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A discrete analogue of generalized Burr-type XII distribution is introduced using a general approach of discretizing a continuous distribution. It may be worth exploring the possibility of developing a discrete version of the six parameter generalized Burr-type XII distribution for use in modeling a discrete data. This distribution is suggested as a suitable reliability model to fit a range of discrete lifetime data, as it is shown that hazard rate function can attain monotonic increasing (deceasing) shape for certain values of parameters. The equivalence of discrete generalized Burr-type XII (DGBD-XII) and continuous generalized Burr-type XII (GBD-XII) distributions has been established. The increasing failure rate property in the discrete setup has been ensured. Various theorems relating this new model to other probability distributions have also been proved.

Keywords: Discrete generalized Burr-type XII distribution, discrete lifetime models, reliability, failure rate

Introduction

In reliability theory a number of continuous life models is now available in the subject to portray the survival behavior of a component or a system. Many continuous life distributions have been studied in details (see for example Kapur & Lamberson, 1997; Lawless, 1982; Sinha, 1986). However, it is sometimes impossible or inconvenient in life testing experiments to measure the life length of a device on a continuous scale. For example the lifetime of an on/off switching device is a discrete random variable, or life length of a device receiving a number of shocks it sustain before it fails is also a discrete random variable.

Recently, the special roles of discrete distributions have received recognition from survival analysts. Many continuous distributions have been discretized. For example, the Geometric and Negative binomial distributions are the discrete

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versions of Exponential and Gamma distributions. Nakagawa (1975) discretized the Weibull distribution. The discrete versions of the normal and Rayleigh distributions were also proposed by Roy (2003, 2004). Discrete analogues of Maxwell, two parameter Burr-type XII and Pareto distributions were also proposed by Krishna and Punder (2007, 2009). Recently the inverse Weibull distribution was also discretized by Jazi, Lai and Alamatsaz (2010). This article addresses the problem of discretization of generalized Burr-type XII (GBD-XII) distribution, because there is a need to find more plausible discrete lifetime distributions to fit to various life time data.

The Discrete Generalized Burr XII Distribution:

Roy (1993) pointed out that the univariate geometric distribution can be viewed as a discrete concentration of a corresponding exponential distribution in the following manner

$$p[X = x] = s(x) - s(x+1) \text{ when } x = 0, 1, 2, \ldots$$

where X is discrete random variable following geometric distribution with probability mass functions as

$$p(x) = \theta^{x}(1-\theta),\ x = 0, 1, 2, \ldots$$

where $s(x)$ represents the survival function of an exponential distribution of the form $s(x) = \exp(-\lambda x)$, clearly $\theta = \exp(-\lambda)$, $0 < \theta < 1$. Thus, one to one correspondences between the geometric distribution and the exponential distribution can be established, the survival functions being of the same form.

The general approach of discretizing a continuous variable is to introduce a greatest integer function of X i.e., $[X]$ (the greatest integer less than or equal to X till it reaches the integer), in order to introduce grouping on a time axis. If the underlying continuous failure time X has the survival function $s(x) = p(X > x)$ and times are grouped into unit intervals, so that the discrete observed variable is $dX = [X]$.

The probability mass function of *dX* can be written as

$$\begin{aligned} p(x) &= p(dX = x) = p(x \le X < x+1) \\ &= \varnothing(x+1) - \varnothing(x) \\ &= s(x) - s(x+1), \qquad x = 0,1,2,\ldots \end{aligned}$$

$\varnothing(x)$ being the cumulative distribution function of random variable *X*.

In reliability theory, many classification properties and measures are directly related to the functional form of the survival function. The increasing failure rate (IFR), decreasing failure rate (DFR), Increasing failure rate average (IFRA), decreasing failure rate average (DFRA), new better than used (NBU), new worse than used (NWU), new better (worse) than used in expectation NBUE (NWUE) and increasing (decreasing) mean residual lifetime IMRL (DMRL) etc. are examples of such class properties as may be seen from Barlow and Proschan (1975). If discretization of a continuous life distribution can retain the same functional form of the survival function then many reliability measures and class properties will remain unchanged. In this sense, the discrete concentration concept is considered herein as a simple approach that can generate a discrete life distribution model. Thus, given any continuous life variable with survival function $s(x)$ a discrete lifetime variable *X* with probability mass function $p(x)$ is defined by

$$p(x) = s(x) - s(x+1), x = 0,1,2,\ldots$$

Using this concept for the purpose of discretizing generalized Burr-type XII distribution, suppose that Y_1 and Y_2 are independently distributed continuous random variables. If Y_1 has an exponential density function $f(y_1,\theta) = \theta e^{-\theta y_1}, y_1 > 0; \theta > 0$ and Y_2 has a gamma distribution with pdf $f(y_2;\alpha,k) = \frac{\alpha^k}{\Gamma(k)} e^{-\alpha y_2} y_2^{k-1}, \alpha > 0; k > 0; y_2 > 0$, then the random variable $X = c\sqrt{\frac{Y_1}{Y_2}}$ has a six parameter generalized Burr-type XII distribution with parameters ($\mu = 0$, $\sigma = 1$, α, θ, c, k) with a density function

$$f(x) = \frac{k\theta\alpha c\alpha^k x^{c-1}}{(\alpha + \theta x^c)k+1}; \qquad x > 0, \theta > 0; c > 0; \alpha > 0; k > 0 \tag{1}$$

The pdf plot for DGB-XII variate X for different values of parameters is shown in Figure 1. It is evident that the distribution of the random variable X is right skewed.

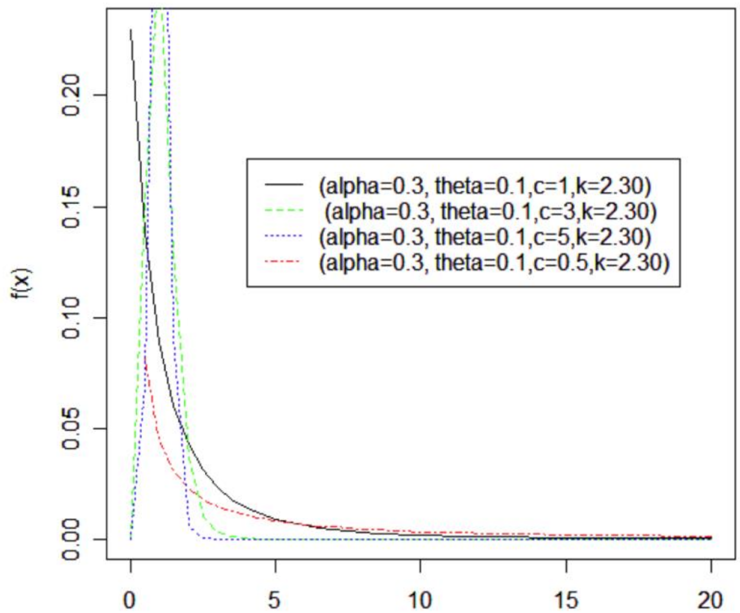

Figure 1. PDF plot for GBD-XII (α, θ, c, k)

Introducing location parameter μ and scale parameter σ in (1) results in

$$f(x_1) = \frac{k\theta c\alpha^k \left(\frac{x_1 - \mu}{\sigma}\right)^{c-1}}{\sigma\left[\alpha + \theta\left(\frac{x_1 - \mu}{\sigma}\right)^c\right]^{k+1}}, \; x > 0, \alpha > 0, c > 0, \theta > 0, \mu > 0, \sigma > 0, k > 0$$

Reliability measures of GBD-XII random variable *X*

Various reliability measures of GBD-XII random variable *X* are given by

Survival Function:

$$
\begin{aligned}
s(x) &= p(X \geq x) = 1 - \int_0^x f(x)\,dx \\
&= 1 - \int_0^x \frac{x^k k\theta c x^{c-1}}{\left(x+\theta x^c\right)k+1}\,dx \\
&= \alpha^k \left(\alpha + \theta x^c\right)^{-k}
\end{aligned}
$$

r^{th} Moment:

$$
E\left(x^r\right) = k\left(\frac{\alpha}{\theta}\right)^{r/c} B\left(\frac{r}{c}+1, k - r/c\right), x > 0; k > r/c; \alpha > 0; \theta > 0; r > 0; c > 0
$$

where $B(a,b) = \int_0^\infty \frac{x^{a-1}}{(1+x)^{a+b}}dx$

Failure Rate:

$$
r(x) = \frac{f(x)}{s(x)} = \frac{k\theta c x^{c-1}}{\left(\alpha + \theta x^c\right)}, \ x > 0; k > 0; \alpha > 0; \theta > 0; c > 0
$$

Second Rate of Failure:

$$
\begin{aligned}
SRF(x) &= \log\left(\frac{S(x)}{S(x+1)}\right) \\
&= -k \log\left[\frac{\alpha + \theta x^c}{\alpha + \theta(x+1)^c}\right] \qquad x > 0; k > 0; \alpha > 0; \theta > 0; c > 0
\end{aligned}
$$

A discrete generalized Burr XII variable, *dX* can be viewed as the discrete concentration of the continuous generalized Burr-type XII variate *X* distribution, where the corresponding probability mass function of *dX* can be written as

$$
\begin{aligned}
p(dX = x) = p(x) &= s(x) - S(x+1) \\
&= \alpha^{k}\left(\alpha+\theta x^{c}\right)^{-k} - \alpha^{k}\left(\alpha+\theta(x+1)^{c}\right)^{-k} \\
&= \alpha^{-\log\beta}\left[\beta^{\log\left(\alpha+\theta x^{c}\right)} - \beta^{\log\left(\alpha+\theta(x+1)^{c}\right)}\right]
\end{aligned}
$$

The pmf of DGBD-XII takes the form

$$
p(x) = \begin{cases} 1-\alpha^{\log\beta}\beta^{(\alpha+\theta)} & x = 0 \\ \alpha^{-\log\beta}\left(\beta^{\log\left(\alpha+\theta x^{c}\right)} - \beta^{\log\left(\alpha+\theta(x+1)^{c}\right)}\right) & x = 1, 2, 3, \ldots \end{cases} \tag{2}
$$

where $\beta = e^{-k}$; $\quad 0 < \beta < 1$

$k > 0; \alpha > 0; c > 0; \beta > 0$

Figures 2 through 5 give the pmf plot of (2) for (α = 0.3; θ = 0.1; c = 4; β = 0.1), (α = 0.3; θ = 0.1; c = 1; β = 0.1), (α = 0.3; θ = 0.01; c = 0.5; β = 0.1), and (α = 1; θ = 1; c = 2; β = 0.5), respectively.

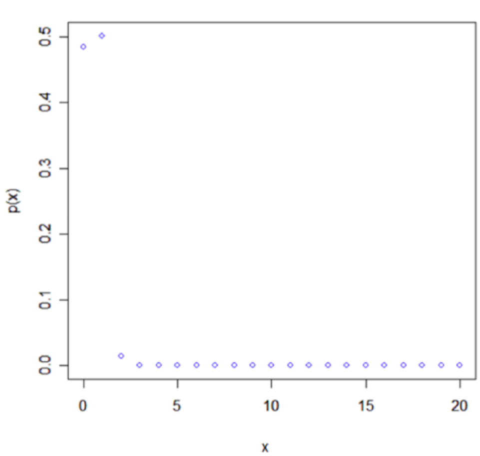

Figure 2. PMF plot for DGBD-XII (α = 0.3; θ = 0.1; c = 4; β = 0.1)

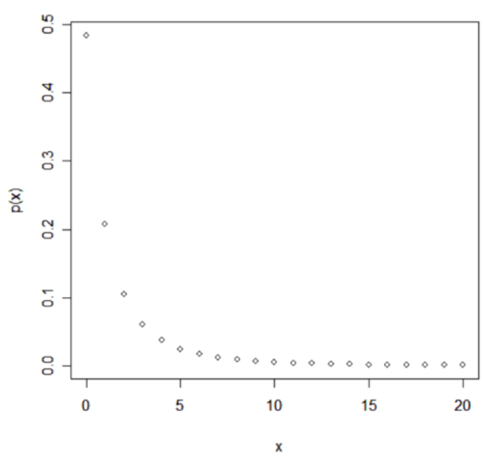

Figure 3. PMF plot for DGBD-XII (α = 0.3; θ = 0.1; c = 1; β = 0.1)

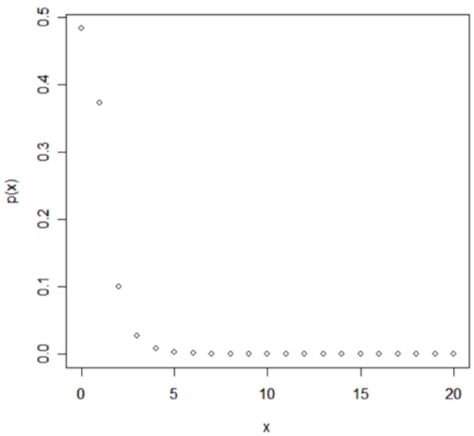

Figure 4. PMF plot for DGBD-XII (α = 0.3; θ = 0.01; c = 0.5; β = 0.1)

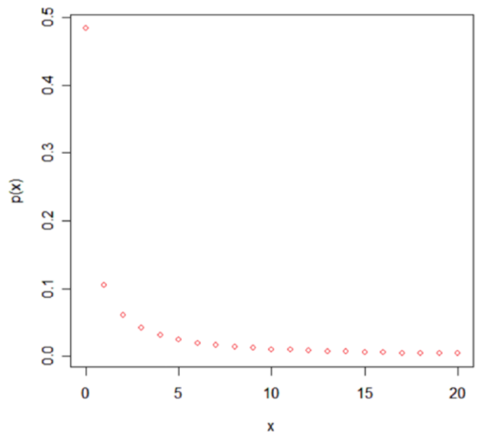

Figure 5. PMF plot for DGBD-XII (α = 1; θ = 1; c = 2; β = 0.5)

The pmf at $x = 0$ is independent of the shape parameter c. It is monotonic decreasing if

$$c \le \frac{\log\left(\dfrac{e^{\varnothing(\beta,\alpha,\theta)} - \alpha}{\theta}\right)}{\log 2} \qquad \text{where } \varnothing(\beta,\alpha,\theta) = \frac{\log\left(2\beta^{\log(\alpha+\theta)} - \alpha^{\log\beta}\right)}{\log\beta}$$

$$\text{where } \beta = e^{-k}; 0 < \beta < 1; k > 0; \alpha > 0; c > 0; \theta > 0$$

otherwise it is no longer monotonic decreasing but is unimodal, having a mode at $x = 1$ i.e., it takes a jump at $x = 1$ and then decreases for all $x \ge 1$.For $\alpha = \theta = c = 1$ pmf of discrete generalized Burr-type XII distribution coincides with discrete pareto distribution and for $\alpha = \theta = 1$ DGBD-XII reduces to discrete Burr-type XII distribution.

To introduce location parameter μ and scale parameter σ then the discretized version of $f(x_1)$ is given as

$$
\begin{aligned}
P(dX_1 = x_1) = p(x_1) &= p(x_1 \le X_1 < x_1 + 1) \\
&= P(x_1 \le \sigma x + \mu < x_1 + 1) \\
&= P\left(\frac{x_1 - \mu}{\sigma} \le X < \frac{x_1 + 1 - \mu}{\sigma}\right) \\
&= \varnothing\left(\frac{x_1 + 1 - \mu}{\sigma}\right) - \varnothing\left(\frac{x_1 - \mu}{\sigma}\right)
\end{aligned}
$$

where $\emptyset(x_1)$ represents cumulative distribution function of random variable X.

The survival function of discrete generalized Burr-type XII random variable dX is given by

$$
\begin{aligned}
s(x) = p(dX \ge x) &= 1 - p(dX < x) \\
&= 1 - \left[p(dX = 0) + p(dX = 1) + \cdots + p(dX = x - 1) \right] \\
&= a^k \beta^{\log\left(x + \theta x^c\right)} = \alpha^{-\log \beta} \beta^{\log\left(x + \theta x^c\right)} \qquad x = 0, 1, 2, \ldots \\
&\qquad a > 0; c > 0; k > 0; \theta > 0
\end{aligned}
$$

Thus, survival function of discrete generalized DGBD-XII is same as continuous GBD-XII for integer points of x.

The failure rate of discrete generalized Burr-type XII random variable is given by

$$
r(x) = \frac{p(x)}{s(x)} = 1 - \beta^{\log}\left[\frac{\alpha + \theta(x+1)^c}{\left(\alpha + \theta x^c\right)}\right] \qquad (3)
$$
$$
x = 0, 1, 2, \ldots; a > 0; c > 0; k > 0; \theta > 0
$$

i.e., the conditional probability that failure occurs at a time x given that the system has not failed by $x - 1$.

Note that $r(0) = r(1)$ gives $c = \dfrac{\log\left(\frac{\theta}{\alpha} + 2\right)}{\log 2} = a$ (for example). If $\alpha = \theta = 1$, the hazard function of discrete generalized Burr-type XII distribution coincides with two parameter discrete Burr-type XII distribution. Note that $r(x)$ is decreasing in x if $0 < c < a$ and for $c = a$; $r(0) = r(1)$ and for $c > a$, $r(0) > r(1)$ i.e., monotonic increasing, and $r(x) < r(x - 1)$ $\forall$ $x > 2$ i.e., $r(x)$ decreases for all $x > 1$, uniformly in β.

For different values of α and θ, c can take different values as:

a) $\theta = .5$; $\alpha = 100$ gives $c = 1.003$
b) $\theta = .99$; $\alpha = .5$ gives $c = 1.993$
c) $\theta = 9999$; $\alpha = .9$ gives $c = 13.43$
d) $\theta = 1$; $\alpha = 1$ gives $c = 1.585$

Taking part a) it could be seen that for

$\theta = .5$; $\alpha = 100$ gives $c = 1.003$ (for example as above)
$r(0) > r(1)$, for $0 < c < a$ (for example when $c = 1$)
and for $c = a$; $r(0) = r(1)$
and for $c > a$, $r(0) < r(1)$ implies a monotonic increasing
and $r(x) < r(x - 1)\ \forall\ x \geq 2$
i.e., $r(x)$ takes a jump at $r = 1$ and decreases for all $x \geq 1$, uniformly in β.

Similarly, for all other cases where c can take different values for different values of α and θ, $r(x)$ will show its monotonicity accordingly as in the above case.

For discrete generalized Burr-type XII distribution i.e., DGBD-XII(α, θ, c, β)

$$SRF(x) = \log\left(\frac{s(x)}{s(x+1)}\right) = \log\left(\frac{\alpha^{-\log\beta}\beta^{\log(\alpha+\theta x^c)}}{\alpha^{-\log\beta}\beta^{\log(\propto+\theta(x+1)^c)}}\right)$$
$$= \log\beta\log\left(\frac{\alpha+\theta x^c}{\alpha+\theta(x+1)^c}\right), \qquad \alpha > 0; c > 0; k > 0; \theta > 0$$

and to see whether $SRF(x)$ shows the same monotonicity as $r(x)$

$$SRF(x) = \log\beta\log\left(\frac{\alpha+\theta x^c}{\alpha+\theta(x+1)^c}\right)$$

For $SRF(0) = SRF(1)$

$$c = \log_2\left(\tfrac{\theta}{\alpha}+2\right) = \frac{\log\left(\frac{\theta}{\alpha}+2\right)}{\log 2} = \alpha \text{ (for example)}$$

Using the same procedure as in (3) it is clear that $SRF(x)$ shows the same monotonicity as that of $r(x)$.

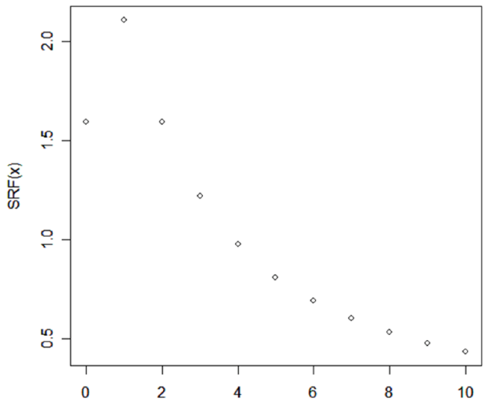

Figure 6. second rate of failure plot of DGBD-XII ($\alpha = 1$; $\theta = 1$; $c = 2$; $\beta = 0.1$)

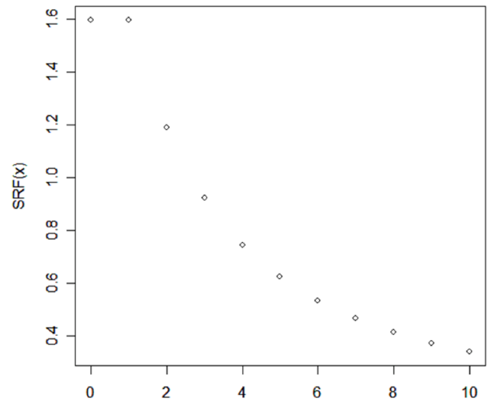

Figure 7. second rate of failure plot of DGBD-XII ($\alpha = 1$; $\theta = 1$; $c = 1.585$; $\beta = 0.1$)

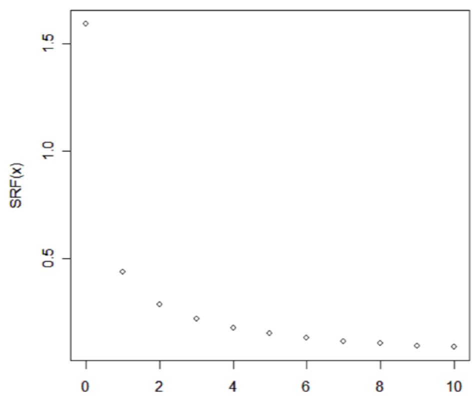

Figure 8. second rate of failure plot of DGBD-XII ($\alpha = 1$; $\theta = 1$; $c = 0.5$; $\beta = 0.1$)

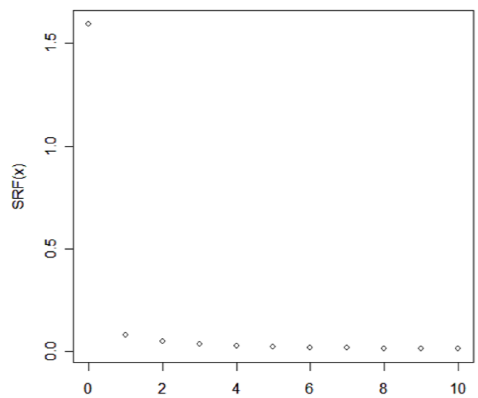

Figure 9. second rate of failure plot of DGBD-XII ($\alpha = 1$; $\theta = 1$; $c = 0.1$; $\beta = 0.1$)

Figures 6 through 9 illustrate the alternative hazard rate plot for DGBD-XII for different values of parameters. Note that $SRF(x)$ is decreasing in x if $0 < c < a$

and for $c = a$; $SRF(0) = SRF(1)$ and for $c > a$, $SRF(0) > SRF(1)$ i.e., monotonic increasing, and $SRF(x) < SRF(x - 1)$ $\forall$ $x > 2$ i.e., $SRF(x)$ decreases for all $x > 1$.

The r^{th} moment of discrete generalized Burr-type XII distribution, i.e. DGBD (α, θ, c, β), is as follows

$$\begin{aligned} E\left(X^r\right) &= \sum_{x=0}^{\infty} x^r p(x) = \sum_{x=0}^{\infty} x^r \left[\beta^{\log\left(\alpha+\theta x^c\right)} - \beta^{\log\left(\alpha+\theta c+x\right)^c}\right]\alpha^{-\log\beta} \\ &= \sum_{x=1}^{\infty}\left[x^r - (x-1)^r\right] S(x) \end{aligned} \tag{4}$$

$$E\left(X^r\right) \le r\sum_{x=1}^{\infty} x^{r-1} s(x) \le \frac{r\alpha^{-\log\beta}}{\theta^k}\sum_{x=1}^{\infty}\frac{1}{x^{ck-r+1}}$$

Now, $E\left(x^r\right) \le r\sum_{x=1}^{\infty}\frac{r\alpha^{-\log\beta}}{\theta^k}\sum_{x=1}^{\infty}\frac{1}{x^{ck-r+1}}$, $E\left(x^r\right)$ will take a finite value if $ck > r$,

and from (4), $E(x) = \sum_{x=1}^{\infty} s(x) = \sum_{x=1}^{\infty}\alpha^{-\log\beta}\beta^{\log\left(\alpha+\theta x^c\right)}$ and

$$\begin{aligned} E\left(x^2\right) &= \sum_{x=1}^{\infty}\left[x^2 - (x-1)^2\right] s(x) \\ &= 2\sum_{x=1}^{\infty} x s(x) - E(x) \\ E\left(x^2\right) &= 2\sum_{x=1}^{\infty}\alpha^{-\log\beta}\beta^{\log\left(\alpha+\theta x^c\right)} + E(x) \end{aligned}$$

which are infinite series and cannot be written as closed form. The parameter β of DGBD-XII (α, θ, c, β) and the parameter k of GBD-XII (α, θ, c, k), are matched via $\beta = e^{-k}$. It is therefore observed from the survival functions of DGBD-XII and GBD-XII distributions

$$\sum_{x=1}^{\infty}\left\{\alpha^{-\log\beta}\beta^{\log\left(\alpha+\theta x^c\right)}\right\} < \int_0^{\infty}\alpha^k\left(\alpha+\theta x^c\right)^{-k}dx < \sum_{x=0}^{\infty}\left\{\alpha^{-\log}\beta^{-\log\beta}\beta^{\left(\alpha+\theta x^c\right)}\right\}$$

In other words, $\mu_d - 1 < \mu_c < \mu_d$ where μ_c and μ_d are the means of the continuous and discrete generalized Burr XII distributions, respectively.

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Appendix: Some theorems related to discrete generalized Burr XII distribution

Lemma 1

If X is a continuous random variable with increasing (decreasing) failure rate IFR (DFR) distribution, then $dX = [X]$ has a discrete increasing (decreasing) failure rate dIFR (dDFR).

Proof: (See Roy and Dasgupta, 2001)

Lemma 2

If X is a non-negative continuous random variable and Y is a non-negative integer valued discrete random variable, then

$$[X] \geq Y \Leftrightarrow X \geq Y$$

Proof: Note that

$$([X] \geq Y) \subseteq (X \geq Y) \subseteq ([X] \geq [Y]) = ([X] \geq Y)$$

where the last equality holds because Y is integer valued. Therefore

$$(X \geq Y) = ([X] \geq Y)$$

Theorem 1

If X ~ DGBD-XII ($\mu = 0$; $\sigma = 1$; α, θ, c, β) then

$$Y = \left[\log\left(\frac{\alpha + \theta x^c}{\alpha} \right) \right] \sim Geo(\beta) \qquad \text{where } \beta = e^{-k}; \alpha > 0; c > 0; k > 0; \theta > 0$$

Proof:

$$
\begin{aligned}
P[Y \geq y] &= P\left[\left[\log\left(\frac{\alpha+\theta x^c}{\alpha}\right)\right] \geq y\right] \\
&= P\left[\log\left(\frac{\alpha+\theta x^c}{\alpha}\right) \geq y\right] && \text{by Lemma 2} \\
P[Y \geq y] &= P\left[X \geq \left(\frac{e^{y-\log\alpha^{-1}}-\alpha}{\theta}\right)^{1/c}\right] \\
&= \alpha^{-\log\beta}\beta^{\log\left[\alpha+\theta\left\{\left(\frac{e^{y-\log\alpha^{-1}}-\alpha}{\theta}\right)^{1/c}\right\}^{c}\right]} \\
&= \alpha^{-\log\beta}\beta^{y-\log\alpha^{-1}} \\
&= \beta^{y} \sim Geo(\beta) && y = 0, 1, 2, \ldots
\end{aligned}
$$

As β^y is the survival function of geometric random variable.

Theorem 2

If X ~ GBD-XII ($\mu = 0$; $\sigma = 1$; α, θ, c, k) then $Y = [X]$ ~ DGBD-XII ($\mu = 0$; $\sigma = 1$; α, θ, c, β); where $\beta = e^{-k}$; $\alpha > 0$; $c > 0$; $k > 0$; $\theta > 0$

Proof:

$$
\begin{aligned}
P[Y \geq y] &= P[[X] \geq y] = P[X \geq y] && \text{by Lemma 2} \\
&= \alpha^{k}\left(\alpha+\theta y^{c}\right)^{-k} \\
&= \alpha^{-\log\beta}\beta^{\log\left(\alpha+\theta y^{c}\right)} && y = 0, 1, 2, \ldots \\
& && \beta = e^{-k};\ 0 < \beta < 1
\end{aligned}
$$

Thus, $Y = [X]$~DGBD-XII (α, θ, c, β)

Theorem 3

If $X \sim exp(k)$, the exponential distribution with failure rate k. Then

$Y = \left[\left(\frac{e^{X+\log\alpha} - \alpha}{\theta}\right)^{1/c}\right] \sim$ DGBD-XII (α, θ, c, β), where $\beta = e^{-k}$; $\alpha > 0$, $c > 0$, $k > 0$, $\theta > 0$.

Proof:

$$
\begin{aligned}
P\left[Y \ge y\right] &= P\left[\left[\left(\frac{e^{X+\log\alpha} - \alpha}{\theta}\right)^{1/c}\right] \ge y\right] \\
&= P\left[\left(\frac{e^{X+\log\alpha} - \alpha}{\theta}\right)^{1/c} \ge y\right] && \text{by Lemma 2} \\
&= P\left[X \ge \log\left(\theta y^c + \alpha\right) - \log\alpha\right] \\
&= e^{-k\left[\log\left(\theta y^c + \alpha\right) - \log\alpha\right]} \\
&= \alpha^k e^{-k\log\left(\alpha + \theta y^c\right)} \\
P\left[Y \ge y\right] &= \alpha^{-\log\beta} \beta^{\log\left(\alpha + \theta y^c\right)} && y = 0, 1, 2, \ldots
\end{aligned}
$$

which is the survival function of DGBD-XII (α, θ, c, β). Thus, $Y \sim$ DGBD-XII (α, θ, c, β).

Theorem 4

Let X be a random variable following continuous generalized Burr XII distribution with $E(X^r) < \infty \ \forall \ r = 1, 2, \ldots$

Then $E(Y^r) < \infty$ where $Y = [X] \sim$ DGB (α, θ, c, k)

Proof: Proof is straightforward, because $0 \le [X] \le X$, so clearly if $E(X^r) < \infty \ \forall \ r = 1, 2, \ldots$, then $E([X]^2) < \infty$.