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Likelihood Ratio Type Test for Linear Failure Rate Distribution vs. Exponential Distribution

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The Linear Failure Rate Distribution (LFRD) is considered. The graphs of its probability density function are examined for selected parameter combinations. Some of them are similar to the well-known exponential distribution. Incidentally exponential distribution is one of the two component models of the LFRD model. In view of the simpler form of exponential model as applicable in inference, looking at the frequency curves of LFRD, a test statistic is proposed based on ratio of likelihood functions containing the standard forms of the density functions of both LFRD and Exponential to discriminate between LFRD and exponential models. The critical values and the powers of the test statistic are developed.

Keywords: Linear failure rate distribution, likelihood ratio type, test statistic, power

Introduction

In reliability studies, series systems are one of many popular system configurations. If a series system has two components having independently distributed lifetime random variables with failure rate functions $h_1(x)$ and $h_2(x)$ then the reliability of the series system is

$$R(x) = exp\left[-\int_0^x \left\{h_1(t) + h_2(t)\right\} dt\right] \tag{1}$$

The corresponding cumulative distribution function, failure density function and failure rate function are respectively given by

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$$F(x)=1-exp\left[-\int_0^x\left\{h_1\left(t\right)+h_2\left(t\right)\right\}dt\right] \tag{2}$$

$$f(x)=\frac{d}{dx}F(x) \tag{3}$$

$$h(x)=\frac{f(x)}{R(x)} \tag{4}$$

Taking $h_1(x), h_2(x)$, as the failure rates of the exponential and Rayleigh distributions in (1) results in the most commonly used Linear Failure Rate Distribution (LFRD). More specifically, if $h_1(x)=a$ and $h_2(x)=bx$ then the failure density function, cumulative distribution function, hazard or failure rate function of LFRD is:

$$f(x)=\left(a+bx\right)e^{-\left(ax+\frac{bx^2}{2}\right)};x>0,a>0,b>0 \tag{5}$$

$$F(x)=1-e^{-\left(ax+\frac{bx^2}{2}\right)};x>0,a>0,b>0 \tag{6}$$

$$h(x)=a+bx \tag{7}$$

Bain (1974) seems to be one of the earliest works that has touched upon LFRD as a model useful for analysis in life testing. Ananda Sen (2005) gave a detailed review along with the distributional characteristics and inferential aspects of LFRD. Some basic features of LFRD are as follows:

Mean:

$$\mu=\sqrt{\frac{2\pi}{b}e^{\frac{a^2}{2b}}\left(1-\phi(\frac{a}{\sqrt{b}}\right)} \tag{8}$$

where ϕ denotes the cumulative distribution function of a standard normal variate.

Variance:

$$\sigma^2 = \frac{2}{b}(1 - a\mu) - \mu^2 \tag{9}$$

Mode:

$$M = \left(\sqrt{\frac{1}{b} - \frac{a}{b}}\right) I\left(a^2 < b\right) \tag{10}$$

where $I(.)$ denotes indicator function.

100 p^th^ Percentile:

$$F^{-1}(p) = \sqrt{\left(\frac{a}{b}\right)^2 - \frac{2\log(1-p)}{b}} - \frac{a}{b} \tag{11}$$

and hence median is

$$M_d = \sqrt{\left(\frac{a}{b}\right)^2 - \frac{2\log(0.5)}{b}} - \frac{a}{b} \tag{12}$$

In biological sciences this is called 50% survival time denoted by t_{50}.

Recurrence relation for raw moments is

$$\mu_k^1 = \frac{a}{k+1}\mu_{k+1}^{'} + \frac{b}{k+2}\mu_{k+2}^{'}; \quad k = 0,1,2... \tag{13}$$

The second, third and fourth non-central moments are

$$\mu_2^{'} = \frac{2}{b}(1 - a\mu) \tag{14}$$

$$\mu_3^{'} = \frac{3}{b}\left(\mu - \frac{a}{b}(1 - a\mu)\right) \tag{15}$$

$$\mu_4' = \frac{8}{b^2} + \frac{4a^2}{b^3} - \mu\left(\frac{12a}{b^2} + \frac{4a^3}{b^3}\right) \tag{16}$$

where μ is the mean of the distribution given by (8).

It can be seen from (10) that LFRD has a non-zero mode only if its parameters a and b satisfy the relation $a^2 < b$ with $a > 0$, $b > 0$.

The graphs of LFRD density function for some combinations of the parameters a, b are shown in the following figures.

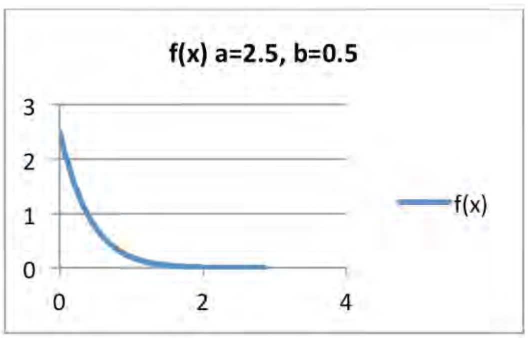

Figure 1. LFRD Density function when $a = 2.5$, $b = 0.5$

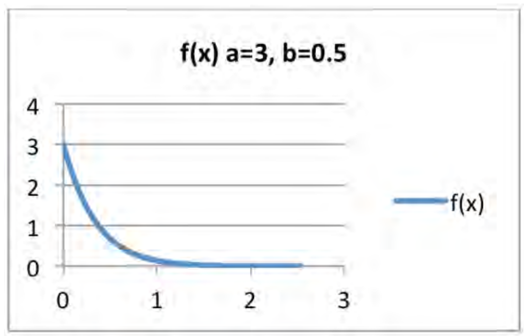

Figure 2. LFRD Density function when $a = 3$, $b = 0.5$

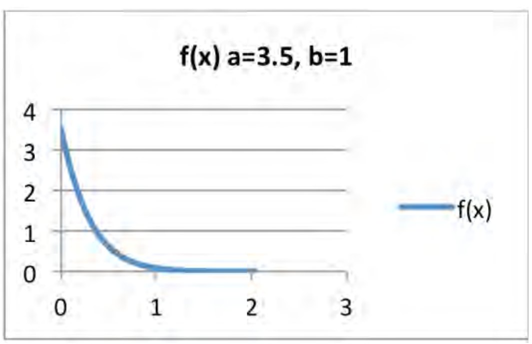

Figure 3. LFRD Density function when $a = 3.5$, $b = 1$

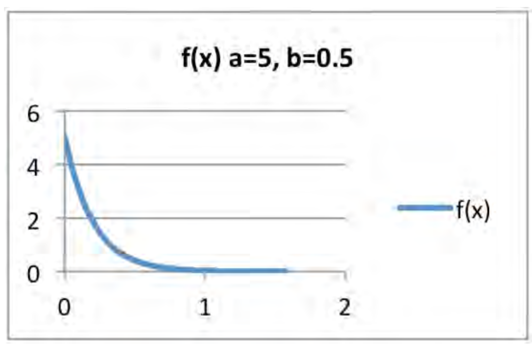

Figure 4. LFRD Density function when $a = 5$, $b = 0.5$

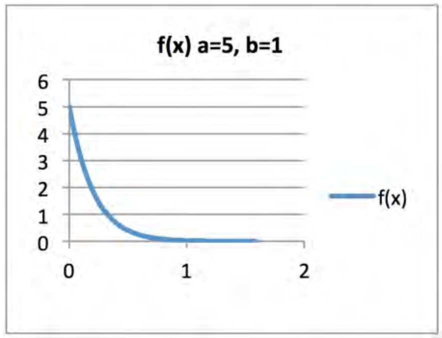

Figure 5. LFRD Density function when $a = 5$, $b = 1$

In Figures 1 – 5, the combinations of a and b are bound by $a^2 > b$, accordingly the mode is zero and the graphs are similar to that of exponential distribution. These characteristics of LFRD and its component distribution-exponential, motivated us to study the discriminatory aspect between LFRD and exponential through statistical test procedures. Such studies of discriminatory problems between probability models are made by Gupta, et al. (2002), Gupta and Kundu (2003a), Gupta and Kundu (2003b), Kundu and Gupta (2004a, 2004b), Kundu and Manglick (2004), Kundu, et al. (2005), Kundu and Manglick (2005), Kundu (2005), Kundu and Raqab (2007), Arabin and Kundu (2009), Arabin and Kundu (2010), Arabin and Kundu (2012a), Arabin and Kundu (2012b) and the references therein. The rest of the article is organised as follows. The methodology of the proposed LR type criterion for testing is described in the next section. The critical values of the test statistic are presented in following section. The aspects of power of the proposed test statistic are given in the final section, with a comparative study.

LR Type Methodology

Consider LFRD as a null population for example, P_0, the exponential model is regarded as an alternative population such as P_1. Let $x_1, x_2, \ldots, x_n$ be a given random sample of size n. Let L_1 denote the value of the likelihood function at the sample $x_1, x_2, \ldots, x_n$ with reference to the population P_1. L_1 is obtained as follows. Considering $x_1, x_2, \ldots, x_n$ as a sample from P_1 with some method of point estimation using the P_1 as the mathematical model, substituting the values of the estimates so obtained and the sample observations $x_1, x_2, \ldots, x_n$ in L_1 results in a value of L_1 from the sample $x_1, x_2, \ldots, x_n$ with respect to P_1. Using the sample $x_1, x_2, \ldots, x_n$ with P_0 as the model one can get estimates of the parameters of P_0 thereby getting the value of the likelihood function in relation to P_0 at $x_1, x_2, \ldots, x_n$ the parameters of P_0 as estimated using $x_1, x_2, \ldots, x_n$. L_0 is thus the value of likelihood function substituting the same sample $x_1, x_2, \ldots, x_n$ and the estimates of P_0. Thus for the same sample $x_1, x_2, \ldots, x_n$, two values of likelihood function with respect to P_0 as well as P_1 were obtained.

Generally in likelihood ratio test procedure the MLEs of the parameters in L_1 and L_0 are substituted thereby getting the value of L_1/L_0 at a given samples $x_1, x_2, \ldots, x_n$ with the parameters of P_1, P_0 estimated by ML method using the respective models. Because likelihood is also joint probability of the sample $x_1, x_2, \ldots, x_n$, had the sample belonged to P_0 the ratio L_1/L_0 tends to be very small. If it is the other way—that is the sample is truly from P_1—then the ratio L_1/L_0 tends to be very large. Hence the ratio L_1/L_0 can be a criterion to test whether the sample $x_1, x_2, \ldots, x_n$ actually belongs to the population P_1 or P_0. If L_1/L_0 is very small it may be stated that the sample belongs to P_0. Thus the ratio L_1/L_0 decides the sample to have belonged to either P_1 or P_0. It is therefore necessary to get critical values for L_1/L_0 to decide whether a given sample belongs to P_1 or P_0. In turn this leads to the knowledge of percentiles of the sampling distribution of L_1/L_0. In the proposed method of testing LFRD vs. exponential, point estimates of the parameters were used in both null and alternative populations using any other point estimation instead of the classical ML method, because MLEs of LFRD parameters are not analytically available. Similar testing processes were adopted by other researchers (Gupta & Kundu, 2003a; Kundu, et al., 2005). The proposed method is named the LR Type Criterion. In the discussion, the methods of point estimation that are considered are Least Squares estimators, Percentiles estimators, and Weighted Least Squares Estimators. The sampling distribution of L_1/L_0 is not mathematically tractable. The percentiles of L_1/L_0 were obtained through Monte-Carlo simulation as described in the following section. For comparison purposes, the following parametric combinations were chosen.

Table 1. Parametric combinations chosen for the study.

Least Squares Estimators		Percentiles Estimators		Weighted Least Squares Estimators	
a	***b***	***a***	***b***	***a***	***b***
0.5	4.0	0.5	4.0	0.5	4.0
2.5	0.5	2.5	0.5	2.5	0.5
3.0	0.5	3.0	0.5	3.0	0.5
3.5	1.0	3.5	1.0	3.5	1.0
5.0	0.5	5.0	0.5	5.0	0.5
5.0	1.0	5.0	1.0	5.0	1.0

LR Type Test Statistic – Critical Values

A random sample of size n is generated from LFRD (P_0) with parameter combinations as specified in the Table 1. Using that sample the parameters of LFRD are estimated by least square method / percentile method / weighted least square method given method of estimation. The estimates so obtained are substituted in P_0 in the respective places of the parameters along with the sample observations used to get those estimates thus having an estimated value of L_0. Using the same sample, the parameters appearing in P_1 are estimated by a least square method / percentile method / weighted least square method in succession using the model P_1 method suitable for P_1. Here because P_1 is an exponential distribution the MLEs of parameters of P_1 were calculated using formulae and expressions suitable for P_1. The estimates of the parameters of P_1 so obtained are then substituted in P_1 along with the sample observations used to get the estimates. Thus estimated likelihood function L_1 are obtained by three separate methods. The ratio L_1/L_0 for different samples with the same parameter combinations as described in the previous section is calculated for each sample. This procedure was repeated 10,000 times for accuracy and precision. Among these 10,000 values, various specified cut off points (percentiles) would form the critical values of L_1/L_0 useful for testing. These are given below in the following Tables 2 and 3, for only the parameters (a=2.5, b=0.5), (a=3, b=0.5). Results of other parameter combinations are available from the authors.

Table 2a: Percentiles of L_1/L_0 :: P_0: LFRD vs P_1: EXP, Least Square Estimation, (a=2.5, b=0.5)

	Least Square Estimation			
n	**5**	**10**	**15**	**20**
0.00100	0.05555	0.00866	0.00641	0.00495
0.00135	0.05579	0.00980	0.00700	0.00551
0.00270	0.05802	0.01389	0.01311	0.01005
0.00500	0.06338	0.01944	0.01997	0.01839
0.01000	0.07127	0.03196	0.03634	0.04059
0.02500	0.09607	0.07852	0.09572	0.09172
0.05000	0.15049	0.17091	0.17663	0.16909
0.10000	0.27829	0.32933	0.33165	0.32362
0.90000	1.45170	1.30077	1.31607	1.35776
0.95000	2.36966	1.56559	1.55775	1.59782
0.97500	4.97214	2.00069	1.86525	1.85212
0.99000	20.67554	3.27230	2.50671	2.34857
0.99500	89.41741	6.02098	3.90709	3.01258
0.99730	206.88170	10.79545	5.50735	4.79198
0.99865	938.89170	20.64189	19.63486	12.46792
0.99000	1441.98200	40.78289	23.69878	36.68090

Table 2b: Percentiles of L_1/L_0 :: P_0: LFRD vs P_1: EXP, Weighted Least Square Estimation, (a=2.5, b=0.5)

	Weighted Least Square Estimation			
n	**5**	**10**	**15**	**20**
0.00100	0.05558	0.00865	0.00560	0.00541
0.00135	0.05639	0.00958	0.00619	0.00693
0.00270	0.06081	0.01370	0.01244	0.00971
0.00500	0.06562	0.01851	0.01819	0.01855
0.01000	0.07279	0.03215	0.03719	0.04195
0.02500	0.09342	0.07797	0.09896	0.09637
0.05000	0.14239	0.16794	0.17843	0.18065
0.10000	0.26167	0.32321	0.33875	0.34647
0.90000	1.42631	1.28926	1.36464	1.46510
0.95000	2.39327	1.58677	1.65037	1.76804
0.97500	5.02094	2.18780	2.14986	2.28297
0.99000	19.63531	3.80238	4.02971	3.72403
0.99500	88.76622	9.38806	9.31526	8.65864
0.99730	222.91150	19.79771	28.90935	27.69206
0.99865	825.53910	58.24844	314.41790	122.53770
0.99000	1537.66000	125.22960	826.64140	388.92530

Table 2c: Percentiles of L_1/L_0 :: P_0: LFRD vs P_1: EXP, Percentile Estimation, (a=2.5, b=0.5)

	Percentile Estimation			
n	**5**	**10**	**15**	**20**
0.00100	0.07292	0.01208	0.00611	0.00468
0.00135	0.07628	0.01361	0.00718	0.00559
0.00270	0.08055	0.01710	0.00989	0.00979
0.00500	0.08669	0.02131	0.01592	0.01877
0.01000	0.09456	0.03308	0.03379	0.03679
0.02500	0.12043	0.07695	0.08074	0.07452
0.05000	0.16311	0.15190	0.15107	0.15069
0.10000	0.24330	0.28240	0.28224	0.28175
0.90000	2.08305	1.52860	1.46043	1.44967
0.95000	4.89041	2.23917	1.97528	1.82456
0.97500	14.79908	4.16817	3.02435	2.50522
0.99000	123.33970	19.42763	7.95037	5.62689
0.99500	748.87240	71.67762	31.90508	13.60665
0.99730	2710.38500	246.98620	100.23880	55.98616
0.99865	71595.25000	623.14900	454.89490	233.64480
0.99000	190377.10000	897.07890	952.26130	833.10900

Table 3a: Percentiles of L_1/L_0 :: P_0: LFRD vs P_1: EXP, Least Square Estimation, (a=3.0, b=0.5)

	Least Square Estimation			
n	**5**	**10**	**15**	**20**
0.00100	0.05603	0.01062	0.00596	0.00513
0.00135	0.05691	0.01139	0.00662	0.00634
0.00270	0.06038	0.01314	0.01177	0.01171
0.00500	0.06443	0.02129	0.02265	0.02324
0.01000	0.06995	0.03725	0.04223	0.04346
0.02500	0.08877	0.09321	0.09890	0.09904
0.05000	0.13725	0.17716	0.17834	0.18288
0.10000	0.26580	0.33410	0.34319	0.33358
0.90000	1.43639	1.33367	1.35457	1.37469
0.95000	2.31841	1.62922	1.59357	1.61963
0.97500	4.98869	2.14252	1.94302	1.93538
0.99000	21.02987	4.00168	2.80106	2.50630
0.99500	80.51004	8.20346	3.78306	3.15834
0.99730	252.88440	20.03408	6.46744	3.97503
0.99865	3116.18000	71.98767	11.33482	6.36183
0.99000	59094.28000	179.53870	17.87834	7.78476

Table 3b: Percentiles of L_1/L_0 :: P_0: LFRD vs P_1: EXP, Weighted Least Square Estimation, (a=3.0, b=0.5)

	Weighted Least Square Estimation			
n	**5**	**10**	**15**	**20**
0.00100	0.05647	0.01066	0.00576	0.00664
0.00135	0.05761	0.01104	0.00669	0.00795
0.00270	0.06264	0.01395	0.01140	0.01456
0.00500	0.06704	0.02150	0.02225	0.02212
0.01000	0.07299	0.03735	0.04243	0.04817
0.02500	0.08761	0.08531	0.09748	0.10723
0.05000	0.12932	0.17186	0.18351	0.19496
0.10000	0.25456	0.32422	0.34730	0.36366
0.90000	1.42080	1.31768	1.38883	1.49770
0.95000	2.32665	1.65014	1.67716	1.86482
0.97500	4.88276	2.28681	2.14252	2.36452
0.99000	20.92875	4.86321	3.67318	3.89300
0.99500	72.31535	11.28078	6.79731	6.48627
0.99730	281.68090	32.09840	21.00146	24.05159
0.99865	2668.58100	204.66170	82.91345	187.71560
0.99000	60999.62000	313.55800	123.97500	744.18340

Table 3c: Percentiles of L_1/L_0 :: P_0: LFRD vs P_1: EXP, Percentile Estimation, (a=3.0, b=0.5)

	Percentile Estimation			
n	**5**	**10**	**15**	**20**
0.00100	0.07245	0.01317	0.00645	0.00460
0.00135	0.07337	0.01435	0.00712	0.00557
0.00270	0.08110	0.01997	0.01176	0.01181
0.00500	0.08790	0.02728	0.01912	0.02171
0.01000	0.09718	0.04102	0.03919	0.03662
0.02500	0.11891	0.08040	0.08506	0.08822
0.05000	0.16062	0.15065	0.15994	0.16028
0.10000	0.25323	0.28654	0.30645	0.29008
0.90000	2.06479	1.54506	1.49559	1.46275
0.95000	5.04268	2.27774	1.96774	1.85958
0.97500	14.98131	4.37480	3.03259	2.57786
0.99000	95.64787	17.19165	7.48988	4.85088
0.99500	765.44120	76.76962	19.14950	14.15270
0.99730	4913.02900	229.08730	59.52394	70.34382
0.99865	343286.90000	526.59070	325.89340	280.56010
0.99000	2031568.00000	1125.17300	478.65110	711.98170

LR Type Test Statistic – Power

The LR type statistic suggested would be meaningful only if it is able to distinguish between the null and alternative populations. As is generally considered, the level of significance was fixed at 0.05. The critical value of L_1/L_0 corresponding to the level of significance 0.05 is (corresponding to the percentile at 0.95) identified from the relevant portion of Tables 2 and 3.

10,000 random samples of size each n = 5 (5) 20, from the alternative population (exponential) are generated. The MLE (reciprocal of sample mean) of the parameter of the alternative population, the individual sample values are substituted in L_1 to get the value of L_1. Using the same sample the value of L_0 as described in the previous section is also computed in order to get 10,000 values of L_1/L_0 for a given sample size, for a given parametric combination and for a given method of point estimation applied to the parameters of P_0. The proportion of values of L_1/L_0 that exceeded the critical value (c_0) out of 10,000 is computed and is considered as the power of the test statistic at level of significance 0.05.

Table 4. Powers of LR Test Criterion at α = 0.05 Parameter Estimates Using P.E., L.S.E., W.L.S.E. Methods

	Estimation Method											
	Percentile				**Least Squares**				**Weighted Least Squares**			
Parameter Combinations	***n*=5**	***n*=10**	***n*=15**	***n*=20**	***n*=5**	***n*=10**	***n*=15**	***n*=20**	***n*=5**	***n*=10**	***n*=15**	***n*=20**
a=2.5, *b*=0.5	0.0539	0.0601	0.0606	0.0729	0.0598	0.0692	0.0735	0.0737	0.0587	0.0672	0.0646	0.0697
a=3, *b*=0.5	0.0516	0.0585	0.0608	0.0704	0.0612	0.0619	0.0676	0.0700	0.0608	0.0607	0.0609	0.0599
a=3.5, *b*=1	0.0534	0.0586	0.0613	0.0726	0.0632	0.0714	0.0740	0.0786	0.0621	0.0668	0.0806	0.0678
a=5, *b*=0.5	0.0505	0.0500	0.0533	0.0608	0.0581	0.0570	0.0592	0.0599	0.0589	0.0559	0.0525	0.0543
a=5, *b*=1	0.0505	0.0540	0.0549	0.0606	0.0920	0.0639	0.0645	0.0619	0.0571	0.0613	0.0534	0.0589
a=0.5, *b*=4	0.0517	0.1126	0.3692	0.6813	0.1987	0.4105	0.6137	0.7472	0.2018	0.0613	0.5280	0.6024

A large value of the power shows that the test statistic is able to distinguish between the null and alternative populations. A small value of the power would show the indistinguishability between P_1 and P_0 as decided by LR type test statistic. The powers so obtained are given in Table 4, treated separately for each method of estimation at a specified level of significance 0.05.

The tabulated power values are very poor touching a maximum of 0.092 at n=5, a=5, b=1. These recorded powers show that the LR type test statistic is not able to discriminate between LFRD and exponential at all the values of n and the respective parametric combinations across the methods of estimation, except the last row of each table. It shows that exponential distribution can be used as an alternative for LFRD without much loss whereas the last row of each table shows that LFRD and exponential stand apart from each other for a=0.5, b=4. It is therefore concluded that the simple and powerful inferential tools available for exponential may be used for LFRD also. The discrimination between LFRD and exponential is clear as evident from the last row of each table.

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