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Variables Sampling Plan For Correlated Data

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The sampling plan for the mean for correlated data is studied. The Operating Characteristic (OC) of the variable sampling plan for mean for correlated data are calculated and compared with the OC of known σ case.

Keywords: Variable sampling plan, correlation coefficient, operating characteristic function

Introduction

Quality control methods are commonly used to determine acceptability of a product with regard to its usefulness at the time it is put into service. It is essential from the consumer point of view, however, that a product return its usefulness for a certain length of time. Acceptance sampling is the testing or the inspection of selected items from a given lot followed by acceptance or rejection of that lot on the basis of the results of the test and its indicator of the lot's quality. It is assumed that a lot's quality is determined by the proportion of defective items in the lot. Further, attention will be restricted to those types of defects that are determined by one sided specification limits. For the purpose of exactness, upper specification limits will usually be discussed, i.e. an item will said to be defective if its measured characteristics are greater than some specified value U . Variable plans, however, require that the characteristic of interest be continuous variable. The characteristic is measured and its actual value is recorded. In variable sampling plans an underlying process distribution form is assumed. Then the proportion defective in the lot can be estimated by estimating the parameters of that distribution. The variable model thus requires more restrictive assumptions on the manufacturing process. If these assumptions can be justified there would be a substantial saving in sample size corresponding to a given sampling list.

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When the standard deviation of the lot quality is known, the criteria for acceptance and the associated mathematical computations get simplified. But, we should examine in each case whether treating the lot standard deviation (σ) as known and giving it a particular value are justified. When products are manufactured by automatic machinery whose inherent variation is known and tested, an example is provided where the lot standard deviation is known. When it is assumed that the lot standard deviation is known, and given a particular value σ , it must be remembered that σ is a constant in calculations and discussions. Also, the previous assumption that the lots are formed in such a way as to ensure homogeneity within lots holds good here also; and we assume that the directly measurable quality X follows the normal law of pattern of variation in the lot; these assumptions must be examined and reviewed from time to time when variables plans with known σ are in use.

Hapuxachi and Macephexsan (1992) studied the effect of serial correlation on acceptance sampling plans by variables by comparing Operating Characteristic (OC) curves, sample size and producers risk, α with that of the independent case when the process standard deviation (σ) is known. When σ is unknown and for large n , sampling plans can be constructed using central limit theorem. Several works have studied the effect of correlated data (see Kaiyang & Hancock, 1990; Seal, 1959; and Qiu et al., 2010). This study examines the sampling plan for mean for correlated data. The OC function of the variable sampling plan for mean for correlated data are calculated and compared with the OC function of known σ case.

Model Description And OC Function For Correlated Data

For a single sampling plan, with known-sigma, the procedure of selection of sample is as in the other single sampling plans. The n units in the sample are measured, and the values $x_1, x_2, x_3, \dots, x_n$ are obtained. The mean \bar{x} is calculated. Since the standard deviation sigma (σ) of the lot is known, σ is used.

Suppose that observations $x_1, x_2, x_3, \dots, x_n$ have a multivariate normal distribution with $E(x_i) = \mu$ and $Var(x_i) = \sigma^2$ and ρ as the common correlation coefficient between any x_i and $x_j, i \neq j$. Then

$$E(\bar{x}) = \mu \tag{1}$$

and

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$$\begin{aligned} \text{Var}(\bar{x}) &= \frac{\sigma^2}{n} [1 + (n-1)\rho] \\ &= \frac{\sigma^2}{n} T^2 \end{aligned} \tag{2}$$

where

$$T^2 = [1 + (n-1)\rho]. \tag{3}$$

In connection with a single sampling variable plan, when data are correlated, the following symbols will be used,

L = Lower specification limit,

U = Upper specification limit,

k = Acceptance parameter,

\bar{x} = Sample mean of correlated data,

ρ = Correlation Coefficient.

$$F(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{4}$$

where $z \sim N(0,1)$.

The OC function of single sampling plan can now be calculated. The acceptance criterion for correlated data mean plan is, for upper specification limit U , accept the lot if

$$\bar{x} + \frac{k\sigma T}{\sqrt{n}} \leq U \tag{5}$$

reject the lot otherwise.

The values of n and k are determined for a given set of values of the producer risk, α and consumer risk, β , AQL and LTPD, by formulae

$$n = \left[\frac{(K_\alpha + K_\beta)}{(K_{p_1} - K_{p_2})} \right]^2 \tag{6}$$

$$k = \left[\frac{K_\alpha K_{p_2} + K_\beta K_{p_1}}{K_\alpha + K_\beta} \right] \quad (7)$$

If p is the proportion defective in the lot

$$\frac{U - \mu}{\sigma} = K_p, \quad (8)$$

where $p = \int_{K_p}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$.

The expression for probability of acceptance (OC) function of the plan in normal case is

$$L(p) = Prob \left[\bar{x} + \sqrt{MSE(\bar{x})} \leq U = \mu + K_p \sigma \frac{T}{\sqrt{n}} \right] \quad (9)$$

where

$$T^2 = [1 + (n - 1)\rho].$$

Following Schilling (1982) the OC function for correlated data works out to be as

$$L(p) = \Phi \left[\frac{\sqrt{n}}{T} (K_p - k) \right] \quad (10)$$

where

$$\Phi(t) = \int_{-\infty}^t \phi(z) dz$$

The usual single sampling plan for known σ is

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$$L(p) = \Phi \left[\frac{\sqrt{n}(K_p - k)}{\sqrt{\rho}} \right] \quad (11)$$

Numerical Illustration and Result

For illustration, consider an example of producers and consumers oriented single sampling plan $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.08$, and $\beta = 0.10$. The values of n and k have been determined from equation (6) and (7) and are 10 and 18.09, respectively. The values of OC function for the above plan have been calculated for correlated data as well as for known standard deviation by using equation (10) and (11). These values of OC function for different values of correlation and usual known standard deviation case are presented in Table 1 and are plotted in Figure 1.

Table 1. Values of OC for different values of ρ

P	$\rho = 0$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.8$	$\rho = 1$
0.005	0.9926	0.8944	0.8056	0.7867	0.7574	0.7354
0.010	0.9500	0.8006	0.7196	0.7042	0.6812	0.6645
0.020	0.7820	0.6553	0.6085	0.6003	0.5882	0.5797
0.030	0.5908	0.5469	0.5323	0.5298	0.5262	0.5236
0.040	0.4271	0.4625	0.4741	0.4761	0.4790	0.4811
0.050	0.3016	0.3949	0.4271	0.4327	0.4408	0.4466
0.060	0.2101	0.3396	0.3878	0.3963	0.4087	0.4176
0.080	0.1000	0.2555	0.3252	0.3380	0.3568	0.3704
0.100	0.0471	0.1953	0.2770	0.2925	0.3159	0.3328
0.120	0.0221	0.1510	0.2384	0.2558	0.2823	0.3017
0.140	0.0104	0.1177	0.2067	0.2253	0.2539	0.2752
0.160	0.0049	0.0923	0.1803	0.1995	0.2296	0.2522
0.180	0.0023	0.0728	0.1579	0.1774	0.2083	0.2319
0.200	0.0011	0.0576	0.1388	0.1582	0.1896	0.2139

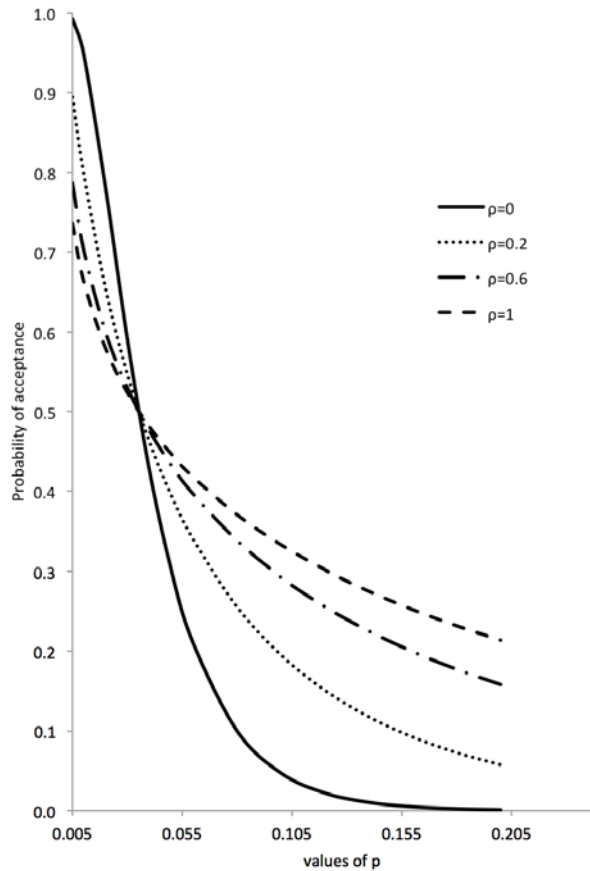


Figure 1. OC Curve for different values of Correlated Data (ρ)

From Figure 1 it is evident that the effect of correlation data on OC increases as ρ increases. As ρ increases, a significant effect is seen in producers risk as well as consumers risk, which is not acceptable. Hence one should maintain the correlation between the observations as low as possible, so as to protect producer as well as consumer.

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