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In some process monitoring situations, consecutive measurements are spaced widely apart in time, making monitoring process aim and spread difficult. This study uses three cases to compare the effectiveness of two such monitoring schemes, i.e., the X chart alone (X-only chart) and the Individuals and Moving Range Chart Combination (X/MR chars), in terms of Average Run Length (ARL) after designing for a common "all OK" (in-control) ARL. The study finds that X chart alone is sufficient (and hence, recommended) in detecting changes in all the 3 cases: changes in the process mean, changes in the process standard deviation, and changes in both process mean and standard deviation.

Keywords Individual chart, X-chart alone, moving range chart, X/MR chart, ARL, Average Run Length, "all OK" ARL

Introduction

In some process monitoring situations, consecutive measurements are spaced widely apart in time. For example, an engineering process may allow only one measurement per day. In some cases, a series of individual items are produced in such a way that no natural subgrouping is possible (Crowder, 1987a). When this happens, exactly how to monitor process aim and spread is not completely obvious. One sensible possibility is to simply plot individual observations on their own chart (X-only chart). Another possibility is to plot a combination of a chart for individual measurements and a moving range chart based on two consecutive observations. Duncan (1974) outlines such a procedure.

The purpose of this study is to compare the effectiveness of these two monitoring schemes, i.e., the X chart alone (X-only chart) and the Individuals and

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Moving Range Chart Combination (X/MR chars), in terms of Average Run Length (ARL) after designing for a common "all OK" (in-control) ARL.

The run length of any process monitoring procedure is the number of sampling periods before an out-of-control signal is given. An out-of-control signal suggests that some change in the process has occurred and that action should be taken to find and correct any assignable causes. The average run length (ARL) is often used to describe the likely performance of a control procedure. A large ARL is desired when the process is stable or in control, and a small ARL otherwise (Crowder, 1987a).

Comparison of monitoring schemes will be made under three sets of circumstances. The first case is where the process mean changes from its standard value, the second case is where the process variability changes, and third case is where both process mean and process variability change from standard values. In each of these three cases, a small ARL is desired, since it will indicate quick detection of the out-of-control situation.

Literature Review

Vardeman and Jobe (1999) discussed the charting of individuals and moving ranges and some other process monitoring techniques that improve on Shewhart charts in situations where it is important to quickly detect small process changes. That is, they also considered EWMA and CUSUM process monitoring schemes. Four types of process monitoring schemes were originally considered in the present study: The X chart alone, Individuals and Moving Range Chart Combinations, EWMA and CUSUM process monitoring schemes. However, because EWMA and CUSUM schemes are known to be better than an X chart alone for detecting small process changes, no further analysis is needed (for EWMA and CUSUM) if the X chart alone is better than Individual and Moving Range Chart Combinations.

Crowder (1987a, 1987b) discussed the Computation of ARLs for Combined Individual Measurement and Moving Range Charts. Numerical procedures and a control chart design strategy are presented. ARLs are given for various choices of the control limits and shifts in the level of the process mean and standard deviation. Also, a Fortran computer program was presented that allows inputting control limits for combined individual measurement and moving range charts and then returns the approximate average run length (ARL) for the normal case with standard deviation 1 and various shifts in the process mean.

Roes, et al. (1993) discussed several options in designing a Shewhart-type control chart for Individual Observations. A number of possible estimators of the standard deviation were considered and a two-stage procedure is suggested for retrospective testing. It was argued that adding a Moving Range Chart has no real added value and, therefore, was ill-advised.

Rigdon, et al. (1994) discussed design strategies for Individuals and Moving Range Control Charts. These authors argue that an X chart alone is nearly as efficient as the combined X/MR chart for detecting changes in the process variability. For the same in-control ARL, the X-only chart is more effective than the X/MR chart combination with moving ranges of $k = 2, 3, 4$ for detecting shifts in the process mean, while the two schemes are about equally effective in detecting changes in the process variability. The size (k) of the window for the moving range has little effect on the ARL for changes in the process variability.

Radson, et al. (1995) considered the possibility of a shift from the in-control standard deviation, σ_0, to a standard deviation level of σ_1, where $\sigma_1 = k \cdot \sigma_0$. In their study, k ranged from 0.1 to 3.0 in increment of 0.1. They demonstrated that the moving range can be used to detect variance reduction. This was best achieved by constructing limits based on the true underlying distribution for the moving range.

Adke and Hong (1997) discussed the X chart and the Moving Range chart for the normally distributed observations when there is a shift in the process variance. They concluded that a Moving Range chart does provide useful information.

Amin (1998) showed that there is no disadvantage in using an X/MR procedure. The discussion is limited to normality assumption.

Marks and Krehbiel (2009) evaluated the Individual Chart and X/MR Chart Combinations for just the first 2 cases mentioned above: mean change only and variability change only, from standard values. But they did not consider the third case, where both mean and variability changes occur, from standard values.

Methods

Several “all OK” (in-control) ARLs are chosen and control limits set to produce them (The “all OK” ARL is the ARL when the process mean and process standard deviation are on-target, i.e., the process mean and process standard deviation are equal to standard values). Then, supposing the mean and/or standard deviation are off-target, new ARLs for both the X/MR Chart Combinations and the X-only Chart are computed and compared (a smaller off-target ARL is preferred since the corresponding charting method then gives quicker detection of non-standard conditions).

X-only Chart Monitoring Scheme

First, an "all OK" (in-control) ARL is chosen. Then, using $\mu_0 = 0$ and $\sigma_0 = 1$ as the standard conditions, control limits for the X-only Chart are

$$UCL_{X-only} = \mu_0 + L\sigma_0 = L$$
$$LCL_{X-only} = \mu_0 + L\sigma_0 = -L$$

where the given "all OK" (in-control) ARL is simply $1/r$ for

$$r = P\left[1^{st}\text{ point plots outside the control limits}\right]$$
$$r = P\left[Z > L\right] + P\left[Z < -L\right]$$

which can be calculated from the standard normal distribution table.

After control limits are set, the off-target ARL of the X-only Chart for any other μ and σ combination, i.e., non-standard conditions, can be computed as follows.

Let

$$\begin{aligned} \mu_{new} &= \mu_0 + \delta = \mu_0 + D\sigma_{new} = \mu_0 + DT\sigma_0 \\ \sigma_{new} &= T\sigma_0 \end{aligned} \tag{1}$$

where D = constant (i.e., a shift in the process mean) and T = constant (i.e., a change in the process standard deviation).

Then,

$$z_1 = \frac{UCL_{X-only} - \mu_{new}}{\frac{\sigma_{new}}{\sqrt{n}}} = \frac{L - \mu_{new}}{\sigma_{new}} = a$$

and

$$z_2 = \frac{LCL_{X-only} - \mu_{new}}{\frac{\sigma_{new}}{\sqrt{n}}} = \frac{-L - \mu_{new}}{\sigma_{new}} = b$$

for $n = 1$, since the natural sample size is one. Finally,

$$r_{new} = P\left[Z > a\right] + P\left[Z < b\right]$$

and the new ARL, i.e., the off-target ARL is $1/r_{new}$.

X/MR Chart Combinations Monitoring Scheme

Crowder (1987a, 1987b) noted that ARLs of X/MR chart combinations can be obtained by solving certain integral equations as follows. ARLs for control schemes are typically evaluated under the model $y_t = \mu + \varepsilon_t$, $t = 1, 2, 3, \ldots$, where ε_t's are independent $N(0, \sigma^2)$ random variables and y is the observation made at time t. Suppose we wish to control the process mean and standard deviation at nominal levels $\mu_0 = 0$ and $\sigma_0 = 1$. The X/MR procedure is to plot on separate charts the individual observations y_t and the successive moving ranges $r_t = |\, y_t - y_{t-1}|$. The process is deemed out-of-control at observation one if $|y_t| > M$ and out-of-control at time $t > 1$ if either $|y_t| > M$ or $r_t > R$, where M and R are specified positive constants.

Let $L(u)$ be the mean additional time until an out-of-control signal, given that the most recent observation is u. If the next observation, y, is more than R units from u or is larger in magnitude than M, an out-of-control signal is given. Otherwise, the run continues with y as the most recent observation. Thus, supposing that observations are normal with mean μ and variance σ^2,

$$\begin{aligned} L(u) &= 1 \cdot P\left(|y-u| > R \text{ or } |y| > M\right) + \int_{(x: x-u \le R \text{ and } |x| \le M)} \left[1 + L(x)\right] \cdot f(x)\,dx \\ &= 1 + \int_{\max\{-M,\, u-R\}}^{\min\{M,\, u+R\}} L(x) f(x)\,dx \end{aligned} \tag{2}$$

where $f(x)$ is the $N(0, \sigma^2)$ density. Now letting T be the run length associated with the procedure and conditioning on y_1,

$$
\begin{aligned}
ARL &= E(T) \\
&= E\left[E\left(T \mid y_1\right)\right] \\
&= E\left[E\left(T \mid y_1\right) \text{ where } \left|y_1 > M\right|\right] + E\left[E\left(T \mid y_1\right) \text{ where } \left|y_1 \le M\right|\right]. \\
&= 1 \cdot P\left(\left|y_1\right| > M\right) + \int_{-M}^{M}\left[1 + L(x) \cdot f(x)\,dx\right] \\
&= 1 + \int_{-M}^{M} L(x) \cdot f(x)\,d
\end{aligned} \tag{3}
$$

Note that $L(x)$ in (3) is unknown. Thus, to approximate the ARL, values of $L(x)$ on the interval (-M, M) are first approximated using (2). The solution to the integral equation (2) can be obtained by replacing the integral equation with a system of linear algebraic equations and solving them numerically using trapezoidal quadrature. The ARL can then be approximated via (3), again using trapezoidal quadrature and the approximations of $L(x)$ at each of the subinterval points. Crowder's published Fortran program (1987b) will compute this ARL.

Vardeman and Jobe (1999) identified several M/R combinations giving each of several choices of "all OK" (in-control) ARLs as shown in Table 1. These M/R combinations will be compared to the X-only possibility.

Table 1. Several M/R combinations for various choices of "all OK" ARLs

"all OK" ARL	Smallest *M* possible				Smallest *R* possible
50	M = 2.33 (R = 4.5+)	M = 2.40 R = 3.66	M = 2.55 R = 3.41	M = 2.80 R = 3.28	(M = 3.3+) R = 3.24
100	M = 2.58 (R = 5.0+)	M = 2.65 R = 4.04	M = 2.80 R = 3.77	M = 3.00 R = 3.67	(M = 3.5+) R = 3.60
250	M = 2.88 (R = 5.5+)	M = 2.95 R = 4.47	M = 3.10 R = 4.22	M = 3.30 R = 4.11	(M = 3.80+) R = 4.05
370	M = 3.00 (R = 6.0+)	M = 3.10 R = 4.40	M = 3.20 R = 4.40	M = 3.40 R = 4.29	(M = 3.8+) R = 4.23
500	M = 3.09 (R = 6.0+)	M = 3.30 R = 4.53	M = 3.30 R = 4.53	M = 3.50 R = 4.42	(M = 4.0+) R = 4.36
750	M = 3.21 (R = 6.0+)	M = 3.45 R = 4.88	M = 3.45 R = 4.66	M = 3.60 R = 4.59	(M = 4.0+) R = 4.55
1000	M = 3.29 (R = 6.5+)	M = 3.40 R = 4.96	M = 3.50 R = 4.82	M = 3.65 R = 4.72	(M = 4.0+) R = 4.65

Source: Vardeman & Jobe (1999)

Because Crowder's published program requires M and R values for inputs and outputs $\sigma = 1$ ARLs for various means, the following procedures are necessary to use the program to evaluate the ARLs we desire.

As before, without loss of generalities, standard values $\mu_0 = 0$ and $\sigma_0 = 1$ are used. From expression (1), then with

$$UCL_X = \mu_0 + M\sigma_0 = M$$
$$LCL_X = \mu_0 - M\sigma_0 = -M$$

and

$$UCL_{MR} = R \cdot \sigma_0 = R \ ,$$

one must use limits

$$M_{new} = \frac{UCL_X - LCL_X}{2 \cdot \sigma_{new}} = \frac{M}{T}$$

and

$$R_{new} = \frac{UCL_{MR}}{\sigma_{new}} = \frac{R}{T}$$

must be used with Crowder's program (1987b).

Notice that for M large (The "Smallest Possible R" values in Table 1), the ARLs can essentially be considered ARLs of the Moving Chart alone (Crowder, 1987a). Similarly, for R large (The "Smallest Possible M" values in Table 1), the ARLs can essentially be considered ARLs of the Individual Chart alone. Therefore, the smallest possible M's from Table 1 are the same as the L's obtained for the Individual Chart alone.

Results

A variety of "all OK" (in control) ARLs, namely 50, 100, 250, 370, 500, 750 and 1000 from Vardeman and Jobe (1999) are selected from this study.

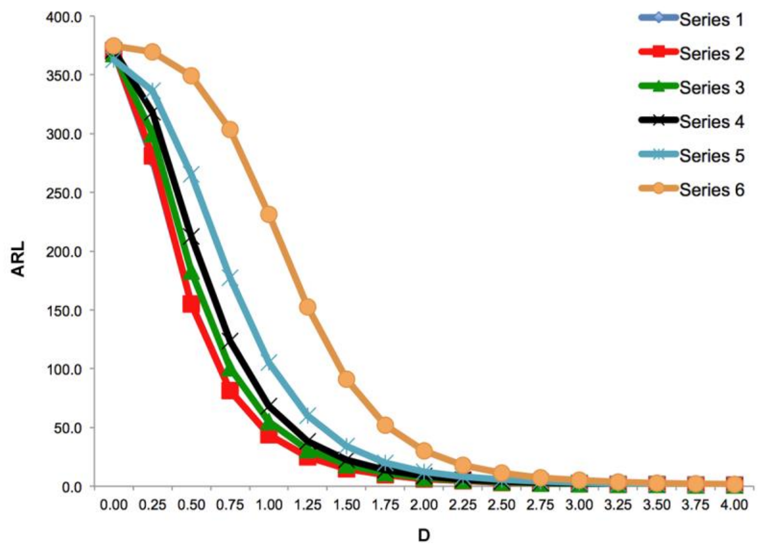
Comparisons of the monitoring schemes under the sets of circumstances mentioned in the Introduction are made in Tables 2-4 and Figures 1-3. Because the results for the different ''all OK'' ARLs all turned out to be similar, only Tables and Figures for the ARL = 370 case are presented here. In Tables 2-4, the smallest ARLs for a given set of parameters are bold, indented, and underlined. Table 2 and the corresponding Figure 1 show the case where only the process mean changes. Table 3 and the corresponding Figure 2 show the case where only the process standard deviation changes. Finally, Table 4 and Figure 3 show the case where both process mean and process standard deviation change from standard values.

Mean Changes Only (Table 2 and Figure 1)

As shown in Table 2, the smallest ARLs (bold, indented, and underlined) fall either in the X-only Chart region or in X/MR Chart Combinations region with *R* large (Smallest Possible *M*). As noted in 'Methods' above, X/MR Chart Combinations with *R* large (Smallest Possible *M*) can essentially be considered the X-only Charts. Therefore, the first 2 "ARL-Columns" (from left) in all the Tables 2-4 are in fact equivalent.

Table 2. Mean Changes – ARL = 370

D	X-only	X/MR Chart Combinations				
		M = 3.00 R = 6.0+	M = 3.10 R = 4.57	M = 3.20 R = 4.40	M = 3.40 R = 4.29	M = 3.8+ R = 4.23
0.00	370.0	370.0	367.8	370.9	363.0	374.8
0.25	***277.8***	280.9	299.7	318.4	336.5	369.6
0.50	156.3	***155.1***	182.9	212.5	265.1	349.2
0.75	81.3	***81.2***	100.5	123.3	177.3	303.6
1.00	***43.9***	43.9	54.9	68.4	105.5	231.5
1.25	***24.9***	24.9	30.9	38.3	59.9	152.7
1.50	15.0	***15.0***	18.2	22.3	34.2	90.8
1.75	9.5	***9.5***	11.3	13.6	20.1	52.0
2.00	***6.3***	6.3	7.4	8.7	12.4	29.9
2.25	4.4	***4.4***	5.1	5.8	8.0	17.8
2.50	3.2	***3.2***	3.7	4.1	5.4	11.1
2.75	2.5	***2.5***	2.8	3.1	3.9	7.2
3.00	***2.0***	2.0	2.2	2.4	2.9	5.0
3.25	***1.7***	1.7	1.8	1.9	2.3	3.6
3.50	***1.4***	1.4	1.5	1.6	1.9	2.7
3.75	***1.3***	1.3	1.3	1.4	1.6	2.1
4.00	1.2	***1.2***	1.2	1.3	1.4	1.7



Series 1 (X-only Chart)
Series 2 (M = 3.00, R = 6.0+)
Series 3 (M = 3.10, R = 4.57)
Series 4 (M = 3.20, R = 4.40)
Series 5 (M = 3.40, R = 4.29)
Series 6 (M = 3.8+, R = 4.23)

Figure 1. Plotted ARL values of Table 2

Theoretically, the ARLs shown in these 2 “ARL-Columns” should be exactly equal. However, small differences might come from the trapezoidal approximation in Crowder’s published program as well as rounding effects, and also the fact that the normal distribution table used in this work shows only 4 decimals places.

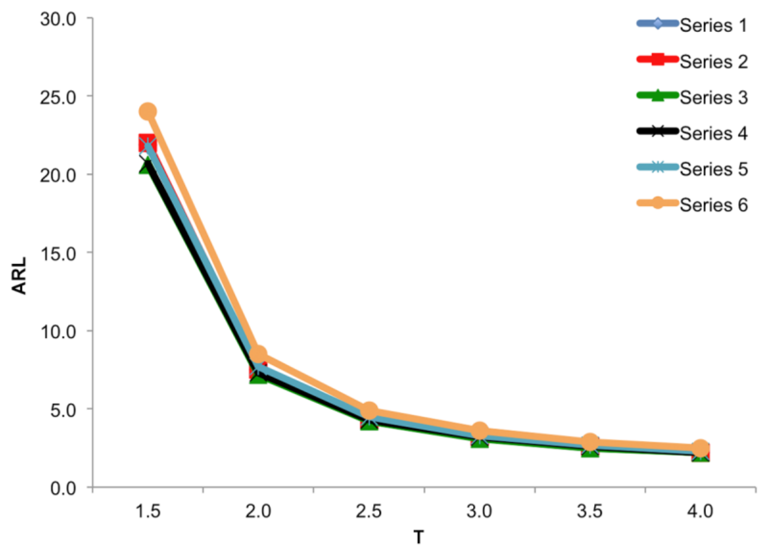
Thus, for Mean Change situation, the X-only Chart is better than the X/MR Chart Combinations since it gives the smallest off-target ARLs (quickest detection of mean process changes).

Sigma Changes Only (Table 3 and Figure 2)

For the second case (Sigma Changes Only), generally, the X/MR Chart with the second smallest *M* give the smallest off-target ARLs most of the time, except for several large changes in the process standard deviation (i.e., several large *T* values). For a large increase in process standard deviation (a large *T*), the Individual Chart alone is better than the X/MR chart Combinations (see Table 3 and Figure 2, for example).

Table 3. Sigma Changes – ARL = 370

T	X-only (L = 3)	X/MR Chart Combinations				
		M = 3.00 R = 6.0+	M = 3.10 R = 4.57	M = 3.20 R = 4.40	M = 3.40 R = 4.29	M = 3.8+ R = 4.23
1.0	370.0	370.0	370.0	370.0	370.0	370.0
1.5	21.9	22.0	***20.6***	20.7	21.8	24.0
2.0	7.5	7.5	***7.2***	7.3	7.7	8.5
2.5	4.3	4.3	***4.2***	4.3	4.5	4.9
3.0	3.2	3.2	***3.1***	3.2	3.3	3.6
3.5	2.6	2.6	***2.5***	2.6	2.7	2.9
4.0	2.2	***2.2***	2.2	2.2	2.3	2.5



Series 1 (X-only Chart)
Series 2 (M = 3.00, R = 6.0+)
Series 3 (M = 3.10, R = 4.57)
Series 4 (M = 3.20, R = 4.40)
Series 5 (M = 3.40, R = 4.29)
Series 6 (M = 3.8+, R = 4.23)

Figure 2. Plotted ARL values of Table 3. ARL = 370 when T = 1 for all series.

Mean and Sigma Change (Table 4 and Figure 3)

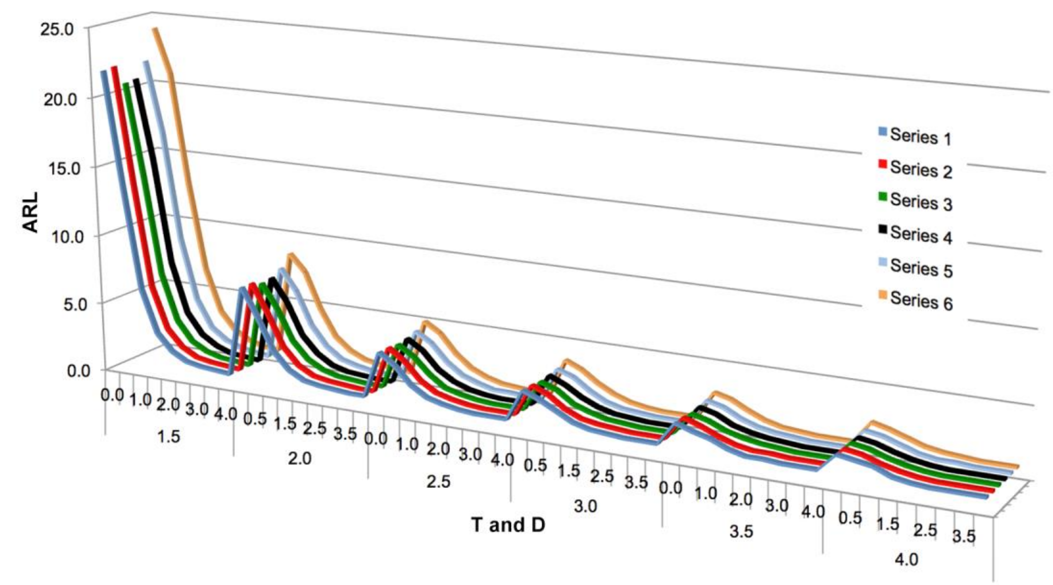
For the third case (Both Mean and Sigma Change), an Individual Chart or X/MR Chart Combination with R large (smallest possible M) gives the smallest off-target ARLs. Only a few exceptions appear for D = 0.5 with some large T values, where the X/MR Chart Combination with the second smallest M gives the smallest off-target ARL.

Where the second smallest M improves on the off-target ARL of the X-only chart, the size of the improvement is clearly quite small. Thus generally speaking, the Individual Chart alone is better than the X/MR Chart Combination (see Table 4 and Figure 3, for example).

Table 4. Mean and Sigma Change – ARL = 370

T	**D**	**X-only (L = 3)**	**X/MR Chart Combinations**				
			M = 3.00 R = 6.0+	M = 3.10 R = 4.57	M = 3.20 R = 4.40	M = 3.40 R = 4.29	M = 3.8+ R = 4.23
	0.0	21.9	22.0	***20.6***	20.7	21.8	24.0
	0.5	13.7	***13.7***	14.0	14.9	16.7	20.8
	1.0	6.3	***6.2***	6.8	7.4	8.9	12.8
	1.5	3.2	***3.2***	3.5	3.8	4.5	6.5
1.5	2.0	2.0	***2.0***	2.1	2.3	2.5	3.4
	2.5	***1.4***	1.4	1.5	1.6	1.7	2.1
	3.0	1.2	***1.2***	1.2	1.2	1.3	1.5
	3.5	***1.1***	1.1	1.1	1.1	1.1	1.2
	4.0	1.0	***1.0***	1.0	1.0	1.0	1.1
	0.0	7.5	7.5	***7.2***	7.3	7.7	8.5
	0.5	5.5	***5.5***	5.5	5.7	6.2	7.3
	1.0	3.2	***3.2***	3.3	3.4	3.8	4.7
	1.5	2.0	***2.0***	2.1	2.2	2.4	2.8
2.0	2.0	1.4	***1.4***	1.5	1.5	1.6	1.9
	2.5	1.2	***1.2***	1.2	1.2	1.3	1.4
	3.0	***1.1***	1.1	1.1	1.1	1.1	1.2
	3.5	1.0	***1.0***	1.0	1.0	1.0	1.1
	4.0	1.0	***1.0***	1.0	1.0	1.0	1.0
	0.0	4.3	4.3	***4.2***	4.3	4.5	4.9
	0.5	***3.5***	3.5	3.5	3.6	3.8	4.2
	1.0	2.3	***2.3***	2.4	2.4	2.6	2.9
	1.5	1.6	***1.6***	1.6	1.7	1.8	2.0
2.5	2.0	1.3	***1.3***	1.3	1.3	1.4	1.5
	2.5	1.1	***1.1***	1.1	1.1	1.1	1.2
	3.0	***1.0***	1.0	1.0	1.0	1.1	1.1
	3.5	1.0	***1.0***	1.0	1.0	1.0	1.0
	4.0	1.0	***1.0***	1.0	1.0	1.0	1.0
	0.0	3.2	3.2	***3.1***	3.2	3.3	3.6
	0.5	2.7	2.7	***2.7***	2.7	2.9	3.1
	1.0	2.1	***1.9***	1.9	2.0	2.1	2.3
	1.5	1.5	***1.4***	1.5	1.5	1.5	1.7
3.0	2.0	1.2	***1.2***	1.2	1.2	1.2	1.3
	2.5	1.1	***1.1***	1.1	1.1	1.1	1.1
	3.0	1.0	***1.0***	1.0	1.0	1.0	1.0
	3.5	1.0	***1.0***	1.0	1.0	1.0	1.0
	4.0	***1.0***	1.0	1.0	1.0	1.0	1.0

T	D	X-only (L = 3)	X/MR Chart Combinations M = 3.00 R = 6.0+	M = 3.10 R = 4.57	M = 3.20 R = 4.40	M = 3.40 R = 4.29	M = 3.8+ R = 4.23
	0.0	2.6	2.6	***2.5***	2.6	2.7	2.9
	0.5	2.2	***2.2***	2.2	2.3	2.4	2.6
	1.0	1.9	***1.7***	1.7	1.7	1.8	2.0
	1.5	1.4	***1.3***	1.4	1.4	1.4	1.5
3.5	2.0	1.1	***1.1***	1.2	1.2	1.2	1.2
	2.5	1.1	***1.1***	1.1	1.1	1.1	1.1
	3.0	1.0	***1.0***	1.0	1.0	1.0	1.0
	3.5	***1.0***	1.0	1.0	1.0	1.0	1.0
	4.0	***1.0***	1.0	1.0	1.0	1.0	1.0
	0.0	2.2	***2.2***	2.2	2.2	2.3	2.5
	0.5	2.0	***2.0***	2.0	2.0	2.1	2.2
	1.0	1.8	***1.6***	1.6	1.6	1.7	1.8
	1.5	1.3	***1.3***	1.3	1.3	1.3	1.4
4.0	2.0	1.1	***1.1***	1.1	1.1	1.1	1.2
	2.5	1.0	***1.0***	1.0	1.0	1.1	1.1
	3.0	1.0	***1.0***	1.0	1.0	1.0	1.0
	3.5	1.0	***1.0***	1.0	1.0	1.0	1.0
	4.0	***1.0***	1.0	1.0	1.0	1.0	1.0



Series 1 (X-only Chart)
Series 2 (M = 3.00, R = 6.0+)
Series 3 (M = 3.10, R = 4.57)
Series 4 (M = 3.20, R = 4.40)
Series 5 (M = 3.40, R = 4.29)
Series 6 (M = 3.8+, R = 4.23)

Figure 3. Plotted ARL values of Table 4

Conclusion

It has been found that X chart alone (X-only Chart) is better than the Individuals and Moving Range Chart Combinations (X/MR Chart Combinations) in detecting changes in the process mean. The Individual Chart alone gives smaller off-target ARLs for detecting changes in the process mean.

For the case where the process standard deviation changes, most of the time, X/MR Chart Combinations (with the 2nd smallest M) are better than the Individual Chart alone. Only for small "all OK" ARL values (ARL = 50 and 100), is the Individual Chart alone better than the X/MR Chart Combinations for large T. Specifically, for "all OK" (in-control) ARL = 50, the X-only Chart is better than X/MR Chart Combination when $T > 2.5$. Also for "all OK" (in-control) ARL = 100, the Individual Chart alone gives smaller off-target ARLs than X/MR Chart Combinations when $T > 3.0$. For large "all OK" (in-control) ARLs (ARL = 250, 370, 500, 750, and 1000), X/MR Chart Combination are better than the Individual Chart alone except for ARL = 370 and 750 with $T = 4.0$, where the Individual Chart alone is better.

Finally, the case where both process mean and process standard deviation change, most of the time the X-only Chart is better than the X/MR Chart Combinations. The X/MR Chart Combination can be better than the X-only Chart only when $D = 0.5$ for some values of large T.

Although the X-only chart can be better, the improvement in the off-target ARLs for the last two cases as described above is not really significant (coming in only the 3rd decimal place or beyond). If we round-off the results to the closest integer value, both results will typically be rounded to the same value. Therefore, in general, we can say that X/MR Chart Combination is "nearly" as efficient as the X-chart alone in detecting changes in the process standard deviation. Also, in general, with the same reason as above, the two monitoring schemes are "about" equally effective detecting changes in both process mean and process standard deviation (case 3).

Recommendation

If changes in only the process mean are of concern, it is definitely better to use the X-only Chart monitoring scheme.

If increases in the process standard deviation are the only ones of concern, the recommendation is to use the X-only Chart (for simplicity) even though the X/MR Chart Combinations is “nearly” as efficient as the Individual Chart alone.

Similarly where one is concerned both process mean and standard deviation changes, the recommendation is to use X-only Chart.

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