling without replacement sample mean, the usual ratio estimator, the linear regression estimator and all the existing modified ratio estimators are the particular cases of the proposed estimator. The bias and the mean squared error of the proposed estimator are derived and are compared with that of existing estimators. The conditions for which the proposed estimator performs better than the existing estimators are also derived. The performance of the proposed estimator is assessed with that of the existing estimators for certain natural populations

Keywords: Auxiliary variable, biases, natural population, mean squared error, parameters

Introduction

Consider a finite population $U = \{ U_1, U_2, \dots , U_N \}$ of N distinct and identifiable units. Let Y be a study variable with value Y_i measured on U_1, $i = 1, 2, 3, \dots , N$ giving a vector $Y = \{ Y_1, Y_2, \dots , Y_N \}$. The problem is to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample selected from the population U. The simple random sample mean is the simplest estimator for estimating the population mean. If an auxiliary variable X, closely related to the study variable Y, is available then one can improve the performance of the estimator of the study variable by using the known values of the population parameters of the auxiliary variable. That is, when the population parameters of the auxiliary variable X such as population mean, coefficient of variation, coefficient of kurtosis, coefficient of skewness etc., are known, then a number of estimators available in the literature (such as ratio, product and linear regression

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estimators and their modifications) perform better than the usual simple random sample mean under certain conditions. Among these estimators, many researchers have used the ratio estimator and its modifications for the estimation of the mean of the study variable (see for example Sisodia and Dwivedi (1981), Kadilar and Cingi (2006a, 2006b), Yan and Tian (2010) and Subramani and Kumarapandiyan (2012a, 2012c)). Before discussing further the existing estimators and the proposed estimators, the notations to be used in this article are described below:

N	Population size
n	Sample size
$f = n/N$	Sampling fraction
Y	Study variable
X	Auxiliary variable
$\bar{X}, \bar{Y}$	Population means
$\bar{x}, \bar{y}$	Sample means
S_x, S_y	Population standard deviations
C_x, C_y	Co-efficient of variations
ρ	Co-efficient of correlation between X and Y
β_1	Co-efficient of skewness of the auxiliary variable
β_2	Co-efficient of kurtosis of the auxiliary variable
M_d	Median of the auxiliary variable
$B(.)$	Bias of the estimator
$MSE(.)$	Mean squared error of the estimator
$\hat{\bar{Y}}_i \left(\hat{\bar{Y}}_{p_j}\right)$	ith existing (jth proposed) modified ratio estimator of $\bar{Y}$

In case of simple random sampling without replacement (SRSWOR), the sample mean $\bar{y}_{srs}$ is used to estimate population mean $\bar{Y}$, which is an unbiased estimator, and its variance is given below:

$$V\left(\bar{y}_{srs}\right) = \frac{(1-f)}{n} S_y^2 \tag{1}$$

The ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is defined as:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \text{ where } \hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x} \tag{2}$$

The bias and mean squared error of the ratio estimator to the first degree of approximation are given below:

$$B\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}\left(C_x^2 - \rho C_x C_y\right) \tag{3}$$

$$MSE\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right) \tag{4}$$

The usual linear regression estimator together with its variance is given below:

$$\hat{\bar{Y}}_{lr} = \bar{y} + \beta\left(\bar{X} - \bar{x}\right) \tag{5}$$

$$V\left(\hat{\bar{Y}}_{lr}\right) = \frac{(1-f)}{n}S_y^2\left(1-\rho^2\right) \tag{6}$$

Sisodia and Dwivedi (1981) have suggested a modified ratio estimator using the co-efficient of variation of auxiliary variable *X* for estimating $\bar{Y}$. When the co-efficient of kurtosis of auxiliary variable *X* is known, Singh et al. (2004) has developed a modified ratio estimator. Singh and Tailor (2003) proposed another estimator for estimating $\bar{Y}$ when the population correlation co-efficient between *X* and *Y* is known. By using the population variance of auxiliary variable *X*, Singh (2003) proposed another modified ratio estimator for estimating population mean. More recently, Yan and Tian (2010) has suggested another modified ratio estimator using the co-efficient of skewness of the auxiliary variable *X*, and Subramani and Kumarapandiyan (2013a) suggested a new modified ratio estimator using known population median of auxiliary variable *X*.

Upadhyaya and Singh (1999) suggested another modified ratio estimator using the linear combination of co-efficient of variation and co-efficient of kurtosis. Singh (2003) used the linear combination of co-efficient of kurtosis and standard deviation and co-efficient of skewness and standard deviation for estimating the populations mean $\bar{Y}$. Motivated by Singh (2003), Yan and Tian (2010) used the linear combination of co-efficient of kurtosis and co-efficient of skewness and co-efficient of variation and co-efficient of skewness. Subramani and Kumarapandiyan (2012a, 2012b, 2012c and 2013b) suggested modified ratio estimators using known median and co-efficient of kurtosis, median and co-efficient of skewness, median and co-efficient of variation and median and co-efficient of correlation.

More detailed discussion about the ratio estimator and its modification can be found in Abdia and Shahbaz (2006), Ahmad et al. (2009), Al-Jararha and Al-Haj Ebrahem (2012), Bhushan (2012), Cochran (1977), Dalabehera and Sahoo (1994), David and Sukhatme (1974), Goodman and Hartley (1958), Gupta and Shabbir (2008), Jhajj et al. (2006), Kadilar and Cingi (2003, 2004), Khoshnevisan et al. (2007), Koyuncu and Kadilar (2009), Kulkarni (1978), Murthy (1967), Naik and Gupta (1991), Olkin (1958), Pathak (1964), Perri (2007), Ray and Sahai (1980), Reddy (1973), Robinson (1987), Sen (1993), Shabbir and Yaab (2003), Sharma and Tailor (2010), Singh and Chaudhary (1986), Singh (2003), Singh and Espejo (2003), Singh and Agnihotri (2008), Singh and Solanki (2012), Singh and Tailor (2003, 2005), Singh et al. (2004, 2008), Sisodia and Dwivedi (1981) , Solanki et al. (2012), Srivenkataramana (1980), Tailor and Sharma (2009), Tin (1965), Upadhyaya and Singh (1999) and Yan and Tian (2010).

The following table contains all modified ratio estimators using known population parameters of the auxiliary variable in which some of the estimators are already suggested in the literature. The remaining estimators are introduced in this article:

Table 1. Modified Ratio estimators with the constant, the bias, and the mean squared errors.

Estimator	**Constant θ_i**	**Bias – B(.)**	**Mean squared error MSE(.)**
$\hat{\bar{Y}}=\bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$ *Sisodia and Dwivedi (1981)*	$\theta_1=\frac{\bar{X}}{\bar{X}+C_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_1^2C_x^2-\theta_1\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_1^2C_x^2-2\theta_1\rho C_xC_y\right)$
$\hat{\bar{Y}}_2=\bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$ *Singh et al. (2004)*	$\theta_2=\frac{\bar{X}}{\bar{X}+\beta_2}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_2^2C_x^2-\theta_2\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_2^2C_x^2-2\theta_2\rho C_xC_y\right)$
$\hat{\bar{Y}}_3=\bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$ *Yan and Tian (2010)*	$\theta_3=\frac{\bar{X}}{\bar{X}+\beta_1}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_3^2C_x^2-\theta_3\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_3^2C_x^2-2\theta_3\rho C_xC_y\right)$
$\hat{\bar{Y}}_4=\bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$ *Singh and Tailor (2003)*	$\theta_4=\frac{\bar{X}}{\bar{X}+\rho}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_4^2C_x^2-\theta_4\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_4^2C_x^2-2\theta_4\rho C_xC_y\right)$

Table 1 Continued

Estimator	Constant θ_i	Bias – B(.)	Mean squared error MSE(.)
$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{\bar{X}+S_x}{\bar{x}+S_x}\right]$ *Singh (2003)*	$\theta_5 = \frac{\bar{X}}{\bar{X}+S_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_5^2 C_x^2 - \theta_5 \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 \rho C_x C_y\right)$
$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\bar{X}+M_d}{\bar{x}+M_d}\right]$ *Subramani and Kumarapandiyan (2013a)*	$\theta_6 = \frac{\bar{X}}{\bar{X}+M_d}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_6^2 C_x^2 - \theta_6 \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 \rho C_x C_y\right)$
$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{\beta_2\bar{X}+C_x}{\beta_2\bar{x}+C_x}\right]$ *Upadhyaya and Singh (1999)*	$\theta_7 = \frac{\beta_2\bar{X}}{\beta_2\bar{X}+C_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_7^2 C_x^2 - \theta_7 \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 \rho C_x C_y\right)$
$\hat{\bar{Y}}_8 = \bar{y}\left[\frac{C_x\bar{X}+\beta_2}{C_x\bar{x}+\beta_2}\right]$ *Upadhyaya and Singh (1999)*	$\theta_8 = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_2}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_8^2 C_x^2 - \theta_8 \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 \rho C_x C_y\right)$
$\hat{\bar{Y}}_9 = \bar{y}\left[\frac{\beta_1\bar{X}+C_x}{\beta_1\bar{x}+C_x}\right]$	$\theta_9 = \frac{\beta_1\bar{X}}{\beta_1\bar{X}+C_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_9^2 C_x^2 - \theta_9 \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 \rho C_x C_y\right)$
$\hat{\bar{Y}}_{10} = \bar{y}\left[\frac{C_x\bar{X}+\beta_1}{C_x\bar{x}+\beta_1}\right]$ *Yan and Tian (2010)*	$\theta_{10} = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_1}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{10}^2 C_x^2 - \theta_{10} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{10}^2 C_x^2 - 2\theta_{10} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{11} = \bar{y}\left[\frac{\rho\bar{X}+C_x}{\rho\bar{x}+C_x}\right]$	$\theta_{11} = \frac{\rho\bar{X}}{\rho\bar{X}+C_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{11}^2 C_x^2 - \theta_{11} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{11}^2 C_x^2 - 2\theta_{11} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{12} = \bar{y}\left[\frac{C_x\bar{X}+\rho}{C_x\bar{x}+\rho}\right]$	$\theta_{12} = \frac{C_x\bar{X}}{C_x\bar{X}+\rho}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{12}^2 C_x^2 - \theta_{12} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{12}^2 C_x^2 - 2\theta_{12} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{13} = \bar{y}\left[\frac{S_x\bar{X}+C_x}{S_x\bar{x}+C_x}\right]$	$\theta_{13} = \frac{S_x\bar{X}}{S_x\bar{X}+C_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{13}^2 C_x^2 - \theta_{13} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{11}^2 C_x^2 - 2\theta_{11} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{14} = \bar{y}\left[\frac{C_x\bar{X}+S_x}{C_x\bar{x}+S_x}\right]$	$\theta_{14} = \frac{C_x\bar{X}}{C_x\bar{X}+S_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{14}^2 C_x^2 - \theta_{14} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{14}^2 C_x^2 - 2\theta_{14} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{15} = \bar{y}\left[\frac{M_d\bar{X}+C_x}{M_d\bar{x}+C_x}\right]$	$\theta_{15} = \frac{M_d\bar{X}}{M_d\bar{X}+C_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{15}^2 C_x^2 - \theta_{15} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{15}^2 C_x^2 - 2\theta_{15} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{16} = \bar{y}\left[\frac{C_x\bar{X}+M_d}{C_x\bar{x}+M_d}\right]$ *Subramani and Kumarapandiyan (2012c)*	$\theta_{16} = \frac{C_x\bar{X}}{C_x\bar{X}+M_d}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{16}^2 C_x^2 - \theta_{16} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{16}^2 C_x^2 - 2\theta_{16} \rho C_x C_y\right)$
$\hat{\bar{Y}}_{17} = \bar{y}\left[\frac{\beta_1\bar{X}+\beta_2}{\beta_1\bar{x}+\beta_2}\right]$ *Yan and Tian (2010)*	$\theta_{17} = \frac{\beta_1\bar{X}}{\beta_1\bar{X}+\beta_2}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{17}^2 C_x^2 - \theta_{17} \rho C_x C_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{17}^2 C_x^2 - 2\theta_{17} \rho C_x C_y\right)$

Table 1 Continued

Estimator	Constant θ_i	Bias – B(.)	Mean squared error MSE(.)
$\hat{\bar{Y}}_{18}=\bar{y}\left[\frac{\beta_2\bar{X}+\beta_1}{\beta_2\bar{x}+\beta_1}\right]$ *Yan and Tian (2010)*	$\theta_{18}=\frac{\beta_2\bar{X}}{\beta_2\bar{X}+\beta_1}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{18}^2C_x^2-\theta_{18}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{18}^2C_x^2-2\theta_{18}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{19}=\bar{y}\left[\frac{\rho\bar{X}+\beta_2}{\rho\bar{x}+\beta_2}\right]$	$\theta_{19}=\frac{\rho\bar{X}}{\rho\bar{X}+\beta_2}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{19}^2C_x^2-\theta_{19}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{19}^2C_x^2-2\theta_{19}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{20}=\bar{y}\left[\frac{\beta_2\bar{X}+\rho}{\beta_2\bar{x}+\rho}\right]$	$\theta_{20}=\frac{\beta_2\bar{X}}{\beta_2\bar{X}+\rho}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{20}^2C_x^2-\theta_{20}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{20}^2C_x^2-2\theta_{20}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{21}=\bar{y}\left[\frac{S_x\bar{X}+\beta_2}{S_x\bar{x}+\beta_2}\right]$	$\theta_{21}=\frac{S_x\bar{X}}{S_x\bar{X}+\beta_2}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{21}^2C_x^2-\theta_{21}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{21}^2C_x^2-2\theta_{21}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{22}=\bar{y}\left[\frac{\beta_2\bar{X}+S_x}{\beta_2\bar{x}+S_x}\right]$ *Singh (2003)*	$\theta_{22}=\frac{\beta_2\bar{X}}{\beta_2\bar{X}+S_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{22}^2C_x^2-\theta_{22}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{22}^2C_x^2-2\theta_{22}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{23}=\bar{y}\left[\frac{M_d\bar{X}+\beta_2}{M_d\bar{x}+\beta_2}\right]$	$\theta_{23}=\frac{M_d\bar{X}}{M_d\bar{X}+\beta_2}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{23}^2C_x^2-\theta_{23}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{23}^2C_x^2-2\theta_{23}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{24}=\bar{y}\left[\frac{\beta_2\bar{X}+M_d}{\beta_2\bar{x}+M_d}\right]$ *Subramani and Kumarapandiyan (2012a)*	$\theta_{24}=\frac{\beta_2\bar{X}}{\beta_2\bar{X}+M_d}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{24}^2C_x^2-\theta_{24}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{24}^2C_x^2-2\theta_{24}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{25}=\bar{y}\left[\frac{\rho\bar{X}+\beta_1}{\rho\bar{x}+\beta_1}\right]$	$\theta_{25}=\frac{\rho\bar{X}}{\rho\bar{X}+\beta_1}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{25}^2C_x^2-\theta_{25}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{25}^2C_x^2-2\theta_{25}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{26}=\bar{y}\left[\frac{\beta_1\bar{X}+\rho}{\beta_1\bar{x}+\rho}\right]$	$\theta_{26}=\frac{\beta_1\bar{X}}{\beta_1\bar{X}+\rho}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{26}^2C_x^2-\theta_{26}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{26}^2C_x^2-2\theta_{26}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{27}=\bar{y}\left[\frac{S_x\bar{X}+\beta_1}{S_x\bar{x}+\beta_1}\right]$	$\theta_{27}=\frac{S_x\bar{X}}{S_x\bar{X}+\beta_1}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{27}^2C_x^2-\theta_{27}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{27}^2C_x^2-2\theta_{27}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{28}=\bar{y}\left[\frac{\beta_1\bar{X}+S_x}{\beta_1\bar{x}+S_x}\right]$ *Singh (2003)*	$\theta_{28}=\frac{\beta_1\bar{X}}{\beta_1\bar{X}+S_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{28}^2C_x^2-\theta_{28}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{28}^2C_x^2-2\theta_{28}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{29}=\bar{y}\left[\frac{M_d\bar{X}+\beta_1}{M_d\bar{x}+\beta_1}\right]$	$\theta_{29}=\frac{M_d\bar{X}}{M_d\bar{X}+\beta_1}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{29}^2C_x^2-\theta_{29}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{29}^2C_x^2-2\theta_{29}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{30}=\bar{y}\left[\frac{\beta_1\bar{X}+M_d}{\beta_1\bar{x}+M_d}\right]$ *Subramani and Kumarapandiyan (2012b)*	$\theta_{30}=\frac{\beta_1\bar{X}}{\beta_1\bar{X}+M_d}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{30}^2C_x^2-\theta_{30}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{30}^2C_x^2-2\theta_{30}\rho C_xC_y\right)$

Table 1 Continued

Estimator	Constant θ_i	Bias – $B(.)$	Mean squared error MSE(.)
$\hat{\bar{Y}}_{31} = \bar{y}\left[\frac{S_x\bar{X}+\rho}{S_x\bar{x}+\rho}\right]$	$\theta_{31} = \frac{S_x\bar{X}}{S_x\bar{X}+\rho}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{31}^2C_x^2 - \theta_{31}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{31}^2C_x^2 - 2\theta_{31}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{32} = \bar{y}\left[\frac{\rho\bar{X}+S_x}{\rho\bar{x}+S_x}\right]$	$\theta_{32} = \frac{\rho\bar{X}}{\rho\bar{X}+S_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{32}^2C_x^2 - \theta_{32}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{32}^2C_x^2 - 2\theta_{32}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{33} = \bar{y}\left[\frac{M_d\bar{X}+\rho}{M_d\bar{x}+\rho}\right]$	$\theta_{33} = \frac{M_d\bar{X}}{M_d\bar{X}+\rho}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{33}^2C_x^2 - \theta_{33}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{33}^2C_x^2 - 2\theta_{33}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{34} = \bar{y}\left[\frac{\rho\bar{X}+M_d}{\rho\bar{x}+M_d}\right]$ *Subramani and Kumarapandiyan (2013b)*	$\theta_{34} = \frac{\rho\bar{X}}{\rho\bar{X}+M_d}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{34}^2C_x^2 - \theta_{34}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{34}^2C_x^2 - 2\theta_{34}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{35} = \bar{y}\left[\frac{M_d\bar{X}+S_x}{M_d\bar{x}+S_x}\right]$	$\theta_{35} = \frac{M_d\bar{X}}{M_d\bar{X}+S_x}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{35}^2C_x^2 - \theta_{35}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{35}^2C_x^2 - 2\theta_{35}\rho C_xC_y\right)$
$\hat{\bar{Y}}_{36} = \bar{y}\left[\frac{S_x\bar{X}+M_d}{S_x\bar{x}+M_d}\right]$	$\theta_{36} = \frac{S_x\bar{X}}{S_x\bar{X}+M_d}$	$\frac{(1-f)}{n}\bar{Y}\left(\theta_{36}^2C_x^2 - \theta_{36}\rho C_xC_y\right)$	$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{36}^2C_x^2 - 2\theta_{36}\rho C_xC_y\right)$

Proposed generalized ratio estimator

As stated earlier, the performance of the estimator of the study variable can be improved by using the known population parameters of the auxiliary variable, which are positively correlated with that of study variable.

The proposed generalized modified ratio estimator for estimating the population mean $\bar{Y}$ is defined as:

$$\hat{\bar{Y}}_{p_i} = \bar{y}\left[\frac{\bar{X}+(1+\alpha)\lambda_i}{\bar{x}+(1+\alpha)\lambda_i}\right]; i = 1, 2, 3, \ldots, 36 \tag{7}$$

The bias and mean squared error of the proposed estimator $\hat{\bar{Y}}_{p_i}$ have been derived (see Appendix A) and are given below:

$$B\left(\hat{\bar{Y}}_{p_i}\right) = \frac{(1-f)}{n}\bar{Y}\left(\theta_{p_i}^2C_x^2 - \theta_{p_i}\rho C_xC_y\right); i = 1, 2, 3, \ldots, 36 \tag{8}$$

$$MSE\left(\hat{\bar{Y}}_{p_i}\right)=\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{p_i}^2C_x^2-2\theta_{p_i}\rho C_xC_y\right);$$
$$\text{where } \theta_{p_i}=\frac{\bar{x}}{\bar{x}+(1-\alpha)\lambda_i}; i=1,2,3,\ldots,36 \tag{9}$$

where $\lambda_1=C_x$, $\lambda_2=\beta_2$, $\lambda_3=\beta_1$, $\lambda_4=\rho$, $\lambda_5=S_x$, $\lambda_6=M_d$, $\lambda_7=C_x/\beta_2$, $\lambda_8=\beta_2/C_x$, $\lambda_9=C_x/\beta_1$, $\lambda_{10}=\beta_1/C_x$, $\lambda_{11}=C_x/\rho$, $\lambda_{12}=\rho/C_x$, $\lambda_{13}=C_x/S_x$, $\lambda_{14}=S_x/C_x$, $\lambda_{15}=C_x/M_d$, $\lambda_{16}=M_d/C_x$, $\lambda_{17}=\beta_2/\beta_1$, $\lambda_{18}=\beta_1/\beta_2$, $\lambda_{19}=\beta_2/\rho$, $\lambda_{20}=\rho/\beta_2$, $\lambda_{21}=\beta_2/S_x$, $\lambda_{22}=S_x/\beta_2$, $\lambda_{23}=\beta_2/M_d$, $\lambda_{24}=M_d/\beta_2$, $\lambda_{25}=\beta_1/\rho$, $\lambda_{26}=\rho/\beta_1$, $\lambda_{27}=\beta_1/S_x$, $\lambda_{28}=S_x/\beta_1$, $\lambda_{29}=\beta_1/M_d$, $\lambda_{30}=M_d/\beta_1$, $\lambda_{31}=\rho/S_x$, $\lambda_{32}=S_x/\rho$, $\lambda_{33}=\rho/M_d$, $\lambda_{34}=M_d/\rho$, $\lambda_{35}=S_x/M_d$, and $\lambda_{36}=M_d/S_x$

Efficiency of the proposed estimator

The variance of SRSWOR sample mean $\bar{y}_{srs}$ is given below:

$$V(\bar{y}_{srs})=\frac{(1-f)}{n}S_y^2 \tag{10}$$

The bias and mean squared error of the usual ratio estimator $\hat{\bar{Y}}_R$ to the first degree of approximation are given below:

$$B\left(\hat{\bar{Y}}_R\right)=\frac{(1-f)}{n}\bar{Y}\left(C_x^2-\rho C_xC_y\right)$$
$$MSE\left(\hat{\bar{Y}}_R\right)=\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+C_x^2-2\rho C_xC_y\right) \tag{11}$$

The bias and the mean squared error of the modified ratio estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_{36}$ listed in the Table 1 are represented in a single class as given below:

$$\hat{\hat{\bar{Y}}}_i = \bar{y}\left[\frac{\bar{X}+\lambda_i}{\bar{x}+\lambda_i}\right]; i = 1,2,3,\ldots,36$$
$$B\left(\hat{\hat{\bar{Y}}}_i\right) = \frac{(1-f)}{n}\bar{Y}\left(\theta_i^2 C_x^2 - \rho\theta_i C_x C_y\right); i = 1,2,3,\ldots,36 \tag{12}$$
$$MSE\left(\hat{\hat{\bar{Y}}}_i\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y\right) \text{ where } \theta_i = \frac{\bar{X}}{\bar{X}+\lambda_i}; i = 1,2,3,\ldots,36$$

As discussed earlier, the bias, the mean squared error and the constant of the proposed modified ratio estimator $\hat{\hat{\bar{Y}}}_{p_i}$ are given below:

$$B\left(\hat{\hat{\bar{Y}}}_{p_i}\right) = \frac{(1-f)}{n}\bar{Y}\left(\theta_{p_i}^2 C_x^2 - \rho\theta_{p_i} C_x C_y\right); i = 1,2,3,\ldots,36$$
$$MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{p_i}^2 C_x^2 - 2\rho\theta_{p_i} C_x C_y\right) \tag{13}$$
$$\text{where } \theta_{p_i} = \frac{\bar{X}}{\bar{X}+(1+\alpha)\lambda_i}; i = 1,2,3,\ldots,36$$

where $\lambda_1 = C_x$, $\lambda_2 = \beta_2$, $\lambda_3 = \beta_1$, $\lambda_4 = \rho$, $\lambda_5 = S_x$, $\lambda_6 = M_d$, $\lambda_7 = C_x / \beta_2$, $\lambda_8 = \beta_2 / C_x$, $\lambda_9 = C_x / \beta_1$, $\lambda_{10} = \beta_1 / C_x$, $\lambda_{11} = C_x / \rho$, $\lambda_{12} = \rho / C_x$, $\lambda_{13} = C_x / S_x$, $\lambda_{14} = S_x / C_x$, $\lambda_{15} = C_x / M_d$, $\lambda_{16} = M_d / C_x$, $\lambda_{17} = \beta_2 / \beta_1$, $\lambda_{18} = \beta_1 / \beta_2$, $\lambda_{19} = \beta_2 / \rho$, $\lambda_{20} = \rho / \beta_2$, $\lambda_{21} = \beta_2 / S_x$, $\lambda_{22} = S_x / \beta_2$, $\lambda_{23} = \beta_2 / M_d$, $\lambda_{24} = M_d / \beta_2$, $\lambda_{25} = \beta_1 / \rho$, $\lambda_{26} = \rho / \beta_1$, $\lambda_{27} = \beta_1 / S_x$, $\lambda_{28} = S_x / \beta_1$, $\lambda_{29} = \beta_1 / M_d$, $\lambda_{30} = M_d / \beta_1$, $\lambda_{31} = \rho / S_x$, $\lambda_{32} = S_x / \rho$, $\lambda_{33} = \rho / M_d$, $\lambda_{34} = M_d / \rho$, $\lambda_{35} = S_x / M_d$, and $\lambda_{36} = M_d / S_x$

From the expressions given in (10) and (13), the conditions (see Appendix B) for which the proposed estimator $\hat{\hat{\bar{Y}}}_{p_i}$ are more efficient than the simple random sampling without replacement (SRSWOR) sample mean $\bar{y}_{srs}$ were derived and are:

$$MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right) \le V\left(\bar{y}_r\right) \text{ if } \theta_{p_i} \le 2\rho\frac{C_y}{C_x} \tag{14}$$

From the expressions given in (11) and (13), the conditions (see Appendix C) for which the proposed estimators $\hat{\bar{Y}}_{p_i}$ are more efficient than the usual ratio estimator $\hat{\bar{Y}}_R$ were derived and are:

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) \le MSE\left(\hat{\bar{Y}}_R\right) \text{ either } \frac{2\rho C_y}{C_x} - 1 \le \theta_{p_i} \le 1 \text{ (or) } 1 \le \theta_{p_i} \le \frac{2\rho C_y}{C_x} - 1 \quad (15)$$

From the expressions given in (12) and (13), the conditions (see Appendix D) for which the proposed estimators $\hat{\bar{Y}}_{p_j}; j = 1, 2, \ldots, 5$ are more efficient than the existing modified ratio estimators given in Class 1, $\bar{Y}_i; i = 1, 2, 3, \ldots, 11$ were derived and are:

$$MSE\left(\hat{\bar{Y}}_{p_j}\right) \le MSE\left(\hat{\bar{Y}}_i\right) \text{ either } \frac{2\rho C_y}{C_x} - \theta_i \le \theta_{p_j} \le \theta_i \text{ (or) } \theta_i \le \theta_{p_j} \le \frac{2\rho C_y}{C_x} - \theta_i \quad (16)$$

The conditions in terms of α in which proposed estimator $\hat{\bar{Y}}_{p_i}$ performs better than the simple random sampling without replacement (SRSWOR) sample mean $\bar{y}_{srs}$ were obtained and are:

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) \le V\left(\bar{y}_r\right) \text{ if } \alpha_i \ge \frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right] \quad (17)$$

From the expression given in (15), the range of α in which proposed estimator $\hat{\bar{Y}}_{p_i}$ performs better than the usual ratio estimator $\hat{\bar{Y}}_R$ is determined and is:

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) \le MSE\left(\hat{\bar{Y}}_R\right) \text{ either } -1 \le \alpha_i \le \frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho - 1\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right]$$

$$\text{(or)} \quad (18)$$

$$\frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho - 1\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right] \le \alpha_i \le -1; i = 1, 2, 3, \ldots, 36$$

From the expression given in (16), the range of α in which proposed estimator $\hat{\bar{Y}}_{p_i}$ performs better than the existing modified ratio estimators listed in Table 1 is:

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) \le MSE\left(\hat{\bar{Y}}_i\right) \text{ either } 0 \le \alpha_i \le \frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho - \theta_i\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right]$$

(or) (19)

$$\frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho - \theta_i\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right] \le \alpha_i \le 0; i = 1, 2, 3, \ldots, 36$$

Particular case:

1) At $\alpha_i = \frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right]; i = 1, 2, 3, \ldots, 36,$ the mean squared error of the proposed estimator $\hat{\bar{Y}}_{p_i}; i = 1, 2, 3, \ldots, 36$ equal to the variance of the SRSWOR sample mean $\bar{y}_{srs}$.

2) At limit point $\alpha_i = \frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho - 1\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right]$ or -1 the mean squared error of the proposed estimator $\hat{\bar{Y}}_{p_i}; i = 1, 2, 3, \ldots, 36$ equal to the mean squared error of the usual ratio estimator $\hat{\bar{Y}}_R$

3) At limit point $\frac{\bar{X}}{\lambda_i}\left[\left(2\frac{C_y}{C_x}\rho - \theta_i\right)^{-1} - 1 - \frac{\lambda_i}{\bar{X}}\right]$ or 0 the mean squared error of the proposed estimator $\hat{\bar{Y}}_{p_i}; i = 1, 2, 3, \ldots, 36$ the mean squared error of the existing modified ratio estimators $\hat{\bar{Y}}_i; i = 1, 2, 3, \ldots, 36$

4) At $\alpha_i = \frac{\bar{X}}{\lambda_i}\left[\left(\frac{C_y}{C_x}\rho\right)^{-1} - 1\right] - 1; i = 1, 2, 3, \ldots, 36,$ the means squared error of the proposed estimator $\hat{\bar{Y}}_{p_i}; i = 1, 2, 3, \ldots, 36$ equal to the variance of the usual linear regression estimator $\hat{\bar{Y}}_{1r}$

Numerical Study

The performance of the proposed generalized modified ratio estimator is assessed with that of the SRSWOR sample mean, the usual ratio estimator and the existing modified ratio estimators listed in Table 1 for certain natural populations. In this connection, four natural populations for the assessment of the performance of the proposed estimators with that of existing estimators were considered. Population 1 is taken from Singh and Chaudhary (1986) given in page 108; population 2 and population 3 are taken from Singh and Chaudhary (1986) given in page 177; population 4 is taken from Cochran (1977) given in page 152. The population parameters and the constants computed from the above populations are given below in Table 2, whereas the range of α in which proposed estimator performs better than the existing estimators, the constants, the biases and the mean squared errors of the existing and proposed estimators for the above populations are respectively given from the Tables 3 to 8.

Table 2. Parameters and constants of the population

Parameters	Population 1	Population 2	Population 3	Population 4
N	70	34	34	49
n	25	20	20	20
$\bar{Y}$	96.7000	856.4118	85.6412	127.7959
$\bar{X}$	175.2671	208.8824	19.9441	103.1429
ρ	0.7293	0.4491	0.4453	0.9817
S_y	60.4714	733.1407	73.3141	123.1212
C_y	0.6254	0.8561	0.8561	0.9634
S_x	140.8572	150.5060	15.0215	104.4051
C_x	0.8037	0.7205	0.7532	1.0122
$\beta_2(x)$	7.0952	0.0974	3.7257	7.5114
$\beta_1(x)$	1.9507	0.9782	1.1823	2.2553
M_d	121.5000	150.0000	14.2500	64.0000

Table 3. Range of *α* in which proposed estimator performs better than the usual ratio estimator

Estimator	***α* range (α_L, α_u)**			
	Population 1	**Population 2**	**Population 3**	**Population 4**
$\hat{\bar{Y}}_{p_1}$	(-1, 1396.1641)	(-1, 4023.1475)	(-1, 2138.4916)	(-1, 14.3885)
$\hat{\bar{Y}}_{p_2}$	(-1, 157.2620)	(-1, 29751.6396)	(-1, 431.5268)	(-1, 1.0738)
$\hat{\bar{Y}}_{p_3}$	(-1, 574.6399)	(-1, 2963.2549)	(-1, 1361.9917)	(-1, 5.9068)
$\hat{\bar{Y}}_{p_4}$	(-1, 1538.6966)	(-1, 6454.9931)	(-1, 3617.8302)	(-1, 14.8665)
$\hat{\bar{Y}}_{p_5}$	(-1, 6.9719)	(-1, 18.2651)	(-1, 106.2772)	(-1, -0.8508)
$\hat{\bar{Y}}_{p_6}$	(-1, 8.2420)	(-1, 18.3301)	(-1, 112.0853)	(-1, -0.7566)
$\hat{\bar{Y}}_{p_7}$	(-1, 9912.1584)	(-1, 391.1699)	(-1, 7970.1040)	(-1, 114.5893)
$\hat{\bar{Y}}_{p_8}$	(-1, 126.1952)	(-1, 21436.6645)	(-1, 324.7792)	(-1, 1.0991)
$\hat{\bar{Y}}_{p_9}$	(-1, 2724.4479)	(-1, 3935.2639)	(-1, 2528.5210)	(-1, 33.7053)
$\hat{\bar{Y}}_{p_{10}}$	(-1, 461.6418)	(-1, 2134.8341)	(-1, 1025.6054)	(-1, 5.9914)
$\hat{\bar{Y}}_{p_{11}}$	(-1, 1017.9517)	(-1, 1806.3268)	(-1, 951.7156)	(-1, 14.1076)
$\hat{\bar{Y}}_{p_{12}}$	(-1, 1236.4542)	(-1, 4650.7357)	(-1, 2724.7029)	(-1, 15.0607)
$\hat{\bar{Y}}_{p_{13}}$	(-1, 196799.6166)	(-1, 605657.2149)	(-1, 32137.3735)	(-1, 1605.6390)
$\hat{\bar{Y}}_{p_{14}}$	(-1, 5.4070)	(-1, 12.8811)	(-1, 79.8012)	(-1, -0.8490)
$\hat{\bar{Y}}_{p_{15}}$	(-1, 169754.4325)	(-1, 603621.1259)	(-1, 30486.7557)	(-1, 983.8649)
$\hat{\bar{Y}}_{p_{16}}$	(-1, 6.4278)	(-1, 12.9279)	(-1, 84.1758)	(-1, -0.7536)
$\hat{\bar{Y}}_{p_{17}}$	(-1, 307.7217)	(-1, 29101.8696)	(-1, 510.3764)	(-1, 3.6769)
$\hat{\bar{Y}}_{p_{18}}$	(-1, 4083.2802)	(-1, 287.8790)	(-1, 5077.0982)	(-1, 50.8800)
$\hat{\bar{Y}}_{p_{19}}$	(-1, 114.4205)	(-1, 13361.518)	(-1, 191.6042)	(-1, 1.0359)
$\hat{\bar{Y}}_{p_{20}}$	(-1, 10923.4555)	(-1, 628.1634)	(-1, 13481.6757)	(-1, 118.1797)
$\hat{\bar{Y}}_{p_{21}}$	(-1, 22291.3463)	(-1, 4477948.8100)	(-1, 6496.2013)	(-1, 215.5109)

Table 3 continued.

Estimator	α range (α_L, α_u)			
	Population 1	**Population 2**	**Population 3**	**Population 4**
$\hat{\bar{Y}}_{p_{22}}$	(-1, 55.5623)	(-1, 0.8775)	(-1, 398.6828)	(-1, 0.1207)
$\hat{\bar{Y}}_{p_{23}}$	(-1, 19227.8366)	(-1, 4462894.9330)	(-1, 6162.5069)	(-1, 131.7205)
$\hat{\bar{Y}}_{p_{24}}$	(-1, 64.5737)	(-1, 0.8838)	(-1, 420.3218)	(-1, 0.8282)
$\hat{\bar{Y}}_{p_{25}}$	(-1, 418.8142)	(-1, 1330.3074)	(-1, 605.9402)	(-1, 5.7807)
$\hat{\bar{Y}}_{p_{26}}$	(-1, 3002.4862)	(-1, 6314.0002)	(-1, 4277.5430)	(-1, 34.7834)
$\hat{\bar{Y}}_{p_{27}}$	81082.0243)	(-1, 446137.0551)	(-1, 20473.1799)	(-1, 720.1090)
$\hat{\bar{Y}}_{p_{28}}$	(-1, 14.5508)	(-1, 17.8444)	(-1, 125.8339)	(-1, -0.6635)
$\hat{\bar{Y}}_{p_{29}}$	(-1, 69939.2477)	(-1, 444637.2376)	(-1, 19421.6318)	(-1, 441.0376)
$\hat{\bar{Y}}_{p_{30}}$	(-1, 17.0283)	(-1, 17.908)	(-1, 132.7007)	(-1, -0.4511)
$\hat{\bar{Y}}_{p_{31}}$	(-1, 216876.3557)	(-1, 971664.4899)	(-1, 54359.2581)	(-1, 1655.5450)
$\hat{\bar{Y}}_{p_{32}}$	(-1, 4.8139)	(-1, 7.6524)	(-1, 46.7706)	(-1, -0.8535)
$\hat{\bar{Y}}_{p_{33}}$	(-1, 187072.1402)	(-1, 968397.9653)	(-1, 51567.3306)	(-1, 1014.4570)
$\hat{\bar{Y}}_{p_{34}}$	(-1, 5.7402)	(-1, 7.6816)	(-1, 49.3569)	(-1, -0.7611)
$\hat{\bar{Y}}_{p_{35}}$	(-1, 967.5869)	(-1, 2888.7708)	(-1, 1527.7007)	(-1, 8.5485)
$\hat{\bar{Y}}_{p_{36}}$	(-1, 1300.7996)	(-1, 2908.2988)	(-1, 1697.7104)	(-1, 24.4109)

Table 4. Range of α in which proposed estimator performs better than the existing modified ratio estimators

Estimator	α range (α_L, α_u)			
	Population 1	Population 2	Population 3	Population 4
$\hat{\bar{Y}}_{p_1}$ w.r.t. $\hat{\bar{Y}}_1$	(0, 1343.3398)	(0, 3813.2012)	(0, 517.1768)	(0, 13.0911)
$\hat{\bar{Y}}_{p_2}$ w.r.t. $\hat{\bar{Y}}_2$	(0, 116.3312)	(0, 29531.8209)	(0, 25.2048)	(-0.0717, 0)
$\hat{\bar{Y}}_{p_3}$ w.r.t. $\hat{\bar{Y}}_3$	(0, 524.4732)	(0, 2757.1363)	(0, 229.5184)	(0, 4.6415)
$\hat{\bar{Y}}_{p_4}$ w.r.t. $\hat{\bar{Y}}_4$	(0, 1485.6902)	(0, 6240.8577)	(0, 1268.9984)	(0, 13.5682)
$\hat{\bar{Y}}_{p_5}$ w.r.t. $\hat{\bar{Y}}_5$	(-0.1011, 0)	(0, 0.4679)	(0, 0.6773)	(-1.2678, 0)
$\hat{\bar{Y}}_{p_6}$ w.r.t. $\hat{\bar{Y}}_6$	(0, 0.2071)	(0, 0.4777)	(0, 0.8631)	(-1.3241, 0)
$\hat{\bar{Y}}_{p_7}$ w.r.t. $\hat{\bar{Y}}_7$	(0, 9857.5942)	(0, 249.4093)	(0, 4332.2435)	(0, 113.2681)
$\hat{\bar{Y}}_{p_8}$ w.r.t. $\hat{\bar{Y}}_8$	(0, 87.6545)	(0, 21217.4712)	(0, 14.0810)	(-0.0482, 0)
$\hat{\bar{Y}}_{p_9}$ w.r.t. $\hat{\bar{Y}}_9$	(0, 2670.6504)	(0, 3725.5608)	(0, 692.7328)	(0, 32.3928)
$\hat{\bar{Y}}_{p_{10}}$ w.r.t. $\hat{\bar{Y}}_{10}$	(0, 412.5020)	(0, 1934.1048)	(0, 135.4434)	(0, 4.7253)
$\hat{\bar{Y}}_{p_{11}}$ w.r.t. $\hat{\bar{Y}}_{11}$	(0, 965.8454)	(0, 1608.9539)	(0, 117.6363)	(0, 12.8106)
$\hat{\bar{Y}}_{p_{12}}$ w.r.t. $\hat{\bar{Y}}_{12}$	(0, 1183.8818)	(0, 4439.3079)	(0, 787.7525)	(0, 13.7621)
$\hat{\bar{Y}}_{p_{13}}$ w.r.t. $\hat{\bar{Y}}_{13}$	(0, 196744.7709)	(0, 605435.8482)	(0, 26599.1484)	(0, 1604.3148)
$\hat{\bar{Y}}_{p_{14}}$ w.r.t. $\hat{\bar{Y}}_{14}$	(-0.4252, 0)	(-0.2370, 0)	(-0.0477, 0)	(-1.2694, 0)
$\hat{\bar{Y}}_{p_{15}}$ w.r.t. $\hat{\bar{Y}}_{15}$	(0, 169699.5893)	(0, 603399.7594)	(0, 24999.7154)	(0, 982.5405)
$\hat{\bar{Y}}_{p_{16}}$ w.r.t. $\hat{\bar{Y}}_{16}$	(-0.2210, 0)	(-0.2317, 0)	(0, 0.0582)	(-1.3250, 0)
$\hat{\bar{Y}}_{p_{17}}$ w.r.t. $\hat{\bar{Y}}_{17}$	(0, 261.0105)	(0, 28882.0870)	(0, 35.2418)	(0, 2.4381)
$\hat{\bar{Y}}_{p_{18}}$ w.r.t. $\hat{\bar{Y}}_{18}$	(0, 4029.1336)	(0, 162.2788)	(0, 2189.8216)	(0, 49.5634)
$\hat{\bar{Y}}_{p_{19}}$ w.r.t. $\hat{\bar{Y}}_{19}$	(0, 77.0150)	(0, 13143.6647)	(0, 4.3618)	(-0.1066, 0)
$\hat{\bar{Y}}_{p_{20}}$ w.r.t. $\hat{\bar{Y}}_{20}$	(0, 10868.8640)	(0, 464.1746)	(0, 9009.6165)	(0, 116.8585)
$\hat{\bar{Y}}_{p_{21}}$ w.r.t. $\hat{\bar{Y}}_{21}$	(0, 22236.6179)	(0, 4477727.3738)	(0, 3199.5909)	(0, 214.1880)

Table 4 continued.

Estimator	*α* range (*α_L*, *α_u*)			
	Population 1	**Population 2**	**Population 3**	**Population 4**
$\hat{\bar{Y}}_{p_{22}}$ w.r.t. $\hat{\bar{Y}}_{22}$	(0, 27.4608)	(-0.9926, 0)	(0, 21.4756)	(-0.9064, 0)
$\hat{\bar{Y}}_{p_{23}}$ w.r.t. $\hat{\bar{Y}}_{23}$	(0, 19173.1292)	(0, 4462673.4962)	(0, 2954.0212)	(0, 130.3989)
$\hat{\bar{Y}}_{p_{24}}$ w.r.t. $\hat{\bar{Y}}_{24}$	(0, 34.4533)	(-0.9925, 0)	(0, 23.9023)	(-0.2968, 0)
$\hat{\bar{Y}}_{p_{25}}$ w.r.t. $\hat{\bar{Y}}_{25}$	(0, 370.1916)	(0, 1140.3196)	(0, 49.3999)	(0, 4.5165)
$\hat{\bar{Y}}_{p_{26}}$ w.r.t. $\hat{\bar{Y}}_{26}$	(0, 2948.5920)	(0, 6100.0225)	(0, 1667.5443)	(0, 33.4704)
$\hat{\bar{Y}}_{p_{27}}$ w.r.t. $\hat{\bar{Y}}_{27}$	(0, 81027.2001)	(0, 445915.7171)	(0, 15429.9892)	(0, 718.7848)
$\hat{\bar{Y}}_{p_{28}}$ w.r.t. $\hat{\bar{Y}}_{28}$	(0, 2.2599)	(0, 0.4053)	(0, 1.3412)	(-1.3376, 0)
$\hat{\bar{Y}}_{p_{29}}$ w.r.t. $\hat{\bar{Y}}_{29}$	(0, 69884.4292)	(0, 444415.8999)	(0, 14444.8014)	(0, 439.7137)
$\hat{\bar{Y}}_{p_{30}}$ w.r.t. $\hat{\bar{Y}}_{30}$	(0, 3.2704)	(0, 0.4146)	(0, 1.6000)	(-1.2834, 0)
$\hat{\bar{Y}}_{p_{31}}$ w.r.t. $\hat{\bar{Y}}_{31}$	(0, 216821.5087)	(0, 971443.0928)	(0, 48401.3982)	(0, 1654.2204)
$\hat{\bar{Y}}_{p_{32}}$ w.r.t. $\hat{\bar{Y}}_{32}$	(-0.5310, 0)	(-0.7110, 0)	(-0.6684, 0)	(-1.2652, 0)
$\hat{\bar{Y}}_{p_{33}}$ w.r.t. $\hat{\bar{Y}}_{33}$	(0, 187017.2953)	(0, 968176.5684)	(0, 45644.6094)	(0, 1013.1325)
$\hat{\bar{Y}}_{p_{34}}$ w.r.t. $\hat{\bar{Y}}_{34}$	(-0.3616, 0)	(-0.7090, 0)	(-0.6313, 0)	(-1.3225, 0)
$\hat{\bar{Y}}_{p_{35}}$ w.r.t. $\hat{\bar{Y}}_{35}$	(0, 915.6162)	(0, 2683.0193)	(0, 283.1533)	(0, 7.2673)
$\hat{\bar{Y}}_{p_{36}}$ w.r.t. $\hat{\bar{Y}}_{36}$	(0, 1248.1186)	(0, 2702.4493)	(0, 342.7819)	(0, 23.1028)

Table 5. Constant, Bias and Mean squared error of the Existing and Proposed estimators for Population 1

Estimator	θ_i	$B\left(\hat{\bar{Y}}_{(.)}\right)$	$MSE\left(\hat{\bar{Y}}_{(.)}\right)$	$MSE\left(\hat{\bar{Y}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\bar{Y}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\bar{Y}}_{p_i}\right)$ at α_a
$\bar{y}_{srs}$	-	-	94.0466	94.0466	-	-	-
$\hat{\bar{Y}}_R$	-	0.6946	73.0773	73.0773	-	-	-
$\hat{\bar{Y}}_{p_1}$	0.9954	0.6842	72.4673	72.4673	0.2448	0.1269	60.1973
$\hat{\bar{Y}}_{p_2}$	0.9611	0.6076	68.0853	68.0853	0.2945	0.1291	55.5981
$\hat{\bar{Y}}_{p_3}$	0.9890	0.6695	71.6173	71.6173	0.2545	0.1279	59.2452
$\hat{\bar{Y}}_{p_4}$	0.9959	0.6851	72.5232	72.5232	0.2442	0.1268	60.2610
$\hat{\bar{Y}}_{p_5}$	0.5544	0.0116	44.0518	44.0518	0.5672	0.0003	44.0253
$\hat{\bar{Y}}_{p_6}$	0.5906	0.0219	44.1080	44.1080	0.5666	0.0009	44.0254
$\hat{\bar{Y}}_{p_7}$	0.9994	0.6932	72.9906	72.9906	0.2389	0.1261	60.7979
$\hat{\bar{Y}}_{p_8}$	0.9520	0.5880	66.9919	66.9919	0.3069	0.1285	54.5701
$\hat{\bar{Y}}_{p_9}$	0.9977	0.6893	72.7631	72.7631	0.2415	0.1264	60.5354
$\hat{\bar{Y}}_{p_{10}}$	0.9863	0.6635	71.2712	71.2712	0.2584	0.1283	58.8657
$\hat{\bar{Y}}_{p_{11}}$	0.9938	0.6803	72.2439	72.2439	0.2474	0.1272	59.9443
$\hat{\bar{Y}}_{p_{12}}$	0.9948	0.6828	72.3894	72.3894	0.2457	0.1270	60.1089
$\hat{\bar{Y}}_{p_{13}}$	1.0000	0.6946	73.0729	73.0729	0.2380	0.1260	60.8934
$\hat{\bar{Y}}_{p_{14}}$	0.5000	0.0542	44.7328	44.7328	0.5595	0.0072	44.0353
$\hat{\bar{Y}}_{p_{15}}$	1.0000	0.6946	73.0722	73.0722	0.2380	0.1260	60.8925
$\hat{\bar{Y}}_{p_{16}}$	0.5369	0.0264	44.1707	44.1707	0.5659	0.0015	44.0257
$\hat{\bar{Y}}_{p_{17}}$	0.9797	0.6485	70.4101	70.4101	0.2682	0.1289	57.9425
$\hat{\bar{Y}}_{p_{18}}$	0.9984	0.6911	72.8672	72.8672	0.2403	0.1263	60.6553
$\hat{\bar{Y}}_{p_{19}}$	0.9474	0.5781	66.4416	66.4416	0.3132	0.1279	54.0709

Table 5 continued.

Estimator	θ_i	$B\left(\hat{\hat{Y}}_{(.)}\right)$	$MSE\left(\hat{\hat{Y}}_{(.)}\right)$	$MSE\left(\hat{\hat{Y}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\hat{Y}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\hat{Y}}_{p_i}\right)$ at α_a
$\hat{\hat{Y}}_{p_{20}}$	0.9994	0.6933	72.9986	72.9986	0.2388	0.1261	60.8072
$\hat{\hat{Y}}_{p_{21}}$	0.9997	0.6940	73.0387	73.0387	0.2383	0.1260	60.8536
$\hat{\hat{Y}}_{p_{22}}$	0.8983	0.4772	61.0160	61.0160	0.3747	0.1160	49.7965
$\hat{\hat{Y}}_{p_{23}}$	0.9997	0.6939	73.0325	73.0325	0.2384	0.1260	60.8465
$\hat{\hat{Y}}_{p_{24}}$	0.9110	0.5026	62.3499	62.3499	0.3596	0.1201	50.7382
$\hat{\hat{Y}}_{p_{25}}$	0.9850	0.6604	71.0929	71.0929	0.2604	0.1284	58.6722
$\hat{\hat{Y}}_{p_{26}}$	0.9979	0.6898	72.7920	72.7920	0.2411	0.1264	60.5687
$\hat{\hat{Y}}_{p_{27}}$	0.9999	0.6945	73.0667	73.0667	0.2380	0.1260	60.8861
$\hat{\hat{Y}}_{p_{28}}$	0.7082	0.1601	47.1006	47.1006	0.5326	0.0298	44.2143
$\hat{\hat{Y}}_{p_{29}}$	0.9999	0.6944	73.0650	73.0650	0.2380	0.1260	60.8841
$\hat{\hat{Y}}_{p_{30}}$	0.7378	0.2018	48.5296	48.5296	0.5164	0.0424	44.4309
$\hat{\hat{Y}}_{p_{31}}$	1.0000	0.6946	73.0733	73.0733	0.2379	0.1260	60.8938
$\hat{\hat{Y}}_{p_{32}}$	0.4757	-0.0701	45.3331	45.3331	0.5527	0.0132	44.0594
$\hat{\hat{Y}}_{p_{33}}$	1.0000	0.6946	73.0727	73.0727	0.2380	0.1260	60.8931
$\hat{\hat{Y}}_{p_{34}}$	0.5127	0.0451	44.4921	44.4921	0.5622	0.0048	44.0296
$\hat{\hat{Y}}_{p_{35}}$	0.9934	0.6796	72.2012	72.2012	0.2478	0.1272	59.8961
$\hat{\hat{Y}}_{p_{36}}$	0.9951	0.6834	72.4230	72.4230	0.2453	0.1269	60.1471

Table 6. Constant, Bias and Mean squared error of the Existing and Proposed estimators for Population 2

Estimator	θ_i	$B\left(\hat{\bar{Y}}_i\right)$	$MSE\left(\hat{\bar{Y}}_i\right)$	$MSE\left(\hat{\bar{Y}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\bar{Y}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\bar{Y}}_{p_i}\right)$ at α_a
$\bar{y}_{srs}$	-	-	11066.0800	11066.0800	-	-	-
$\hat{\bar{Y}}_R$	-	4.2694	10539.2700	10539.2700	-	-	-
$\hat{\bar{Y}}_{p_1}$	0.9966	4.2233	10514.2250	10514.2250	0.1319	0.4851	10098.8070
$\hat{\bar{Y}}_{p_2}$	0.9995	4.2631	10535.8620	10535.8620	0.1268	0.4721	10131.5920
$\hat{\bar{Y}}_{p_3}$	0.9953	4.2070	10505.3560	10505.3560	0.1340	0.4903	10085.4900
$\hat{\bar{Y}}_{p_4}$	0.9979	4.2406	10523.6170	10523.6170	0.1297	0.4795	10112.9870
$\hat{\bar{Y}}_{p_5}$	0.5812	0.2533	8851.7250	8851.7250	0.5294	0.0206	8834.0910
$\hat{\bar{Y}}_{p_6}$	0.5820	0.2581	8852.3420	8852.3420	0.5292	0.0213	8834.1010
$\hat{\bar{Y}}_{p_7}$	0.9658	3.8212	10298.4430	10298.4430	0.1835	0.5881	9794.7990
$\hat{\bar{Y}}_{p_8}$	0.9994	4.2607	10534.5420	10534.5420	0.1271	0.4729	10129.5790
$\hat{\bar{Y}}_{p_9}$	0.9965	4.2223	10513.6700	10513.6700	0.1321	0.4854	10097.9710
$\hat{\bar{Y}}_{p_{10}}$	0.9935	4.1831	10492.3780	10492.3780	0.1371	0.4977	10066.1290
$\hat{\bar{Y}}_{p_{11}}$	0.9924	4.1676	10483.9890	10483.9890	0.1392	0.5024	10053.6950
$\hat{\bar{Y}}_{p_{12}}$	0.9970	4.2295	10517.5830	10517.5830	0.1311	0.4831	10103.8680
$\hat{\bar{Y}}_{p_{13}}$	1.0000	4.2691	10539.1030	10539.1030	0.1260	0.4701	10136.5390
$\hat{\bar{Y}}_{p_{14}}$	0.5000	0.1538	8842.8000	8842.8000	0.5315	0.0103	8833.9850
$\hat{\bar{Y}}_{p_{15}}$	1.0000	4.2691	10539.1020	10539.1020	0.1260	0.4701	10136.5380
$\hat{\bar{Y}}_{p_{16}}$	0.5008	0.1502	8842.3620	8842.3620	0.5316	0.0098	8833.9810
$\hat{\bar{Y}}_{p_{17}}$	0.9995	4.2630	10535.7860	10535.7860	0.1268	0.4721	10131.4760
$\hat{\bar{Y}}_{p_{18}}$	0.9542	3.6732	10220.4740	10220.4740	0.2021	0.6133	9695.2120
$\hat{\bar{Y}}_{p_{19}}$	0.9990	4.2555	10531.6900	10531.6900	0.1277	0.4746	10125.2380

Table 6 continued

Estimator	θ_i	$B\left(\hat{\hat{Y}}_i\right)$	$MSE\left(\hat{\hat{Y}}_i\right)$	$MSE\left(\hat{\hat{Y}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\hat{Y}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\hat{Y}}_{p_i}\right)$ at α_a
$\hat{\hat{Y}}_{p_{20}}$	0.9784	3.9839	10385.0680	10385.0680	0.1628	0.5526	9911.8290
$\hat{\hat{Y}}_{p_{21}}$	1.0000	4.2693	10539.2480	10539.2480	0.1259	0.4700	10136.7600
$\hat{\hat{Y}}_{p_{22}}$	0.1191	0.4520	10180.5970	10180.5970	0.2117	0.6238	9646.3850
$\hat{\hat{Y}}_{p_{23}}$	1.0000	4.2693	10539.2480	10539.2480	0.1259	0.4700	10136.7600
$\hat{\hat{Y}}_{p_{24}}$	0.1195	0.4530	10178.2990	10178.2990	0.2122	0.6243	9643.6140
$\hat{\hat{Y}}_{p_{25}}$	0.9897	4.1318	10464.6450	10464.6450	0.1438	0.5130	10025.2640
$\hat{\hat{Y}}_{p_{26}}$	0.9978	4.2400	10523.2690	10523.2690	0.1298	0.4797	10112.4600
$\hat{\hat{Y}}_{p_{27}}$	1.0000	4.2690	10539.0430	10539.0430	0.1260	0.4701	10136.4470
$\hat{\hat{Y}}_{p_{28}}$	0.5758	0.2226	8847.9320	8847.9320	0.5303	0.0162	8834.0370
$\hat{\hat{Y}}_{p_{29}}$	1.0000	4.2690	10539.0420	10539.0420	0.1260	0.4701	10136.4460
$\hat{\hat{Y}}_{p_{30}}$	0.5767	0.2273	8848.4820	8848.4820	0.5301	0.0169	8834.0440
$\hat{\hat{Y}}_{p_{31}}$	1.0000	4.2692	10539.1660	10539.1660	0.1260	0.4700	10136.6350
$\hat{\hat{Y}}_{p_{32}}$	0.3840	0.5259	9009.4490	9009.4490	0.4916	0.1888	8847.7480
$\hat{\hat{Y}}_{p_{33}}$	1.0000	4.2692	10539.1660	10539.1660	0.1260	0.4700	10136.6350
$\hat{\hat{Y}}_{p_{34}}$	0.3848	0.5242	9007.5850	9007.5850	0.4921	0.1870	8847.4570
$\hat{\hat{Y}}_{p_{35}}$	0.9952	4.2054	10504.4910	10504.4910	0.1343	0.4908	10084.1940
$\hat{\hat{Y}}_{p_{36}}$	0.9953	4.2058	10504.7220	10504.7220	0.1342	0.4906	10084.5400

Table 7. Constant, Bias and Mean squared error of the Existing and Proposed estimators for Population 3

Estimator	θ_i	$B\left(\hat{\bar{Y}}_i\right)$	$MSE\left(\hat{\bar{Y}}_i\right)$	$MSE\left(\hat{\bar{Y}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\bar{Y}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\bar{Y}}_{p_i}\right)$ at α_a
$\bar{y}_{srs}$	-	-	379.4085	379.4085	-	-	-
$\hat{\bar{Y}}_R$	-	1.6938	375.8179	375.8179	-	-	-
$\hat{\bar{Y}}_{p_1}$	0.9636	1.5119	365.6490	365.6490	0.0926	0.1313	354.4061
$\hat{\bar{Y}}_{p_2}$	0.8426	0.9723	337.4291	337.4291	0.2824	0.2167	318.8736
$\hat{\bar{Y}}_{p_3}$	0.9440	1.4178	360.5017	360.5017	0.1272	0.1653	346.3464
$\hat{\bar{Y}}_{p_4}$	0.9782	1.5835	369.6218	369.6218	0.0658	0.0994	361.1082
$\hat{\bar{Y}}_{p_5}$	0.5704	0.1257	305.3883	305.3883	0.4979	0.0139	304.1944
$\hat{\bar{Y}}_{p_6}$	0.5833	0.1543	305.9229	305.9229	0.4944	0.0199	304.2154
$\hat{\bar{Y}}_{p_7}$	0.9900	1.6427	372.9362	372.9362	0.0435	0.0691	367.0206
$\hat{\bar{Y}}_{p_8}$	0.8013	0.8111	329.7622	329.7622	0.3340	0.1972	312.8772
$\hat{\bar{Y}}_{p_9}$	0.9690	1.5385	367.1190	367.1190	0.0827	0.1201	356.8370
$\hat{\bar{Y}}_{p_{10}}$	0.9270	1.3383	356.2136	356.2136	0.1560	0.1873	340.1698
$\hat{\bar{Y}}_{p_{11}}$	0.9218	1.3142	354.9318	354.9318	0.1647	0.1928	338.4184
$\hat{\bar{Y}}_{p_{12}}$	0.9712	1.5491	367.7088	367.7088	0.0787	0.1154	357.8285
$\hat{\bar{Y}}_{p_{13}}$	0.9975	1.6810	375.0921	375.0921	0.0290	0.0475	371.0234
$\hat{\bar{Y}}_{p_{14}}$	0.5000	0.0105	304.1857	304.1857	0.5060	0.0001	304.1748
$\hat{\bar{Y}}_{p_{15}}$	0.9974	1.6803	375.0531	375.0531	0.0293	0.0479	370.9498
$\hat{\bar{Y}}_{p_{16}}$	0.5132	0.0124	304.1895	304.1895	0.5060	0.0002	304.1748
$\hat{\bar{Y}}_{p_{17}}$	0.8636	1.0586	341.7007	341.7007	0.2537	0.2196	322.8924
$\hat{\bar{Y}}_{p_{18}}$	0.9843	1.6144	371.3460	371.3460	0.0542	0.0841	364.1476
$\hat{\bar{Y}}_{p_{20}}$	0.9940	1.6634	374.1000	374.1000	0.0357	0.0576	369.1661

Table 7 continued

Estimator	θ_i	$B\left(\hat{\hat{\bar{Y}}}_i\right)$	$MSE\left(\hat{\hat{\bar{Y}}}_i\right)$	$MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\hat{\bar{Y}}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right)$ at α_a
$\hat{\hat{\bar{Y}}}_{p_{21}}$	0.9877	1.6314	372.2986	372.2986	0.0478	0.0752	365.8606
$\hat{\hat{\bar{Y}}}_{p_{22}}$	0.8318	0.9292	335.3364	335.3364	0.2965	0.2132	317.0819
$\hat{\hat{\bar{Y}}}_{p_{23}}$	0.9871	1.6281	372.1130	372.1130	0.0491	0.0769	365.5250
$\hat{\hat{\bar{Y}}}_{p_{24}}$	0.8391	0.9582	336.7384	336.7384	0.2871	0.2157	318.2694
$\hat{\hat{\bar{Y}}}_{p_{25}}$	0.8825	1.1392	345.7872	345.7872	0.2262	0.2171	327.1910
$\hat{\hat{\bar{Y}}}_{p_{26}}$	0.9815	1.6000	370.5415	370.5415	0.0597	0.0913	362.7196
$\hat{\hat{\bar{Y}}}_{p_{27}}$	0.9961	1.6737	374.6820	374.6820	0.0318	0.0517	370.2524
$\hat{\hat{\bar{Y}}}_{p_{28}}$	0.6109	0.2194	307.3971	307.3971	0.4844	0.0360	304.3128
$\hat{\hat{\bar{Y}}}_{p_{29}}$	0.9959	1.6726	374.6210	374.6210	0.0322	0.0523	370.1382
$\hat{\hat{\bar{Y}}}_{p_{30}}$	0.6233	0.2505	308.2092	308.2092	0.4790	0.0446	304.3911
$\hat{\hat{\bar{Y}}}_{p_{31}}$	0.9985	1.6862	375.3879	375.3879	0.0270	0.0444	371.5822
$\hat{\hat{\bar{Y}}}_{p_{32}}$	0.3716	0.1715	309.4927	309.4927	0.4703	0.0577	304.5507
$\hat{\hat{\bar{Y}}}_{p_{33}}$	0.9984	1.6858	375.3647	375.3647	0.0272	0.0447	371.5383
$\hat{\hat{\bar{Y}}}_{p_{34}}$	0.3839	0.1609	308.5583	308.5583	0.4766	0.0482	304.4302
$\hat{\hat{\bar{Y}}}_{p_{35}}$	0.9498	1.4452	361.9937	361.9937	0.1172	0.1563	348.6100
$\hat{\hat{\bar{Y}}}_{p_{36}}$	0.9546	1.4682	363.2505	363.2505	0.1087	0.1482	350.5627

Table 8. Constant, Bias and Mean squared error of the Existing and Proposed estimators for Population 4

Estimator	θ_i	$B\left(\hat{\hat{\bar{Y}}}_i\right)$	$MSE\left(\hat{\hat{\bar{Y}}}_i\right)$	$MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\hat{\bar{Y}}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right)$ at α_a
$\bar{y}_{srs}$	-	-	448.5780	448.5780	-	-	-
$\hat{\bar{Y}}_R$	-	0.2542	18.3619	18.3619	-	-	-
$\hat{\hat{\bar{Y}}}_{p_1}$	0.9903	0.2144	17.7773	17.7773	0.9311	0.0121	16.2363
$\hat{\hat{\bar{Y}}}_{p_2}$	0.9321	0.0082	16.2333	16.2333	0.9344	0.0000	16.2307
$\hat{\hat{\bar{Y}}}_{p_3}$	0.9786	0.1676	17.1984	17.1984	0.9323	0.0076	16.2329
$\hat{\hat{\bar{Y}}}_{p_4}$	0.9906	0.2156	17.7934	17.7934	0.9310	0.0122	16.2364
$\hat{\hat{\bar{Y}}}_{p_5}$	0.4970	0.8423	110.9857	110.9857	0.7296	0.5790	36.9976
$\hat{\hat{\bar{Y}}}_{p_6}$	0.6171	0.7587	66.0867	66.0867	0.8266	0.3451	21.9799
$\hat{\hat{\bar{Y}}}_{p_7}$	0.9987	0.2488	18.2780	18.2780	0.9300	0.0159	16.2404
$\hat{\hat{\bar{Y}}}_{p_8}$	0.9329	0.0055	16.2319	16.2319	0.9344	0.0000	16.2307
$\hat{\hat{\bar{Y}}}_{p_9}$	0.9957	0.2364	18.0897	18.0897	0.9304	0.0145	16.2387
$\hat{\hat{\bar{Y}}}_{p_{10}}$	0.9789	0.1686	17.2095	17.2095	0.9323	0.0076	16.2330
$\hat{\hat{\bar{Y}}}_{p_{11}}$	0.9901	0.2137	17.7674	17.7674	0.9311	0.0120	16.2362
$\hat{\hat{\bar{Y}}}_{p_{12}}$	0.9907	0.2161	17.7996	17.7996	0.9310	0.0122	16.2364
$\hat{\hat{\bar{Y}}}_{p_{13}}$	0.9999	0.2538	18.3558	18.3558	0.9298	0.0165	16.2412
$\hat{\hat{\bar{Y}}}_{p_{14}}$	0.5000	0.8416	109.6730	109.6730	0.7324	0.5732	36.4262
$\hat{\hat{\bar{Y}}}_{p_{15}}$	0.9998	0.2536	18.3520	18.3520	0.9298	0.0165	16.2411
$\hat{\hat{\bar{Y}}}_{p_{16}}$	0.6200	0.7553	65.1890	65.1890	0.8286	0.3397	21.7747
$\hat{\hat{\bar{Y}}}_{p_{17}}$	0.9687	0.1288	16.8141	16.8141	0.9331	0.0046	16.2315
$\hat{\hat{\bar{Y}}}_{p_{18}}$	0.9971	0.2423	18.1775	18.1775	0.9302	0.0152	16.2395
$\hat{\hat{\bar{Y}}}_{p_{19}}$	0.9309	0.0125	16.2366	16.2366	0.9344	0.0000	16.2307

Table 8 continued

Estimator	θ_i	$B\left(\hat{\hat{Y}}_i\right)$	$MSE\left(\hat{\hat{Y}}_i\right)$	$MSE\left(\hat{\hat{Y}}_{p_i}\right)$ at α_L & α_u	θ_{p_i} at α_a	Bias$\left(\hat{\hat{Y}}_{p_i}\right)$ at α_a	$MSE\left(\hat{\hat{Y}}_{p_i}\right)$ at α_a
$\hat{\hat{Y}}_{p_{20}}$	0.9987	0.2490	18.2805	18.2805	0.9300	0.0160	16.2405
$\hat{\hat{Y}}_{p_{21}}$	0.9993	0.2513	18.3169	18.3169	0.9299	0.0162	16.2408
$\hat{\hat{Y}}_{p_{22}}$	0.8812	0.1815	17.6298	17.6298	0.9314	0.0109	16.2353
$\hat{\hat{Y}}_{p_{23}}$	0.9989	0.2495	18.2887	18.2887	0.9299	0.0160	16.2405
$\hat{\hat{Y}}_{p_{24}}$	0.9237	0.0383	16.2874	16.2874	0.9343	0.0004	16.2307
$\hat{\hat{Y}}_{p_{25}}$	0.9782	0.1661	17.1814	17.1814	0.9323	0.0074	16.2328
$\hat{\hat{Y}}_{p_{26}}$	0.9958	0.2369	18.0976	18.0976	0.9304	0.0145	16.2388
$\hat{\hat{Y}}_{p_{27}}$	0.9998	0.2533	18.3483	18.3483	0.9298	0.0165	16.2411
$\hat{\hat{Y}}_{p_{28}}$	0.6902	0.6531	45.7570	45.7570	0.8706	0.2153	18.2472
$\hat{\hat{Y}}_{p_{29}}$	0.9997	0.2528	18.3398	18.3398	0.9298	0.0164	16.2410
$\hat{\hat{Y}}_{p_{30}}$	0.7842	0.4563	27.3969	27.3969	0.9103	0.0851	16.5191
$\hat{\hat{Y}}_{p_{31}}$	0.9999	0.2538	18.3560	18.3560	0.9298	0.0165	16.2412
$\hat{\hat{Y}}_{p_{32}}$	0.4924	0.8433	112.9917	112.9917	0.7253	0.5877	37.8862
$\hat{\hat{Y}}_{p_{33}}$	0.9999	0.2536	18.3523	18.3523	0.9298	0.0165	16.2411
$\hat{\hat{Y}}_{p_{34}}$	0.6127	0.7637	67.4673	67.4673	0.8237	0.3534	22.3027
$\hat{\hat{Y}}_{p_{35}}$	0.9844	0.1909	17.4704	17.4704	0.9317	0.0097	16.2343
$\hat{\hat{Y}}_{p_{36}}$	0.9941	0.2299	17.9954	17.9954	0.9306	0.0138	16.2379

From the values of Table 5—Table 8, it is observed that the bias of the proposed modified ratio estimator $\hat{\hat{Y}}_{p_j}$; $j = 1, 2, \ldots, 36$ is less than the bias of the usual ratio estimator and the existing modified ratio estimators $\hat{\hat{Y}}_i; i = 1, 2, 3, \ldots, 36$. Similarly, the mean squared error of the proposed modified ratio estimator $\hat{\hat{Y}}_{p_j}$; $j = 1, 2, \ldots, 36$

is less than the variance of SRSWOR sample mean, the mean squared error of the usual ratio estimator and the existing modified ratio estimators $\hat{\bar{Y}}_{p_j}$; $j = 1, 2, \ldots, 36$ for all four populations.

Conclusion

In this article, a generalized modified ratio estimator has been suggested using the known population parameters of the auxiliary variable. Moreover, many modified ratio estimators have been introduced in this article, and have not been discussed earlier in the literature. The bias and mean squared error of the proposed generalized modified ratio estimator are obtained. Furthermore, the conditions have been derived for which the proposed estimator is more efficient than the existing estimators, and it is shown that the SRSWOR sample mean, the usual ratio estimator, the linear regression and the existing modified ratio estimators are particular cases of the proposed estimator. The performances of the proposed estimator are also assessed for some known populations. It is observed that the bias and the mean squared errors of the proposed estimators are less than the bias and the mean squared error of the existing estimators. Moreover, the proposed estimator will be a generalized modified ratio estimator for estimating the population mean of the study variable using the known population parameters of the auxiliary variable.

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Appendix A

An expression for the bias and mean squared error of the proposed estimators $\hat{\bar{Y}}_{p_j}; i = 1, 2, 3, \ldots, 36$ was derived to first order of approximation with the following notations:

Let us define $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$. Further, $\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$ and from the definition of e_0 and e_1:

$$E[e_0] = E[e_1] = 0$$

$$E\left[e_0^2\right] = \frac{(1-f)}{n} C_y^2$$

$$E\left[e_1^2\right] = \frac{(1-f)}{n} C_x^2$$

$$E[e_0 e_1] = \frac{(1-f)}{n} \rho C_y C_x \text{ where } C_x = \frac{S_x}{\bar{X}}, C_y = \frac{S_y}{\bar{Y}} \text{ and } \rho = \frac{S_{xy}}{S_x S_y}$$

The bias of a class of proposed estimators $\hat{\bar{Y}}_{p_i}; i = 1, 2, 3, \ldots, 36$ is derived and is:

$$\hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(\bar{x} + (1+\alpha)\lambda_i\right)} \left(\bar{X} + (1+\alpha)\lambda_i\right); i = 1, 2, 3, \ldots, 36$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(\bar{X} + e_1 \bar{X} + (1+\alpha)\lambda_i\right)} \left(\bar{X} + (1+\alpha)\lambda_i\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(\bar{X} + (1+\alpha)\lambda_i\right)\left(1 + \frac{e_1 \bar{X}}{\left(\bar{X} + (1+\alpha)\lambda_i\right)}\right)} \left(\bar{X} + (1+\alpha)\lambda_i\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(1 + \theta_{p_i} e_1\right)} \text{ where } \theta_{p_i} = \frac{\bar{X}}{\bar{X} + (1+\alpha)\lambda_i}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{y}\left(1 + \theta_{p_i} e_1\right)^{-1}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{y}\left(1 - \theta_{p_i} e_1 + \theta_{p_i}^2 e_1^2 - \theta_{p_i}^3 e_1^3 + \ldots\right)$$

Neglecting the terms more than 2[nd] order, results in

$$\hat{\bar{Y}}_{p_i} = \bar{y}\left(1-\theta_{p_i}e_1+\theta_{p_i}^2e_1^2\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \left(\bar{Y}\left(1+e_0\right)\right)\left(1-\theta_{p_i}e_1+\theta_{p_i}^2e_1^2\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \left(\bar{Y}+\bar{Y}e_0\right)\left(1-\theta_{p_i}e_1+\theta_{p_i}^2e_1^2\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}+\bar{Y}e_0-\bar{Y}\theta_{p_i}e_1-\bar{Y}\theta_{p_i}e_0e_1+\bar{Y}\theta_{p_i}^2e_1^2+\bar{Y}\theta_{p_i}^2e_0e_1^2$$

Neglecting the terms more than 3[rd] order, results in

$$\hat{\bar{Y}}_{p_i} = \bar{Y}+\bar{Y}e_0-\bar{Y}\theta_{p_i}e_1-\bar{Y}\theta_{p_i}e_0e_1+\bar{Y}\theta_{p_i}^2e_1^2$$

$$\Rightarrow \hat{\bar{Y}}_{p_i}-\bar{Y} = \bar{Y}e_0-\bar{Y}\theta_{p_i}e_1-\bar{Y}\theta_{p_i}e_0e_1+\bar{Y}\theta_{p_i}^2e_1^2$$

Taking expectation on both sides, results in

$$E\left(\hat{\bar{Y}}_{p_i}-\bar{Y}\right) = \bar{Y}E\left(e_0\right)-\bar{Y}\theta_{p_i}E\left(e_1\right)-\bar{Y}\theta_{p_i}E\left(e_0e_1\right)+\bar{Y}\theta_{p_i}^2E\left(e_1^2\right)$$

$$\Rightarrow \text{Bias}\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}\theta_{p_i}^2E\left(e_1^2\right)-\bar{Y}\theta_{p_i}E\left(e_0e_1\right)$$

$$\Rightarrow \text{Bias}\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}\theta_{p_i}^2\frac{(1-f)}{n}C_x^2-\bar{Y}\theta_{p_i}\frac{(1-f)}{n}\rho C_yC_x$$

$$\Rightarrow \text{Bias}\left(\hat{\bar{Y}}_{p_i}\right) = \frac{(1-f)}{n}\left(\bar{Y}\theta_{p_i}^2C_x^2-\bar{Y}\theta_{p_i}\rho C_yC_x\right)$$

$$\Rightarrow \text{Bias}\left(\hat{\bar{Y}}_{p_i}\right) = \frac{(1-f)}{n}\bar{Y}\left(\theta_{p_i}^2C_x^2-\theta_{p_i}\rho C_yC_x\right) \text{ where } \theta_{p_i} == \frac{\bar{X}}{\bar{X}+(1+\alpha)\lambda_i}$$

The mean squared error of the proposed estimator $\hat{\bar{Y}}_{p_i}; i=1,2,3,\ldots,36$ to first order of approximation is derived and is:

$$\hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(\bar{x}+(1+\alpha)\lambda_i\right)}\left(\bar{X}+(1+\alpha)\lambda_i\right); i=1,2,3,\ldots,36$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(\bar{X}+e_1\bar{X}+(1+\alpha)\lambda_i\right)}\left(\bar{X}+(1+\alpha)\lambda_i\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(\bar{X}+(1+\alpha)\lambda_i\right)\left(1+\frac{e_1\bar{X}}{\left(\bar{X}+(1+\alpha)\lambda_i\right)}\right)}\left(\bar{X}+(1+\alpha)\lambda_i\right)$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \frac{\bar{y}}{\left(1+\theta_{p_i}e_1\right)} \text{ where } \theta_{p_i} = \frac{\bar{X}}{\bar{X}+(1+\alpha)\lambda_i}$$

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} = \bar{y}\left(1+\theta_{p_i} e_1\right)^{-1}$$

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} = \bar{y}\left(1-\theta_{p_i} e_1 + \theta_{p_i}^2 e_1^2 - \theta_{p_i}^3 e_1^3 + \ldots\right)$$

Neglecting the terms more than 1st order, results in

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} = \bar{y}\left(1-\theta_{p_i} e_1\right)$$

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} = \left(\bar{Y}\left(1+e_0\right)\right)\left(1-\theta_{p_i} e_1\right)$$

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} = \left(\bar{Y}+\bar{Y}e_0\right)\left(1-\theta_{p_i} e_1\right)$$

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} = \bar{Y}+\bar{Y}e_0-\bar{Y}\theta_{p_i} e_1-\bar{Y}\theta_{p_i} e_0 e_1$$

$$\Rightarrow \hat{\hat{\bar{Y}}}_{p_i} - \bar{Y} = \bar{Y}e_0-\bar{Y}\theta_{p_i} e_1-\bar{Y}\theta_{p_i} e_0 e_1$$

Squaring both sides

$$\Rightarrow \left(\hat{\hat{\bar{Y}}}_{p_i} - \bar{Y}\right)^2 = \left(\bar{Y}e_0-\bar{Y}\theta_{p_i} e_1-\bar{Y}\theta_{p_i} e_0 e_1\right)^2$$

Neglecting the terms more than 2nd order, results in

$$\left(\hat{\hat{\bar{Y}}}_{p_i} - \bar{Y}\right)^2 = \bar{Y}^2 e_0{}^2 - \bar{Y}^2\theta_{p_i}^2 e_1^2 - 2\bar{Y}^2\theta_{p_i} e_0 e_1$$

Taking expectation on both sides results in:

$$E\left(\hat{\hat{\bar{Y}}}_{p_i} - \bar{Y}\right)^2 = E\left(\bar{Y}^2 e_0^2\right) + \bar{Y}^2\theta_{p_i}^2 E\left(e_1^2\right) - 2\bar{Y}^2\theta_{p_i} E\left(e_0 e_1\right)$$

$$\Rightarrow MSE\left(\hat{\hat{\bar{Y}}}_{p_i}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 C_y^2 + \bar{Y}^2\theta_{p_i}^2 C_x^2 - 2\bar{Y}^2\theta_{p_i}\rho C_y C_x\right); i = 1,2,3,\ldots,36$$

where $\theta_{p_i} = \dfrac{\bar{X}}{\bar{X}+(1+\alpha)\lambda_i}$

Appendix B

The conditions for which proposed estimator $\hat{\hat{\bar{Y}}}_{p_i}$ perform better than the SRSWOR sample mean are derived and are given below:

For $MSE\left(\hat{\hat{\bar{Y}}}_{p_j}\right) \le V\left(\bar{y}_r\right)$

$$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\right)\leq\frac{(1-f)}{n}S_y^2$$
$$\Rightarrow\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\right)\leq\frac{(1-f)}{n}\bar{Y}^2 C_y^2$$
$$\Rightarrow\left(C_y^2+\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\right)\leq C_y^2$$
$$\Rightarrow\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\leq C_y^2$$
$$\Rightarrow\theta_{p_i}^2 C_x^2\leq 2\rho\theta_{p_i} C_x C_y$$
$$\Rightarrow\theta_{p_i} C_x\leq 2\rho C_y$$
$$\Rightarrow\theta_{p_i}\leq 2\rho\frac{C_y}{C_x}$$

That is, $MSE\left(\hat{\hat{Y}}_{p_i}\right)\leq V\left(\bar{y}_r\right)$ if $\theta_{p_i}\leq 2\rho\frac{C_y}{C_x}$

Appendix C

The conditions for which proposed estimator $\hat{\hat{\bar{Y}}}_{p_i}$ perform better than the usual ratio estimator are derived and are given below:

For $MSE\left(\hat{\hat{Y}}_{p_j}\right)\leq MSE\left(\hat{\hat{Y}}_R\right)$

$$\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\right)\leq\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+C_x^2-2\rho C_x C_y\right)$$
$$\Rightarrow\left(C_y^2+\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\right)\leq\left(C_y^2+C_x^2-2\rho C_x C_y\right)$$
$$\Rightarrow\theta_{p_i}^2 C_x^2-2\rho\theta_{p_i} C_x C_y\leq C_x^2-2\rho C_x C_y$$
$$\Rightarrow\theta_{p_i}^2 C_x^2-C_x^2-2\rho\theta_{p_i} C_x C_y+2\rho C_x C_y\leq 0$$
$$\Rightarrow\left(\theta_{p_i}^2-1\right)C_x^2-2\rho C_x C_y\left(\theta_p-1\right)\leq 0$$
$$\Rightarrow\left(\theta_{p_i}-1\right)\left(\left(\theta_{p_i}+1\right)C_x^2-2\rho C_x C_y\right)\leq 0$$

Condition 1: $\left(\theta_{p_i}-1\right)\leq 0$ and $\left(\left(\theta_{p_i}+1\right)C_x^2-2\rho C_x C_y\right)\geq 0$

$$\Rightarrow \theta_{p_i} \leq 1 \text{ and } \left(\theta_{p_i} + 1\right) C_x^2 \geq 2\rho C_x C_y$$

$$\Rightarrow \theta_{p_i} \leq 1 \text{ and } \theta_{p_i} \geq \frac{2\rho C_y}{C_x} - 1$$

$$\Rightarrow \frac{2\rho C_y}{C_x} - 1 \leq \theta_{p_i} \leq 1$$

Condition 2: $\left(\theta_{p_i} - 1\right) \geq 0$ and $\left(\left(\theta_{p_i} + 1\right) C_x^2 - 2\rho C_x C_y\right) \leq 0$

$$\Rightarrow \theta_{p_i} \geq 1 \text{ and } \left(\theta_{p_i} + 1\right) C_x^2 - 2\rho C_x C_y$$

$$\Rightarrow \theta_{p_i} \geq 1 \text{ and } \theta_{p_i} \leq \frac{2\rho C_y}{C_x} - 1$$

$$\Rightarrow 1 \leq \theta_{p_i} \leq \frac{2\rho C_y}{C_x} - 1$$

That is, $MSE\left(\hat{\bar{Y}}_{p_i}\right) \leq MSE\left(\hat{\bar{Y}}_R\right)$ either $\frac{2\rho C_y}{C_x} - 1 \leq \theta_{p_i} \leq 1$ (or) $1 \leq \theta_{p_i} \leq \frac{2\rho C_y}{C_x} - 1$

Appendix D

The conditions for which proposed estimator $\hat{\bar{Y}}_{p_i}$ perform better than the existing modified ratio estimators (Class 1) are derived and are given below:

For $MSE\left(\hat{\bar{Y}}_{p_j}\right) \leq MSE\left(\hat{\bar{Y}}_i\right); i = 1, 2, 3, \ldots, 36$

$$\frac{(1-f)}{n} \bar{Y}^2 \left(C_y^2 + \theta_{p_i}^2 C_x^2 - 2\rho\theta_{p_i} C_x C_y\right) \leq \frac{(1-f)}{n} \bar{Y}^2 \left(C_y^2 + \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y\right)$$

$$\Rightarrow \left(C_y^2 + \theta_{p_i}^2 C_x^2 - 2\rho\theta_{p_i} C_x C_y\right) \leq \left(C_y^2 + \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y\right)$$

$$\Rightarrow \theta_{p_i}^2 C_x^2 - 2\rho\theta_{p_i} C_x C_y \leq \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y$$

$$\Rightarrow \theta_{p_i}^2 C_x^2 - \theta_i^2 C_x^2 - 2\rho\theta_{p_i} C_x C_y + 2\rho\theta_i C_x C_y \leq 0$$

$$\Rightarrow \left(\theta_{p_i}^2 - \theta_i^2\right) C_x^2 - 2\rho\theta_{p_i} C_x C_y \left(\theta_{p_i} - \theta_i\right) \leq 0$$

$$\Rightarrow \left(\theta_{p_i} - \theta_i\right)\left(\left(\theta_{p_i} + \theta_i\right) C_x^2 - 2\rho C_x C_y\right) \leq 0$$

Condition 1: $\left(\theta_{p_i} - \theta_i\right) \leq 0$ and $\left(\left(\theta_p + \theta_i\right) C_x^2 - 2\rho C_x C_y\right) \geq 0$

$$\Rightarrow \theta_{p_i} \le \theta_i \text{ and } \left(\theta_{p_i} + \theta_i\right) C_x^2 \ge 2\rho C_x C_y$$

$$\Rightarrow \theta_{p_i} \le \theta_i \text{ and } \theta_{p_i} \ge \frac{2\rho C_y}{C_x} - \theta_i$$

$$\Rightarrow \frac{2\rho C_y}{C_x} - \theta_i \le \theta_{p_i} \le \theta_i$$

Condition 2: $\left(\theta_{p_i} - \theta_i\right) \ge 0$ and $\left(\left(\theta_{p_i} + \theta_i\right) C_x^2 - 2\rho C_x C_y\right) \le 0$

$$\Rightarrow \theta_{p_i} \ge \theta_i \text{ and } \left(\theta_{p_i} + \theta_i\right) C_x^2 - 2\rho C_x C_y$$

$$\Rightarrow \theta_{p_i} \ge \theta_i \text{ and } \theta_{p_i} \le \frac{2\rho C_y}{C_x} - \theta_i$$

$$\Rightarrow \theta_i \le \theta_{p_i} \le \frac{2\rho C_y}{C_x} - \theta_i$$

That is, $MSE\left(\hat{\bar{Y}}_{p_i}\right) \le MSE\left(\hat{\bar{Y}}_i\right)$ either

$\frac{2\rho C_y}{C_x} - \theta_i \le \theta_{p_i} \le \theta_i$ (or) $\theta_i \le \theta_{p_i} \le \frac{2\rho C_y}{C_x} - \theta_i$