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Consider the two parameter modified ratio estimators for the estimation of finite population mean using the skewness, kurtosis and correlation coefficient of two auxiliary variables. The efficiencies of the proposed modified ratio estimators are assessed with that of the simple random sampling without replacement (SRSWOR) sample mean and some of the existing ratio estimators in terms of mean squared errors. The entire above is explained with the help of certain natural populations available in the literature.

Keywords: Mean squared error; natural populations; percentage relative efficiency; simple random sampling

Introduction

In survey sampling, consider the problem of estimating the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ for a finite population $U = \{U_1, U_2, \ldots, U_N\}$ of N distinct and identifiable units, where the value Y_i is measured on U_i, $i = 1,2,3,\ldots,N$. Normally the population mean is estimated by the sample mean obtained from a random sample of size n drawn by simple random sampling without replacement (SRSWOR) from a finite population, when there is no auxiliary information available. Suppose that there is an auxiliary variable X available that is positively correlated with a study variable Y, in this case, either a ratio estimator or linear regression estimator may be used to improve the efficiency of the SRSWOR

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sample mean under certain conditions (see, Cochran (1977) and Murthy (1967) for example). Further improvements can be achieved on the ratio estimator by using known parameters such as skewness, kurtosis, quartiles and coefficient of variation of the auxiliary variable; the resulting estimators are called modified ratio estimators. For further details on the modified ratio estimators, readers are referred to Kadilar and Cingi (2004, 2009), Singh and Tailor (2003, 2005), Singh (2003), Sisodia and Dwivedi (1981), Subramani (2013), Subramani and Kumarapandiyan (2012a, b, c, 2013), Upadhyaya and Singh (1999), and Yan and Tian (2010).

If two auxiliary variables exist, then several modified ratio estimators have been proposed by linking together ratio estimators, product estimators and regression estimators in order to obtain more efficient estimators. For more detailed discussion about ratio estimators and their modifications using two auxiliary variables readers are referred to: Abu-Dayyeh et al. (2003), Bandyopadhyay (1980), Cochran (1940), Kadilar and Cingi (2004, 2005), Khare et al. (2013), Murthy (1967) , Naik and Gupta (1991), Olkin (1958), Perri (2004, 2007), Rao and Mudholkar (1967), Raj (1965), Sahoo and Swain (1980), Singh (2003), Singh (1965, 1967), Singh and Tailor (2003, 2005), Srivenkataramana (1980), Srivenkataramana and Tracy (1981), Tailor et al. (2011), and Tracy et al. (1996).

Existing Estimators with and without auxiliary variables

If $(y_1, y_2, \ldots, y_n)$ is a random sample of size n drawn from a population of size N using SRSWOR, then the population mean $\bar{Y}$ can be estimated by the sample mean $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, which is an unbiased estimator, and its variance is given by:

$$V(\bar{y}) = \frac{(1-f)}{n} S_y^2, \text{ where } S_y^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (Y_i - \bar{Y})^2, f = \frac{n}{N}. \tag{1}$$

The ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is defined as

$$\hat{\bar{Y}}_R = \tfrac{\bar{y}}{\bar{x}} \bar{X} = \hat{R}\bar{X}. \tag{2}$$

The mean squared error of the ratio estimator $\hat{\bar{Y}}_R$ to the first degree of approximation is:

$$MSE(\hat{\bar{Y}}_R) = \frac{(1-f)}{n} \bar{Y}^2 (C_y^2 + C_X^2 - 2p_{xy} C_x C_y) . \quad (3)$$

Singh (2003) suggested a ratio estimator with two auxiliary variables for estimating a population mean:

$$\hat{\bar{Y}}_1 = \bar{y} (\frac{\bar{X}_1}{x_1})(\frac{\bar{X}_2}{x_2}) . \quad (4)$$

The mean squared error of $\hat{\bar{Y}}_1$ to the first order of approximation is:

$$MSE(\hat{\bar{Y}}_1) = \frac{(1-f)}{n} \bar{Y}^2 (C_y^2 + C_{X_1}^2 + C_{X_2}^2 - 2p_{yx_1} C_{x_2} C_y + 2p_{x_1x_2} C_{x_1} C_{x_2}) \quad (5)$$

Singh and Tailor (2005) suggested the following modified ratio cum product estimator with known correlation coefficient between auxiliary variables:

$$\hat{\bar{Y}}_2 = \bar{y} \left(\frac{\bar{X}_1 + p_{x_1x_2}}{\bar{x}_1 + p_{x_1x_2}} \right) \left(\frac{\bar{x}_2 + p_{x_1x_2}}{\bar{X}_2 + p_{x_1x_2}} \right) . \quad (6)$$

The mean squared error of $\hat{\bar{Y}}_2$ to the first order of approximation is:

$$MSE(\hat{\bar{Y}}_2) = \frac{1-f}{n} \bar{Y}^2 \left[C_y^2 + \mu_1^* C_{x_1}^2 \left(\mu_1^* - 2K_{yx_1} \right) + \mu_2^* C_{x_2}^2 \left(\mu_2^* + 2(K_{yx_2} - \mu_1^* K_{x_1x_2}) \right) \right] \quad (7)$$

where

$$K_{yx_1} = \rho_{yx_1} \frac{C_y}{C_{x_1}}, K_{yx_2} = \rho_{yx_2} \frac{C_y}{C_{x_2}}, K_{x_1y_2} = \rho_{x_1y_2} \frac{C_{x_1}}{C_{x_2}}, \mu_1^* = \frac{\bar{X}_1}{\bar{X}_1 + \rho_{x_1x_2}} \text{ and } \mu_2^* = \frac{\bar{X}_2}{\bar{X}_2 + \rho_{x_1x_2}}$$

and $\rho_{x_1x_2}$ is the coefficient of correlation between X_1 and X_2.

Kadilar and Cingi (2005) proposed a new ratio estimator using two auxiliary variables as:

$$\hat{\bar{Y}}_3 = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)^{\alpha_1}\left(\frac{\bar{X}_2}{\bar{x}_2}\right)^{\alpha_2} + b_1(\bar{X}_1 - \bar{x}_1) + b_2(\bar{X}_2 - \bar{x}_2)\,. \tag{8}$$

The mean squared error of $\bar{Y}_3$ to the first order of approximation is:

$$MSE\left(\hat{\bar{Y}}_3\right) \cong \frac{1-f}{n}\left\{\begin{array}{l} S_y^2 + \left(\alpha_1 R_1 + B_1\right)^2 S_{x_1}^2 \\ +\ \left(\alpha_2 R_2 + B_2\right)^2 S_{x_1}^2 \\ -\ 2\left(\alpha_1 R_1 + B_1\right) S_{yx_1} \\ -\ 2\left(\alpha_2 R_2 + B_2\right) S_{yx_2} \\ +\ 2\left(\alpha_1 R_1 + B_1\right)\left(\alpha_2 R_2 + B_2\right) S_{x_1 x_2} \end{array}\right\} \tag{9}$$

where $B_1 = \frac{S_{xy}}{S_{x_1}^2}, B_2 = \frac{S_{xy}}{S_{x_2}^2}, R_1 = \frac{\bar{Y}}{\bar{X}_1}$ and $R_2 = \frac{\bar{Y}}{\bar{X}_2}$.

Perri (2007) suggested some modified ratio cum product estimators using two auxiliary variables for estimating the population mean:

$$\hat{\bar{Y}}_4 = \bar{y}\,\frac{\hat{t}_2}{\hat{t}_1}\,\frac{\bar{X}_1}{\bar{X}_2},\ \hat{\bar{Y}}_5 = \bar{y}\,\frac{\bar{X}_1}{\hat{t}_1}\,\frac{\bar{X}_2}{\hat{t}_2} \text{ and } \hat{\bar{Y}}_6 = \bar{y}\,\frac{\hat{t}_1}{\hat{t}_2}\,\frac{\bar{X}_2}{\bar{X}_1}, \tag{10}$$

where $\hat{t}_1 = \bar{x}_1 + \alpha_1(\bar{X}_1 - \bar{x}_1)$ and $\hat{t}_2 = \bar{x}_2 + \alpha_2(\bar{X}_2 - \bar{x}_2)$.

The mean squared errors of $\hat{\bar{Y}}_4,\ \hat{\bar{Y}}_5,\ \hat{\bar{Y}}_6$ to the first order of approximation are:

$$MSE\left(\hat{\bar{Y}}_4\right) = \frac{1-f}{n}\left[S_y^2 + \gamma_{x_1}^2 + \gamma_{x_2}^2 - 2\left(\gamma_{yx_1} - \gamma_{yx_2} + \gamma_{x_1x_2}\right)\right] \tag{11}$$

$$MSE\left(\hat{\bar{Y}}_5\right) = \frac{1-f}{n}\left[S_y^2 + \gamma_{x_1}^2 + \gamma_{x_2}^2 - 2\left(\gamma_{yx_1} + \gamma_{yx_2} - \gamma_{x_1x_2}\right)\right] \tag{12}$$

$$MSE\left(\hat{\bar{Y}}_6\right) = \frac{1-f}{n}\left[S_y^2 + \gamma_{x_1}^2 + \gamma_{x_2}^2 + 2\left(\gamma_{yx_1} - \gamma_{yx_2} - \gamma_{x_1x_2}\right)\right] \tag{13}$$

where $\gamma_{x_1x_2} = (1-\alpha_1)(1-\alpha_2)R_1R_2S_{x_1x_2}$, $\gamma_{x_1} = (1-\alpha_1)R_1S_{x_1}$, $\gamma_{x_2} = (1-\alpha_2)R_2S_{x_2}$ $\gamma_{yx_1} = (1-\alpha_1)R_1S_{yx_1}$. and $\gamma_{yx_2} = (1-\alpha_2)R_2S_{yx_2}$.

This article is concerned with estimating the population mean of a study variable Y by two parameter modified ratio estimators with known correlation coefficient, skewness and kurtosis of two auxiliary variables X_1 and X_2.

Proposed Two Parameter Modified Ratio Estimators

Whenever one or two auxiliary variables exist, a number of estimators including ratio, regression, product and chain ratio type estimators and their linear combinations have been proposed in the literature. These estimators are improved by using the known values of parameters such as skewness, kurtosis and coefficient of variation of the auxiliary variables. All of these estimators are functions of the ratio, product, regression estimators and their linear combinations; hence, an attempt is made herein to introduce the weighted average of the ratio estimators whenever there are two auxiliary variables available. As a result, two parameter modified ratio estimators with known correlation coefficient, skewness, kurtosis and their linear combinations of two auxiliary variables are proposed.

When the coefficient of kurtosis $\beta_2(X_1)$ of the auxiliary variable X_1, and $\beta_2(X_2)$ of the auxiliary variable X_2 is known, the following two parameter modified ratio estimator is proposed:

$$\hat{\bar{Y}}_{SP1} = \bar{y}\left(\frac{\alpha_1\left[\bar{X}_1 + \beta_2(X_1)\right] + \alpha_2\left[\bar{X}_2 + \beta_2(X_2)\right]}{\alpha_1\left[\bar{x}_1 + \beta_2(X_1)\right] + \alpha_2\left[\bar{x}_2 + \beta_2(X_2)\right]}\right). \tag{14}$$

Using the linear combinations of coefficient of kurtosis $\beta_2(X_1)$ of the auxiliary variable X_1, $\beta_2(X_2)$ of the auxiliary variable X_2 and correlation coefficient $\rho_{x_1x_2}$ between X_1 and X_2, the following two parameter modified ratio estimators are proposed:

$$\hat{\bar{Y}}_{SP2} = \bar{y}\left(\frac{\alpha_1\left[\rho_{x_1x_2}\bar{X}_1 + \beta_2(X_1)\right] + \alpha_2\left[\rho_{x_1x_2}\bar{X}_2 + \beta_2(X_2)\right]}{\alpha_1\left[\rho_{x_1x_2}\bar{x}_1 + \beta_2(X_1)\right] + \alpha_2\left[\rho_{x_1x_2}\bar{x}_2 + \beta_2(X_2)\right]}\right) \tag{15}$$

and

$$\hat{\bar{Y}}_{SP3} = \bar{y}\left(\frac{\alpha_1\left[\beta_2(X_1)\bar{X}_1 + \rho_{x_1x_2}\right] + \alpha_2\left[\beta_2(X_2)\bar{X}_2 + \rho_{x_1x_2}\right]}{\alpha_1\left[\beta_2(X_1)\bar{x}_1 + \rho_{x_1x_2}\right] + \alpha_2\left[\beta_2(X_2)\bar{x}_2 + \rho_{x_1x_2}\right]}\right). \tag{16}$$

Using the linear combinations of coefficient of skewness $\beta_1(X_1)$ of the auxiliary variable X_1, $\beta_1(X_2)$ of the auxiliary variable X_2, coefficient of kurtosis $\beta_2(X_1)$ of the auxiliary variable X_1 and $\beta_2(X_2)$ of the auxiliary variable X_2 the following two parameter modified ratio estimators are proposed:

$$\hat{\bar{Y}}_{SP4} = \bar{y}\left(\frac{\alpha_1\left[\beta_2(X_1)\bar{X}_1 + \beta_1(X_1)\right] + \alpha_2\left[\beta_2(X_2)\bar{X}_2 + \beta_1(X_2)\right]}{\alpha_1\left[\beta_2(X_1)\bar{x}_1 + \beta_1(X_1)\right] + \alpha_2\left[\beta_2(X_2)\bar{x}_2 + \beta_1(X_2)\right]}\right) \tag{17}$$

and

$$\hat{\bar{Y}}_{SP5} = \bar{y}\left(\frac{\alpha_1\left[\beta_1(X_1)\bar{X}_1 + \beta_2(X_1)\right] + \alpha_2\left[\beta_1(X_2)\bar{X}_2 + \beta_2(X_2)\right]}{\alpha_1\left[\beta_1(X_1)\bar{x}_1 + \beta_2(X_1)\right] + \alpha_2\left[\beta_1(X_2)\bar{x}_2 + \beta_2(X_2)\right]}\right). \tag{18}$$

In general, the estimators proposed in (14) to (18) can be defined as particular cases of the estimator:

$$\hat{\bar{Y}}_{SPT} = \bar{y}\left(\frac{\alpha_1\left[\bar{X}_1 + T_1\right] + \alpha_2\left[\bar{X}_2 + T_2\right]}{\alpha_1\left[\bar{x}_1 + T_1\right] + \alpha_2\left[\bar{x}_2 + T_2\right]}\right). \tag{19}$$

For suitable choices of T_1 and T_2 in (19), the estimators defined in (14) to (18) are obtained.

Suppose that,

i. if $T_1 = \beta_2(X_1)$ and $T_2 = \beta_2(X_2)$ in (19), then $\hat{\bar{Y}}_{SPT}$ becomes $\hat{\bar{Y}}_{SP1}$ as defined in (14);

ii. if $T_1 = \dfrac{\beta_2(X_1)}{\rho_{x_1x_2}}$ and $T_2 = \dfrac{\beta_2(X_2)}{\rho_{x_1x_2}}$ in (19), then $\hat{\bar{Y}}_{SPT}$ becomes $\hat{\bar{Y}}_{SP2}$ as defined in (15);

iii. if $T_1 = \dfrac{\rho_{x_1x_2}}{\beta_2(X_1)}$ and $T_2 = \dfrac{\rho_{x_1x_2}}{\beta_2(X_2)}$ in (19), then $\hat{\bar{Y}}_{SPT}$ becomes $\hat{\bar{Y}}_{SP3}$ as defined in (16);

iv. if $T_1 = \frac{\beta_1(X_1)}{\beta_2(X_1)}$ and $T_2 = \frac{\beta_1(X_2)}{\beta_2(X_2)}$ in (19), then $\hat{\bar{Y}}_{SPT}$ becomes $\hat{\bar{Y}}_{SP4}$ as defined in (17); and

v. if $T_1 = \frac{\beta_2(X_1)}{\beta_1(X_1)}$ and $T_2 = \frac{\beta_2(X_2)}{\beta_1(X_2)}$ in (19), then $\hat{\bar{Y}}_{SPT}$ becomes $\hat{\bar{Y}}_{SP5}$ as defined in (18).

Derivation of Mean Squared Error of the proposed estimators

The mean squared error of the proposed estimator $\hat{\bar{Y}}_{SPT}$ is derived as follows. If $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$, $e_1 = \frac{\bar{x}_1 - \bar{X}_1}{\bar{X}_1}$, and $e_2 = \frac{\bar{x}_2 - \bar{X}_2}{\bar{X}_2}$, then $\bar{y} = \bar{Y}(1+e_0)$, $\bar{x}_1 = \bar{X}_1(1+e_1)$, and $\bar{x}_2 = \bar{X}_2(1+e_2)$. From the definition of e_0 and e_1, $E\left[e_0\right] = E\left[e_1\right] = 0$ is obtained where $E\left[e_0^2\right] = \frac{(1-f)}{n} C_y^2$, $E\left[e_1^2\right] = \frac{1-f}{n} C_{x_1}^2$, $E\left[e_2^2\right] = \frac{1-f}{n} C_{x_2}^2$, $E(e_0 e_1) = \frac{1-f}{n} \rho_{yx_1} C_y C_{x_1}$, $E(e_0 e_2) = \frac{1-f}{n} \rho_{yx_2} C_y C_{x_2}$ and $E(e_1 e_2) = \frac{1-f}{n} \rho_{x_1 x_2} C_{x_1} C_{x_2}$.

The proposed estimator $\hat{\bar{Y}}_{SPT}$ can be written in terms of e_0, e_1 and e_2 as:

$$\hat{\bar{Y}}_{SPT} = \bar{Y}(1+e_0)\left(\frac{\alpha_1[\bar{X}_1 + T_1] + \alpha_2[\bar{X}_2 + T_2]}{\alpha_1[\bar{X}_1(1+e_1) + T_1] + \alpha_2[\bar{X}_2(1+e_2) + T_2]} \right)$$

$$\Rightarrow \hat{\bar{Y}}_{SPT} = \bar{Y}(1+e_0)\left(\frac{\alpha_1\bar{X}_1 + \alpha_2\bar{X}_2 + \alpha_1 T_1 + \alpha_2 T_2}{\alpha_1\bar{X}_1 + \alpha_2\bar{X}_2 + \alpha_1 T_1 + \alpha_2 T_2 + \alpha_1\bar{X}_1 e_1 + \alpha_2\bar{X}_2 e_2} \right)$$

$$\Rightarrow \hat{\bar{Y}}_{SPT} = \bar{Y}(1+e_0)\left(\frac{1}{1+\theta_1' e_1 + \theta_2' e_2} \right), \theta_1' = \frac{\alpha_1\bar{X}_1}{\alpha_1[\bar{X}_1 + T_1] + \alpha_2[\bar{X}_2 + T_2]} \text{ and } \theta_2' = \frac{\alpha_2\bar{X}_2}{\alpha_1[\bar{X}_1 + T_1] + \alpha_2[\bar{X}_2 + T_2]}$$

$$\Rightarrow \hat{\bar{Y}}_{SPT} = \bar{Y}(1+e_0)\left(1+\theta_1' e_1 + \theta_2' e_2\right)^{-1}$$

$$\Rightarrow \hat{\bar{Y}}_{SPT} = \bar{Y}(1+e_0)\left(1-\theta_1' e_1 - \theta_2' e_2 + \left(\theta_1' e_1 + \theta_2' e_2\right)^2\right)$$

$$\Rightarrow \hat{\bar{Y}}_{SPT} = \bar{Y}(1+e_0)\left(1-\theta_1' e_1 - \theta_2' e_2 + \theta_1'^2 e_1^2 + \theta_2'^2 e_2^2 + 2\theta_1' e_1 \theta_2' e_2\right)$$

Neglecting higher order terms

$$\hat{\hat{Y}}_{SPT} - Y = \bar{Y}e_0 - \bar{Y}\theta_1' e_1 - \bar{Y}\theta_2' e_2 + \bar{Y}\theta_1' e_1 \theta_2' e_2 - \bar{Y}\theta_1' e_0 e_1 - \bar{Y}\theta_2' e_0 e_2$$

and squaring and taking expectations on both sides results in:

$$MSE(\hat{\hat{Y}}_{SPT}) = E(\hat{\hat{Y}}_{SPT} - \bar{Y})^2 = \bar{Y}^2 E(e_0 - \theta_1' e_1 - \theta_2' e_2)^2$$

$$\Rightarrow MSE(\hat{\hat{Y}}_{SPT}) = \bar{Y}^2 E\left(e_0^2 + \theta_1'^2 e_1^2 + \theta_2'^2 e_2^2 - 2\theta_1' e_0 e_1 - 2\theta_2' e_0 e_2 + 2\theta_1'\theta_2' e_1 e_2\right)$$

$$\Rightarrow MSE(\hat{\hat{Y}}_{SPT}) = \bar{Y}^2 \left\{E(e_0^2) + \theta_1'^2 E(e_1^2) + \theta_2'^2 E(e_2^2) - 2\theta_1' E(e_0 e_1) - 2\theta_1' E(e_0 e_2) + 2\theta_1'\theta_2' E(e_1 e_2)\right\}$$

$$MSE(\hat{\hat{Y}}_{SPT}) = \frac{1-f}{n}\bar{Y}^2 \left\{C_y^2 + \theta_1'^2 C_{x_1}^2 + \theta_2'^2 C_{x_2}^2 - 2\theta_1' \rho_{yx_1} C_y C_{x_1} - 2\theta_2' \rho_{yx_2} C_y C_{x_2} + 2\theta_1'\theta_2' \rho_{x_1 x_2} C_{x_1} C_{x_2}\right\} \quad (20)$$

The proposed modified ratio estimator $\hat{\hat{Y}}_{SPT}$ can be easily generalized to include several auxiliary variables. If X_1, X_2,…,X_k are k auxiliary variables that are positively correlated with a study variable Y, then the generalized modified ratio estimator is defined as

$$\hat{\hat{Y}}_{GSPT} = \bar{y}\left(\frac{\alpha_1[\bar{X}_1 + T_1] + \alpha_2[\bar{X}_2 + T_2] + \alpha_3[\bar{X}_3 + T_3] + \ldots + \alpha_k[\bar{X}_k + T_k]}{\alpha_1[\bar{X}_1 + T_1] + \alpha_2[\bar{X}_2 + T_2] + \alpha_3[\bar{X}_3 + T_3] + \ldots + \alpha_k[\bar{X}_k + T_k]}\right)$$

where α_1 , α_2 , …, α_k are the weights and the T_1 , T_2 ,…, T_k are the known parameters of the auxiliary variables.

Efficiency Comparisons

The efficiencies of the proposed estimators for estimating the finite population mean are assessed with that of SRSWOR sample mean and other existing estimators, as previously proposed.

From expressions (20) and (1), the proposed estimators $\hat{\hat{Y}}_{SPT}$ are more efficient than the SRSWOR sample mean $\bar{y}_r$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq V(\bar{y}_r)$$

if

$$\theta_1'^2 C_{x_1}^2 + \theta_2'^2 C_{x_2}^2 \leq 2\left(\theta_1' \rho_{yx_1} C_y C_{x_1} + \theta_2' \rho_{yx_2} C_y C_{x_2} - \theta_1' \theta_2' \rho_{x_1 x_2} C_{x_1} C_{x_2}\right) \tag{21}$$

From expressions (20) and (5), the proposed estimators $\hat{\bar{Y}}_{SPT}$ are more efficient than the existing ratio estimator $\hat{\bar{Y}}_1$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq MSE(\hat{\bar{Y}}_1)$$

if

$$\left(\theta_1'^2 - 1\right) C_{x_1}^2 + \left(\theta_2'^2 - 1\right) C_{x_2}^2 \leq 2\left\{(\theta_1' - 1)\rho_{yx_1} C_y C_{x_1} + (\theta_2' - 1)\rho_{yx_2} C_y C_{x_2} - (\theta_1'\theta_2' - 1)\rho_{x_1 x_2} C_{x_1} C_{x_2}\right\} \tag{22}$$

From expressions (20) and (7), the proposed estimators $\hat{\bar{Y}}_{SPT}$ are more efficient than the existing ratio estimator $\hat{\bar{Y}}_2$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq MSE(\hat{\bar{Y}}_2)$$

if

$$\begin{aligned}&\left(\theta_1'^2 - \mu_1^{*2}\right) C_{x_1}^2 + \left(\theta_1'^2 - \mu_2^{*2}\right) C_{x_2}^2 \leq \\ &2\left\{(\theta_1' - \mu_1^*)\rho_{yx_1} C_y C_{x_1} + (\theta_2' - \mu_2^*)\rho_{yx_2} C_y C_{x_2} - (\theta_1'\theta_2' + \mu_1^*\mu_2^*)\rho_{x_1 x_2} C_{x_1} C_{x_2}\right\}\end{aligned} \tag{23}$$

From expressions (20) and (9), the proposed estimators $\hat{\bar{Y}}_{SPT}$ are more efficient than the existing ratio estimator $\hat{\bar{Y}}_3$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq MSE(\hat{\bar{Y}}_3)$$

if

$$\begin{aligned}&\alpha_1^2\left(R_{sp}'^{\,2} - R_1^2\right) S_{x_1}^2 + \alpha_2^2\left(R_{sp}'^{\,2} - R_2^2\right) S_{x_2}^2 + B_1 S_{yx_1} + B_2 S_{yx_2} \leq \\ &2\left\{R_{sp}'(\alpha_1 S_{yx_1} + \alpha_2 S_{yx_2}) - \left[\alpha_1\alpha_2 R_{sp}'^{\,2} - (\alpha_1 R_1 + B_1)(\alpha_2 R_2 + B_2)\right] S_{x_1 x_2}\right\}\end{aligned} \tag{24}$$

From expressions (20) and (11), the proposed estimators $\hat{\bar{Y}}_{SPT}$ are more efficient than the existing ratio estimator $\hat{\bar{Y}}_4$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq MSE(\hat{\bar{Y}}_4)$$

if

$$\begin{aligned} &\left[\alpha_1^2 R_{sp}'^{\,2} - (1-\alpha_1)^2 R_1^2\right] S_{x_1}^2 + \left[\alpha_2^2 R_{sp}'^{\,2} - (1-\alpha_2)^2 R_2^2\right] S_{x_2}^2 \leq \\ &2\Big\{ S_{yx_1}\left[\alpha_1 R'_{sp} - (1-\alpha_1) R_1\right] + S_{yx_2}\left[\alpha_2 R'_{sp} + (1-\alpha_2) R_2\right] \\ &\qquad - S_{x_1x_2}\left[\alpha_1\alpha_2 R_{sp}'^{\,2} + R_1 R_2 (1-\alpha_1)(1-\alpha_2)\right]\Big\} \end{aligned} \tag{25}$$

From expressions (20) and (12), the proposed estimators $\hat{\bar{Y}}_{SPT}$ are more efficient than the existing ratio estimator $\hat{\bar{Y}}_5$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq MSE(\hat{\bar{Y}}_5)$$

if

$$\begin{aligned} &\left[\alpha_1^2 R_{sp}'^{\,2} - (1-\alpha_1)^2 R_1^2\right] S_{x_1}^2 + \left[\alpha_2^2 R_{sp}'^{\,2} - (1-\alpha_2)^2 R_2^2\right] S_{x_2}^2 \leq \\ &2\Big\{ S_{yx_1}\left[\alpha_1 R'_{sp} - (1-\alpha_1) R_1\right] + S_{yx_2}\left[\alpha_2 R'_{sp} - (1-\alpha_2) R_2\right] \\ &\qquad - S_{x_1x_2}\left[\alpha_1\alpha_2 R_{sp}'^{\,2} - R_1 R_2 (1-\alpha_1)(1-\alpha_2)\right]\Big\} \end{aligned} \tag{26}$$

From expressions (20) and (13), the proposed estimators $\hat{\bar{Y}}_{SPT}$ are more efficient than the existing ratio estimator $\hat{\bar{Y}}_6$. The derived conditions are:

$$MSE\left(\hat{\bar{Y}}_{SPT}\right) \leq MSE(\hat{\bar{Y}}_6)$$

if

$$\begin{aligned} &\left[\alpha_1^2 R_{sp}'^{\,2} - (1-\alpha_1)^2 R_1^2\right] S_{x_1}^2 + \left[\alpha_2^2 R_{sp}'^{\,2} - (1-\alpha_2)^2 R_2^2\right] S_{x_2}^2 \leq \\ &2\Big\{ S_{yx_1}\left[\alpha_1 R'_{sp} + (1-\alpha_1) R_1\right] + S_{yx_2}\left[\alpha_2 R'_{sp} - (1-\alpha_2) R_2\right] \\ &\qquad - S_{x_1x_2}\left[\alpha_1\alpha_2 R_{sp}'^{\,2} + R_1 R_2 (1-\alpha_1)(1-\alpha_2)\right]\Big\} \end{aligned} \tag{27}$$

where $R'_{sp} = \dfrac{\bar{Y}}{\alpha_1(\bar{X}_1 + T_1) + \alpha_2(\bar{X}_2 + T_2)}$

Numerical Study

The performance of the proposed two parameter modified ratio estimators have been compared with that of the SRSWOR sample mean and some existing modified ratio estimators algebraically. However, the proposed estimators perform well compared to the existing estimators only under certain conditions and - for numerical comparisons - they are assessed for certain natural populations. In this connection, two natural populations were considered to assess the performance of the proposed estimators with that of existing estimators. Population 1 is from Singh and Chaudhary (1986, p. 177) and population 2 is from Kadilar and Cingi (2009, p. 117). The description of the study and auxiliary variables for the two populations are shown in Table 1.

Table 1. Description of the study variable and auxiliary variable

Population	Study Variable Y	Auxiliary Variable X_1	Auxiliary Variable X_2
1	Area under wheat in 1974	Area under wheat in1971	Area under wheat in1973
2	Length of the fish	Length of the head	Length of the fin

The population parameters and constants computed for the two populations are given in Tables 2-4.

Table 2. Parameters and Constants of the Populations

Parameter	N	n	$\bar{Y}$	$\bar{X}_1$	$\bar{X}_2$	ρ_{yx_1}	ρ_{yx_2}	$\rho_{x_1x_2}$	β_{11}
Pop. 1	34.00	20.00	856.41	208.88	199.44	0.45	0.45	0.98	0.87
Pop. 2	25.00	10.00	75.28	14.30	6.82	0.99	0.89	0.92	1.24

Parameter	β_{12}	β_{21}	β_{22}	S_y	C_y	S_{x_1}	S_{x_2}	C_{x_1}	C_{x_2}
Pop. 1	1.28	2.91	3.73	733.14	0.86	150.51	150.22	0.72	0.75
Pop. 2	0.86	4.26	4.35	17.27	0.23	3.17	1.53	0.22	0.22

Table 3. Variance/Mean squared error of the existing and proposed estimators for Population 1

Existing Estimators		
$\hat{\bar{Y}}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$
37940.84	90847.02	40145.19

		Existing Estimators				**Proposed Estimators**				
α_1	α_2	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$	$\hat{\bar{Y}}_{SP1}$	$\hat{\bar{Y}}_{SP2}$	$\hat{\bar{Y}}_{SP3}$	$\hat{\bar{Y}}_{SP4}$	$\hat{\bar{Y}}_{SP5}$
0.0	1.0	67310.24	64818.97	64818.97	64818.97	37057.66	37047.45	37541.57	37466.93	37396.04
0.1	0.9	62385.73	60005.70	60005.90	60005.94	36843.39	36834.06	37275.19	37210.98	37138.09
0.2	0.8	58048.59	56317.41	56317.77	56317.84	36654.14	36645.64	37036.69	36982.32	36907.47
0.3	0.7	54298.80	53754.11	53754.56	53754.66	36489.42	36481.71	36825.28	36780.23	36703.44
0.4	0.6	51136.38	52315.78	52316.28	52316.42	36348.74	36341.77	36640.21	36604.01	36525.27
0.5	0.5	48561.32	52002.43	52002.92	52003.10	36231.61	36225.37	36480.75	36452.97	36372.28
0.6	0.4	46573.62	52814.07	52814.49	52814.70	36137.58	36132.02	36346.17	36326.46	36243.79
0.7	0.3	45173.28	54750.68	54750.99	54751.23	36066.20	36061.30	36235.78	36223.84	36139.12
0.8	0.2	44360.30	57812.27	57812.42	57812.69	36017.00	36012.74	36148.92	36144.46	36057.65
0.9	0.1	44134.68	61998.84	61998.77	61999.08	35989.55	35985.91	36084.91	36087.73	35998.74
1.0	0.0	44496.43	67310.39	67310.05	67310.39	35983.42	35980.39	36043.13	36053.04	35961.79

Table 4. Variance/Mean squared error of the existing and proposed estimators for Population 2

Existing Estimators		
$\hat{\bar{Y}}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$
17.90	17.58	17.58

		Existing Estimators				Proposed Estimators				
α_1	α_2	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$	$\hat{\bar{Y}}_{SP1}$	$\hat{\bar{Y}}_{SP2}$	$\hat{\bar{Y}}_{SP3}$	$\hat{\bar{Y}}_{SP4}$	$\hat{\bar{Y}}_{SP5}$
0.0	1.0	35.07	34.61	34.61	34.61	5.32	5.54	3.89	3.90	5.72
0.1	0.9	32.15	31.58	31.62	31.64	4.50	4.72	2.84	2.84	4.72
0.2	0.8	29.57	29.24	29.31	29.34	3.85	4.07	2.12	2.12	3.92
0.3	0.7	27.33	27.58	27.67	27.71	3.32	3.53	1.62	1.62	3.28
0.4	0.6	25.42	26.60	26.71	26.75	2.89	3.10	1.26	1.26	2.77
0.5	0.5	23.85	26.31	26.41	26.47	2.54	2.74	1.01	1.01	2.36
0.6	0.4	22.62	26.71	26.79	26.86	2.26	2.44	0.83	0.83	2.03
0.7	0.3	21.72	27.78	27.83	27.92	2.02	2.19	0.70	0.70	1.76
0.8	0.2	21.16	29.55	29.55	29.65	1.83	1.99	0.61	0.61	1.55
0.9	0.1	20.94	31.99	31.94	32.05	1.67	1.81	0.55	0.55	1.38
1.0	0.0	21.05	35.12	35.00	35.12	1.53	1.67	0.51	0.51	1.25

From the values in Tables 3 and 4, the mean squared error of the proposed modified ratio estimators $\hat{\bar{Y}}_{SPj,} j = 1,2,3,4,5$ are less than the variance of SRSWOR sample mean, the mean squared error of the existing modified ratio estimators $\hat{\bar{Y}}_j; j = 1,2,3,...,6$. Further, to show the efficiency of the proposed estimators, the percentage relative efficiencies (PRE's) of the proposed estimators with respect to the existing estimators is computed by:

$$PRE\left(\hat{\bar{Y}}_{SPj}\right) = \frac{MSE(.)}{MSE\left(\hat{\bar{Y}}_{SPj}\right)} * 100 .$$

Table 5. PRE of the proposed estimator $\hat{\bar{Y}}_{SPj}$ for Population 1

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	Modified Ratio Estimators					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.0	1.0	$\hat{\bar{Y}}_{SP1}$	102.38	245.15	108.33	181.64	174.91	174.91	174.91
		$\hat{\bar{Y}}_{SP2}$	102.41	245.22	108.36	181.69	174.96	174.96	174.96
		$\hat{\bar{Y}}_{SP3}$	101.06	241.99	106.94	179.30	172.66	172.66	172.66
		$\hat{\bar{Y}}_{SP4}$	101.26	242.47	107.15	179.65	173.00	173.00	173.00
		$\hat{\bar{Y}}_{SP5}$	101.46	242.93	107.35	179.99	173.33	173.33	173.33
0.1	0.9	$\hat{\bar{Y}}_{SP1}$	102.98	246.58	108.96	169.33	162.87	162.87	162.87
		$\hat{\bar{Y}}_{SP2}$	103.00	246.64	108.99	169.37	162.91	162.91	162.91
		$\hat{\bar{Y}}_{SP3}$	101.79	243.72	107.70	167.37	160.98	160.98	160.98
		$\hat{\bar{Y}}_{SP4}$	101.96	244.14	107.89	167.65	161.26	161.26	161.26
		$\hat{\bar{Y}}_{SP5}$	102.16	244.62	108.10	167.98	161.57	161.58	161.58
0.2	0.8	$\hat{\bar{Y}}_{SP1}$	103.51	247.85	109.52	158.37	153.65	153.65	153.65
		$\hat{\bar{Y}}_{SP2}$	103.53	247.91	109.55	158.41	153.68	153.68	153.68
		$\hat{\bar{Y}}_{SP3}$	102.44	245.29	108.39	156.73	152.06	152.06	152.06
		$\hat{\bar{Y}}_{SP4}$	102.59	245.65	108.55	156.96	152.28	152.28	152.28
		$\hat{\bar{Y}}_{SP5}$	102.80	246.15	108.77	157.28	152.59	152.59	152.59
0.3	0.7	$\hat{\bar{Y}}_{SP1}$	103.98	248.97	110.02	148.81	147.31	147.32	147.32
		$\hat{\bar{Y}}_{SP2}$	104.00	249.02	110.04	148.84	147.35	147.35	147.35
		$\hat{\bar{Y}}_{SP3}$	103.03	246.70	109.02	147.45	145.97	145.97	145.97
		$\hat{\bar{Y}}_{SP4}$	103.16	247.00	109.15	147.63	146.15	146.15	146.15
		$\hat{\bar{Y}}_{SP5}$	103.37	247.52	109.38	147.94	146.46	146.46	146.46

Table 5, continued

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	*Modified Ratio Estimators*					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.4	0.6	$\hat{\bar{Y}}_{SP1}$	104.38	249.93	110.44	140.68	143.93	143.93	143.93
		$\hat{\bar{Y}}_{SP2}$	104.40	249.98	110.47	140.71	143.95	143.96	143.96
		$\hat{\bar{Y}}_{SP3}$	103.55	247.94	109.57	139.56	142.78	142.78	142.78
		$\hat{\bar{Y}}_{SP4}$	103.65	248.19	109.67	139.70	142.92	142.92	142.93
		$\hat{\bar{Y}}_{SP5}$	103.88	248.72	109.91	140.00	143.23	143.23	143.23
0.5	0.5	$\hat{\bar{Y}}_{SP1}$	104.72	250.74	110.80	134.03	143.53	143.53	143.53
		$\hat{\bar{Y}}_{SP2}$	104.74	250.78	110.82	134.05	143.55	143.55	143.55
		$\hat{\bar{Y}}_{SP3}$	104.00	249.03	110.04	133.11	142.55	142.55	142.55
		$\hat{\bar{Y}}_{SP4}$	104.08	249.22	110.13	133.22	142.66	142.66	142.66
		$\hat{\bar{Y}}_{SP5}$	104.31	249.77	110.37	133.51	142.97	142.97	142.97
0.6	0.4	$\hat{\bar{Y}}_{SP1}$	104.99	251.39	111.09	128.88	146.15	146.15	146.15
		$\hat{\bar{Y}}_{SP2}$	105.01	251.43	111.11	128.90	146.17	146.17	146.17
		$\hat{\bar{Y}}_{SP3}$	104.39	249.95	110.45	128.14	145.31	145.31	145.31
		$\hat{\bar{Y}}_{SP4}$	104.44	250.08	110.51	128.21	145.39	145.39	145.39
		$\hat{\bar{Y}}_{SP5}$	104.68	250.66	110.76	128.50	145.72	145.72	145.72
0.7	0.3	$\hat{\bar{Y}}_{SP1}$	104.99	251.38	111.09	125.00	151.50	151.50	151.50
		$\hat{\bar{Y}}_{SP2}$	104.74	250.79	110.83	124.71	151.15	151.15	151.15
		$\hat{\bar{Y}}_{SP3}$	104.71	250.71	110.79	124.66	151.10	151.10	151.10
		$\hat{\bar{Y}}_{SP4}$	105.21	251.92	111.32	125.27	151.83	151.83	151.83
		$\hat{\bar{Y}}_{SP5}$	105.20	251.89	111.31	125.25	151.81	151.81	151.81

Table 5, continued

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	*Modified Ratio Estimators*					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.8	0.2	$\hat{\bar{Y}}_{SP1}$	105.34	252.23	111.46	123.16	160.51	160.51	160.52
		$\hat{\bar{Y}}_{SP2}$	105.35	252.26	111.47	123.18	160.53	160.53	160.53
		$\hat{\bar{Y}}_{SP3}$	104.96	251.31	111.06	122.72	159.93	159.93	159.93
		$\hat{\bar{Y}}_{SP4}$	104.97	251.34	111.07	122.73	159.95	159.95	159.95
		$\hat{\bar{Y}}_{SP5}$	105.22	251.95	111.34	123.03	160.33	160.33	160.33
0.9	0.1	$\hat{\bar{Y}}_{SP1}$	105.42	252.43	111.55	122.63	172.27	172.27	172.27
		$\hat{\bar{Y}}_{SP2}$	105.43	252.45	111.56	122.64	172.29	172.29	172.29
		$\hat{\bar{Y}}_{SP3}$	105.14	251.76	111.25	122.31	171.81	171.81	171.81
		$\hat{\bar{Y}}_{SP4}$	105.14	251.74	111.24	122.30	171.80	171.80	171.80
		$\hat{\bar{Y}}_{SP5}$	105.39	252.36	111.52	122.60	172.23	172.22	172.23
1.0	0.0	$\hat{\bar{Y}}_{SP1}$	105.44	252.47	111.57	123.66	187.06	187.06	187.06
		$\hat{\bar{Y}}_{SP2}$	105.45	252.49	111.58	123.67	187.08	187.07	187.08
		$\hat{\bar{Y}}_{SP3}$	105.27	252.05	111.38	123.45	186.75	186.75	186.75
		$\hat{\bar{Y}}_{SP4}$	105.24	251.98	111.35	123.42	186.70	186.70	186.70
		$\hat{\bar{Y}}_{SP5}$	105.50	252.62	111.63	123.73	187.17	187.17	187.17

Table 5 shows the following ranges for the PRE of the proposed estimators:

- from 101.06 to 105.50 in comparison with the SRSWOR sample mean;
- from 241.99 to 252.62 in comparison with the existing estimator $\hat{\bar{Y}}_1$ defined in (4);
- from 106.94 to 111.63 in comparison with the existing estimator $\hat{\bar{Y}}_2$ defined in (6);
- from 122.30 to 181.69 in comparison with the existing estimator $\hat{\bar{Y}}_3$ defined in (8);
- from 142.55 to 187.17 in comparison with the existing estimator $\hat{\bar{Y}}_4$, $\hat{\bar{Y}}_5, \hat{\bar{Y}}_6$ defined in (10).

Based on these comparisons, it is concluded that the proposed estimators perform better than the SRSWOR sample mean and other existing ratio estimators for the natural population 1 considered in this study.

Table 6. PRE of the proposed estimator $\hat{\bar{Y}}_{SPj}$ for Population 2

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	***Modified Ratio Estimators***					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.0	1.0	$\hat{\bar{Y}}_{SP1}$	336.47	330.45	330.45	659.21	650.56	650.56	650.56
		$\hat{\bar{Y}}_{SP2}$	323.10	317.33	317.33	633.03	624.73	624.73	624.73
		$\hat{\bar{Y}}_{SP3}$	460.15	451.93	451.93	901.54	889.72	889.72	889.72
		$\hat{\bar{Y}}_{SP4}$	458.97	450.77	450.77	899.23	887.44	887.44	887.44
		$\hat{\bar{Y}}_{SP5}$	312.94	307.34	307.34	613.11	605.07	605.07	605.07

Table 6, continued

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	Modified Ratio Estimators					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.1	0.9	$\hat{\bar{Y}}_{SP1}$	397.78	390.67	390.67	714.44	701.78	702.67	703.11
		$\hat{\bar{Y}}_{SP2}$	379.24	372.46	372.46	681.14	669.07	669.92	670.34
		$\hat{\bar{Y}}_{SP3}$	630.28	619.01	619.01	1132.04	1111.97	1113.38	1114.08
		$\hat{\bar{Y}}_{SP4}$	630.28	619.01	619.01	1132.04	1111.97	1113.38	1114.08
		$\hat{\bar{Y}}_{SP5}$	379.24	372.46	372.46	681.14	669.07	669.92	670.34
0.2	0.8	$\hat{\bar{Y}}_{SP1}$	464.94	456.62	456.62	768.05	759.48	761.30	762.08
		$\hat{\bar{Y}}_{SP2}$	439.80	431.94	431.94	726.54	718.43	720.15	720.88
		$\hat{\bar{Y}}_{SP3}$	844.34	829.25	829.25	1394.81	1379.25	1382.55	1383.96
		$\hat{\bar{Y}}_{SP4}$	844.34	829.25	829.25	1394.81	1379.25	1382.55	1383.96
		$\hat{\bar{Y}}_{SP5}$	456.63	448.47	448.47	754.34	745.92	747.70	748.47
0.3	0.7	$\hat{\bar{Y}}_{SP1}$	539.16	529.52	529.52	823.19	830.72	833.43	834.64
		$\hat{\bar{Y}}_{SP2}$	507.08	498.02	498.02	774.22	781.30	783.85	784.99
		$\hat{\bar{Y}}_{SP3}$	1104.94	1085.19	1085.19	1687.04	1702.47	1708.02	1710.49
		$\hat{\bar{Y}}_{SP4}$	1104.94	1085.19	1085.19	1687.04	1702.47	1708.02	1710.49
		$\hat{\bar{Y}}_{SP5}$	545.73	535.98	535.98	833.23	840.85	843.60	844.82
0.4	0.6	$\hat{\bar{Y}}_{SP1}$	619.38	608.30	608.30	879.58	920.42	924.22	925.61
		$\hat{\bar{Y}}_{SP2}$	577.42	567.10	567.10	820.00	858.06	861.61	862.90
		$\hat{\bar{Y}}_{SP3}$	1420.63	1395.24	1395.24	2017.46	2111.11	2119.84	2123.02
		$\hat{\bar{Y}}_{SP4}$	1420.63	1395.24	1395.24	2017.46	2111.11	2119.84	2123.02
		$\hat{\bar{Y}}_{SP5}$	646.21	634.66	634.66	917.69	960.29	964.26	965.70

Table 6, continued

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	Modified Ratio Estimators					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.5	0.5	$\hat{\bar{Y}}_{SP1}$	704.72	692.13	692.13	938.98	1035.83	1039.76	1042.13
		$\hat{\bar{Y}}_{SP2}$	653.28	641.61	641.61	870.44	960.22	963.87	966.06
		$\hat{\bar{Y}}_{SP3}$	1772.28	1740.59	1740.59	2361.39	2604.95	2614.85	2620.79
		$\hat{\bar{Y}}_{SP4}$	2361.39	2604.95	2614.85	1772.28	1740.59	1740.59	1772.28
		$\hat{\bar{Y}}_{SP5}$	758.47	744.92	744.92	1010.59	1114.83	1119.07	1121.61
0.6	0.4	$\hat{\bar{Y}}_{SP1}$	792.04	777.88	777.88	1000.88	1181.86	1185.40	1188.50
		$\hat{\bar{Y}}_{SP2}$	733.61	720.49	720.49	927.05	1094.67	1097.95	1100.82
		$\hat{\bar{Y}}_{SP3}$	2156.63	2118.07	2118.07	2725.30	3218.07	3227.71	3236.14
		$\hat{\bar{Y}}_{SP4}$	2156.63	2118.07	2118.07	2725.30	3218.07	3227.71	3236.14
		$\hat{\bar{Y}}_{SP5}$	881.77	866.01	866.01	1114.29	1315.76	1319.70	1323.15
0.7	0.3	$\hat{\bar{Y}}_{SP1}$	886.14	870.30	870.30	1075.25	1375.25	1377.72	1382.18
		$\hat{\bar{Y}}_{SP2}$	817.35	802.74	802.74	991.78	1268.49	1270.78	1274.89
		$\hat{\bar{Y}}_{SP3}$	2557.14	2511.43	2511.43	3102.86	3968.57	3975.71	3988.57
		$\hat{\bar{Y}}_{SP4}$	2557.14	2511.43	2511.43	3102.86	3968.57	3975.71	3988.57
		$\hat{\bar{Y}}_{SP5}$	1017.05	998.86	998.86	1234.09	1578.41	1581.25	1586.36
0.8	0.2	$\hat{\bar{Y}}_{SP1}$	978.14	960.66	960.66	1156.28	1614.75	1614.75	1620.22
		$\hat{\bar{Y}}_{SP2}$	899.50	883.42	883.42	1063.32	1484.92	1484.92	1489.95
		$\hat{\bar{Y}}_{SP3}$	2934.43	2881.97	2881.97	3468.85	4844.26	4844.26	4860.66
		$\hat{\bar{Y}}_{SP4}$	2934.43	2881.97	2881.97	3468.85	4844.26	4844.26	4860.66
		$\hat{\bar{Y}}_{SP5}$	1154.84	1134.19	1134.19	1365.16	1906.45	1906.45	1912.90

Table 6, continued

α_1	α_2	Proposed Estimators	Existing Estimators						
			SRSWOR	***Modified Ratio Estimators***					
			$\bar{y}_r$	$\hat{\bar{Y}}_1$	$\hat{\bar{Y}}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$
0.9	0.1	$\hat{\bar{Y}}_{SP1}$	1071.86	1052.69	1052.69	1253.89	1915.57	1912.57	1919.16
		$\hat{\bar{Y}}_{SP2}$	988.95	971.27	971.27	1156.91	1767.40	1764.64	1770.72
		$\hat{\bar{Y}}_{SP3}$	3254.55	3196.36	3196.36	3807.27	5816.36	5807.27	5827.27
		$\hat{\bar{Y}}_{SP4}$	3254.55	3196.36	3196.36	3807.27	5816.36	5807.27	5827.27
		$\hat{\bar{Y}}_{SP5}$	1297.10	1273.91	1273.91	1517.39	2318.12	2314.49	2322.46
1.0	0.0	$\hat{\bar{Y}}_{SP1}$	1169.93	1149.02	1149.02	1375.82	2295.42	2287.58	2295.42
		$\hat{\bar{Y}}_{SP2}$	1071.86	1052.69	1052.69	1260.48	2102.99	2095.81	2102.99
		$\hat{\bar{Y}}_{SP3}$	3509.80	3447.06	3447.06	4127.45	6886.27	6862.75	6886.27
		$\hat{\bar{Y}}_{SP4}$	3509.80	3447.06	3447.06	4127.45	6886.27	6862.75	6886.27
		$\hat{\bar{Y}}_{SP5}$	1432.00	1406.40	1406.40	1684.00	2809.60	2800.00	2809.60

Table 6 shows the following ranges for the PRE of the proposed estimators:

- from 312.94 to 3509.80 in comparison with SRSWOR sample mean;
- from 307.34 to 3447.06 in comparison with the existing estimator $\hat{\bar{Y}}_1$ defined in (4) and $\hat{\bar{Y}}_2$ defined in (6);
- from 613.11 to 4127.45 in comparison with the existing estimator $\hat{\bar{Y}}_3$ defined in (8);
- from 605.07 to 6886.27 in comparison with the existing estimator $\hat{\bar{Y}}_4$ defined in (10);
- from 605.07 to 6862.75 in comparison with the existing estimator $\hat{\bar{Y}}_5$ defined in (10);
- from 605.07 to 6886.27 in comparison with the existing estimator $\hat{\bar{Y}}_6$ defined in (10).

Based on these comparisons, it may be concluded that the proposed estimators perform better than the SRSWOR sample mean and other existing ratio estimators for the natural population 2 considered in this study.

Conclusion

This article proposed two parameter modified ratio estimators with known correlation coefficient, skewness and kurtosis of the auxiliary variables and their linear combinations. The mean squared errors of the proposed estimators were derived and compared with that of SRSWOR sample mean, the classical ratio estimator and the existing modified ratio estimators. The performance of the proposed estimators was also assessed with that of the existing estimators for certain natural populations. It was observed from the numerical comparisons that the mean squared errors of the proposed estimators are less than the mean squared error of the existing estimators. Further it was shown that the PREs of the proposed estimators, with respect to existing estimators, range from 101.06 to 6886.27. Hence, the proposed modified ratio estimators are strongly recommended and may be preferred over existing estimators for practical applications.

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