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Estimation of Gumbel Parameters under Ranked Set Sampling

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Consider the MLEs (maximum likelihood estimators) of the parameters of the Gumbel distribution using SRS (simple random sample) and RSS (ranked set sample) and the MOMEs (method of moment estimators) and REGs (regression estimators) based on SRS. A comparison between these estimators using bias and MSE (mean square error) was performed using simulation. It appears that the MLE based on RSS can be a robust competitor to the MLE based on SRS.

Keywords: Ranked set sampling; simple random sampling, parameters, Gumbel distribution, maximum likelihood estimator, bias, mean square error, regression estimator, method of moment estimator.

Introduction

There are many areas of application of the Gumbel distribution including environmental sciences, system reliability, and hydrology. In hydrology, for example, the Gumbel distribution may be used to represent the distribution of the minimum level of a river in a particular year based on minimum values for the past few years. It is useful for predicting the occurrence of extreme earthquakes, floods, and other natural disasters. The potential applicability of the Gumbel distribution to represent the distribution of minima relates to extreme value theory, which indicates that it is likely to be useful if the distribution of the underlying sample data is of the normal or exponential type.

The problem of estimation of the unknown parameters of the Gumbel distribution is considered by many authors under simple random sampling. Maciunas et al. (1979) considered the estimates of the parameters of the Gumbel distribution by the methods of probability weighted moments, moments, and

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maximum likelihood. They used both independent and serially correlated Gumbel numbers to derive the results from Monte Carlo experiments. They found the method of probability weighted moments estimator is more efficient than the estimators. Leese (1973), derived the MLE (maximum likelihood estimator) of Gumbel distribution parameters in case of censored samples and he gave expressions for their large-sample standard errors. Fiorentino and Gabriele (1984), given some modifications of the MLE the Gumbel distribution parameters to reduce the bias of the estimators. Phien (1987) estimated the parameters of the Gumbel distribution by moments, MLE, maximum entropy and probability weighted moments. He derived the asymptotic variance-covariance matrix of the MLEs and used simulation to compare between the various estimators. He found that the MLE is best in terms of the root MSE (mean square error). Corsini et al. (1995), discussed the MLE and Cramer-Rao (CR) bounds for the location and scale parameters of the Gumbel distribution. Mousa et al. (2002), found the Bayesian estimation for the two parameters of the Gumbel distribution based on record values.

RSS as introduced by McIntyre (1952) is an ingenious sampling technique for selecting a sample which is more informative than a SRS to estimate the population mean. He used of RSS technique to estimate the mean pasture and forage yields. RSS technique is very useful when visual ranking of population units is less expensive than their actual quantifications. Therefore, selecting a sample based on RSS can reduce the cost and increase the efficiency of estimation.

The basic idea behind selecting a sample under RSS can be described as follows: Select m random samples each of size m, using a visual inspection or any cheap method to rank the units within each sample with respect to the variable of interest. Then select, for actual measurement, the i^{th} smallest unit from the i^{th} sample, $i = 1, \ldots, m$. In this way, a total of m measured units are obtained, one from each sample. The procedure could be repeated r times until a sample of $n = mr$ measurements are obtained. These mr measurements form RSS. Takahasi and Wakimoto (1968) gave the theoretical background for RSS. They showed that the mean of an RSS is an unbiased estimator of the population mean with variance smaller than that of the mean of a SRS. Dell and Clutter (1972) showed that the RSS mean remains unbiased and more efficient than the SRS mean for estimating the population even if ranking is not perfect. A comprehensive survey about developments in RSS can be found in Chen (2000) and Muttlak and Al-Saleh (2000).

Because there are many attractive applications of Gumbel distribution, it is of interest to conduct a statistical inference for the Gumbel distribution. The statistical inference includes the study of some properties of Gumbel distribution, emphasizing on estimation of Gumbel parameters. The estimation of the location and scale parameters, denoted as α and β respectively, of the Gumbel distribution under SRS and RSS is studied. The Gumbel parameters were estimated by using several methods of estimation in both cases of SRS and RSS such as maximum likelihood, moments and regression. Furthermore, the performance of these estimators is investigated and compared through simulation. Bias, mean square error (MSE) and efficiency of these estimators were used for comparison.

Parameter Estimation Using SRS

The cdf and pdf of the random variable X which has a Gumbel distribution with parameters α and β are given respectively by

$$F(x;\alpha,\beta)=\exp\left(-\exp\left(-\frac{x-\alpha}{\beta}\right)\right), \tag{1}$$

$$f(x;\alpha,\beta)=\frac{1}{\beta}\exp\left(-\frac{x-\alpha}{\beta}-\exp\left(-\frac{x-\alpha}{\beta}\right)\right), \tag{2}$$

where α is the location parameter and β is the scale parameter, $\beta > 0$, x and $\alpha \in (-\infty, \infty)$.

Let $X_1, X_2, \ldots, X_n$ be a random sample from X. The MLEs, MOMEs (method of moment estimators) and REGs (regression estimators) will be examined in case of both parameters are unknown based on $X_1, X_2, \ldots, X_n$.

MLEs

Let $X_1, X_2, \ldots, X_n$ be a random sample from (2). The log-likelihood function is given by

$$l(\alpha,\beta)=-n\log(\beta)-\sum_{i=1}^{n}\left(\frac{x_i-\alpha}{\beta}\right)-\sum_{i=1}^{n}\exp\left(-\frac{x_i-\alpha}{\beta}\right). \tag{3}$$

After taking the derivatives with respect to α and β equating to 0, the MLEs are obtained as

$$\hat{\beta}_{MLE,S} = \bar{x} - \sum_{i=1}^{n} x_i w_i \text{ and } \hat{\alpha}_{MLE,S} = -\hat{\beta}_{MLE,S} \log(\bar{z}). \tag{4}$$

where $z_i = \exp\left(-\dfrac{x_i}{\hat{\beta}_{mle,S}}\right)$, $\bar{z} = \dfrac{1}{n}\sum_{i=1}^{n} z_i$ and $w_i = \dfrac{z_i}{n\bar{z}}$.

MOMEs

The mean and variance for Gumbel distribution are given by

$$\mu = \alpha + \gamma\beta \text{ and } \sigma^2 = \frac{\pi^2}{6}\beta^2. \tag{5}$$

The moment estimators of the two parameters are

$$\hat{\beta}_{MOME,S} = \frac{\sqrt{6}}{\pi} s \text{ and } \hat{\alpha}_{MOME,S} = \bar{x} - \gamma\, \hat{\beta}_{MOME,S} \tag{6}$$

where s, $\bar{x}$ are the sample standard deviation and mean, respectively, and γ = 0.57721566 is Euler's constant.

REGs

Let $y = F(x;\alpha,\beta) = \exp\left(-\exp\left(-\dfrac{x-\alpha}{\beta}\right)\right) \Rightarrow \ln \text{y} = -\exp\left(-\dfrac{x-\alpha}{\beta}\right)$

$$\Rightarrow -\ln \text{y} = \exp\left(-\frac{x-\alpha}{\beta}\right) \Rightarrow \text{t} = \ln\left(\text{-}\ln \text{y}\right) = -\left(\frac{x-\alpha}{\beta}\right) \Rightarrow t = a\,x + b$$

where $a = \dfrac{-1}{\beta}$ and $b = \dfrac{\alpha}{\beta}$.

The regression estimators of the two parameters are

$$\hat{\beta}_{REG,S} = \frac{-1}{\hat{a}} \text{ and } \hat{\alpha}_{REG,S} = \hat{\beta}_{REG,S}\left(t - \hat{a}\overline{x}\right) \tag{7}$$

where $\hat{a} = \dfrac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(t_i - \overline{t}\right)}{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}$.

Parameter Estimation Under RSS

MLEs

Let $X_{(i:m)j}$, $i = 1, \ldots, m$ and $j = 1, \ldots, r$ denote the i^{th} order statistics from the i^{th} set of size m of the j^{th} cycle be the RSS data for X with sample size $n = mr$.

Using (1) and (2), the pdf of $X_{(i:m)j}$ is given by (Arnold et al.,1992)

$f_{i:m}(X_{(i:m)j}) = c\left(F(X_{(i:m)j})\right)^{i\text{-}1}\left(1\text{-}F(X_{(i:m)j})\right)^{m\text{-}i} f(X_{(i:m)j})$, where $c = \dfrac{1}{B(i, m\text{-}i+1)}$,

$f(X_{(i:m)j}) = \dfrac{1}{\beta}\exp\left(-\left(\dfrac{X_{(i:m)j} - \alpha}{\beta}\right) - \exp\left(-\left(\dfrac{X_{(i:m)j} - \alpha}{\beta}\right)\right)\right)$ and

$F(X_{(i:m)j}) = \exp\left(-\exp\left(-\left(\dfrac{X_{(i:m)j} - \alpha}{\beta}\right)\right)\right)$. Then the likelihood function is given by

$$\begin{aligned}
l(\alpha,\beta) &= \prod_{i=1}^{r}\prod_{j=1}^{m} f_{i:m}(X_{(i:m)j}) \\
&= \prod_{i=1}^{r}\prod_{j=1}^{m}\left[c\left(F(X_{(i:m)j})\right)^{i\text{-}1}\left(1\text{-}F(X_{(i:m)j})\right)^{m\text{-}i} f(X_{(i:m)j})\right] \\
&= c^{mr}\prod_{i=1}^{r}\prod_{j=1}^{m}\left[\left(F(X_{(i:m)j})\right)^{i\text{-}1}\left(1\text{-}F(X_{(i:m)j})\right)^{m\text{-}i} f(X_{(i:m)j})\right].
\end{aligned}$$

The log-likelihood function is given by

$$\begin{aligned} L(\alpha,\ \beta) &= mr\log\ c + \sum_{j=1}^{r}\sum_{i=1}^{m}(i-1)\log F(X_{(i:m)j}) \\ &+ \sum_{j=1}^{r}\sum_{i=1}^{m}(m-i)\log(1-F(X_{(i:m)j})) + \sum_{j=1}^{r}\sum_{i=1}^{m}\log f(X_{(i:m)j}). \end{aligned} \tag{8}$$

Taking the derivatives of (8) with respect to α and β respectively, and equating the resulting quantities to zero. Because there is no explicit solution for (8), the equations need to be solved numerically to find $\hat{\alpha}_{MLE,R}$ and $\hat{\beta}_{MLE,R}$.

Ad-hoc Estimators

These are the same as the estimators in (6) and (7) with SRS replaced by RSS

Estimator Comparison

A comparison between all above estimators for both parameters of the Gumbel distribution was carried out under SRS and RSS using simulation. The package R has been used to conduct the simulation. The following values of the parameters and sample sizes have been considered: $\alpha = 0.5$, $\beta = 1$; $\alpha = 1$, $\beta = 0.5$; $\alpha = 1$, $\beta = 1$; $\alpha = 1$, $\beta = 2$; $\alpha = 2$, $\beta = 1$, $n = 12$ and $n = 24$.

For each n, a set $(m;r)$ is decided such that $n = mr$. The bias and the MSE are computed for all the estimators under consideration. The efficiency between all estimators with respect to the MLE based on SRS are calculated where the efficiency of the estimator is defined as

$$eff\,(\hat{\theta}_2, \hat{\theta}_1) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)}$$

where

$$MSE\left(\hat{\theta}\right) = \frac{1}{10,000}\sum_{t=1}^{10,000}\left(\hat{\theta}_{2t} - \theta_2\right)^2.$$

If $eff\left(\hat{\theta}_2, \hat{\theta}_1\right) > 1$ then $\hat{\theta}_2$ is better than $\hat{\theta}_1$.

The bias of the estimators is reported in Tables 1 and 3 and the efficiencies of the estimators is reported in Tables 2 and 4.

Table 1. The bias and MSE of estimators of *α*

			Bias					MSE				
(α, b)	n	n=mr	$\hat{\alpha}_{mle,S}$	$\hat{\alpha}_{moe,S}$	$\hat{\alpha}_{reg,S}$	$\hat{\alpha}_{mle,R}$	$\hat{\alpha}_{moe,R}$	$\hat{\alpha}_{mle,S}$	$\hat{\alpha}_{moe,S}$	$\hat{\alpha}_{reg,S}$	$\hat{\alpha}_{mle,R}$	$\hat{\alpha}_{moe,R}$
		m=2, *r*=6			0.121	-1.156	-1.141	1.647	1.499	0.058	1.522	1.486
	12	*m*=3, *r*=4	-1.183	-1.130	-1.153		-1.146				1.480	1.469
(1,1)		*m*=4, *r*=3			-1.140		-1.146				1.427	1.444
		m=2, *r*=12			-1.187	1.601	-1.149	1.601	1.424	0.033	1.515	1.415
	24	*m*=3, *r*=8	-1.213	0.103	-1.179		-1.152				1.473	1.405
		m=4, *r*=6			-1.167		-1.152				1.430	1.393
		m=2, *r*=6			2.268	5.841	-2.285	5.841	5.907	2.205	4.212	5.943
	12	*m*=3, *r*=4	-1.728	-1.471	2.174		-2.283				4.211	5.799
(1,2)		*m*=4, *r*=3			2.092		-2.292				4.211	5.793
		m=2, *r*=12			-2.047	4.211	-2.298	4.211	5.666	2.296	5.810	5.636
	24	*m*=3, *r*=8	2.321	-1.508	2.135		-2.306				4.211	5.628
		m=4, *r*=6			2.174		-2.302				4.211	5.567
		m=2, *r*=6			-1.167	1.666	-1.144	1.666	1.519	3.793	1.549	1.486
	12	*m*=3, *r*=4	-1.728	1.901	-1.160		-1.141				1.499	1.446
(2,1)		*m*=4, *r*=3			-1.153		-1.151				1.456	1.457
		m=2, *r*=12			-1.203	1.640	-1.140	1.640	1.432	3.793	1.549	1.389
	24	*m*=3, *r*=8	-1.229	1.921	-1.182		-1.148				1.480	1.395
		m=4, *r*=6			-1.176		-1.152				1.453	1.394
		m=2, *r*=6			-1.139	1.558	-1.144	1.558	1.516	0.362	1.485	1.490
	12	*m*=3, *r*=4	-1.143	-0.576	-1.131		-1.141				1.427	1.457
(0.5,1)		*m*=4, *r*=3			-1.129		-1.148				1.410	1.448
		m=2, *r*=12			-1.168	1.557	-1.140	1.557	1.432	0.389	1.469	1.392
	24	*m*=3, *r*=8	-1.185	-0.612	-1.159		-1.142				1.436	1.381
		m=4, *r*=6			-1.137		-1.149				1.371	1.388
		m=2, *r*=6			-0.589	0.419	-0.571	0.419	0.376	0.205	0.393	0.371
	12	*m*=3, *r*=4	-0.598	0.375	-0.575		-0.569				0.368	0.361
(1,0.5)		*m*=4, *r*=3			-0.577		-0.572				0.364	0.360
		m=2, *r*=12			-0.602	0.419	-0.576	0.419	0.353	0.157	0.387	0.356
	24	*m*=3, *r*=8	-0.619	0.344	-0.597		-0.573				0.378	0.347
		m=4, *r*=6			-0.585		-0.575				0.359	0.347

Table 2. The efficiency of estimators of α

(α, b)	n	n=mr	$\hat{\alpha}_{mle,S}$	$\hat{\alpha}_{moe,S}$	$\hat{\alpha}_{reg,S}$	$\hat{\alpha}_{mle,R}$	$\hat{\alpha}_{moe,R}$
(1,1)	12	m=2, r=6	1	1.099	28.397	1.082	1.108
		m=3, r=4				1.113	1.121
		m=4, r=3				1.154	1.141
	24	m=2, r=12	1	1.124	48.515	1.057	1.131
		m=3, r=8				1.087	1.140
		m=4, r=6				1.120	1.149
(1,2)	12	m=2, r=6	1	0.989	2.649	1.387	0.983
		m=3, r=4				1.387	1.007
		m=4, r=3				1.387	1.008
	24	m=2, r=12	1	0.743	1.834	0.725	0.747
		m=3, r=8				1.000	0.748
		m=4, r=6				1.000	0.756
(2,1)	12	m=2, r=6	1	1.097	0.439	1.076	1.121
		m=3, r=4				1.111	1.152
		m=4, r=3				1.144	1.143
	24	m=2, r=12	1	1.145	0.432	1.059	1.181
		m=3, r=8				1.108	1.176
		m=4, r=6				1.129	1.176
(0.5,1)	12	m=2, r=6	1	1.028	4.304	1.049	1.046
		m=3, r=4				1.092	1.069
		m=4, r=3				1.105	1.076
	24	m=2, r=12	1	1.087	4.003	1.060	1.119
		m=3, r=8				1.084	1.127
		m=4, r=6				1.136	1.122
(1,0.5)	12	m=2, r=6	1	1.114	2.044	1.066	1.129
		m=3, r=4				1.139	1.161
		m=4, r=3				1.151	1.164
	24	m=2, r=12	1	1.187	2.669	1.083	1.177
		m=3, r=8				1.108	1.207
		m=4, r=6				1.167	1.207

Table 3. The bias and MSE of estimators of β

			Bias					MSE				
(α, b)	n	n=mr	$\hat{\alpha}_{mle,S}$	$\hat{\alpha}_{moe,S}$	$\hat{\alpha}_{reg,S}$	$\hat{\alpha}_{mle,R}$	$\hat{\alpha}_{moe,R}$	$\hat{\alpha}_{mle,S}$	$\hat{\alpha}_{moe,S}$	$\hat{\alpha}_{reg,S}$	$\hat{\alpha}_{mle,R}$	$\hat{\alpha}_{moe,R}$
		m=2, *r*=6			0.889	0.296	-0.023	0.327	0.077	0.963	0.337	0.079
	12	*m*=3, *r*=4	0.273	-0.042		0.303	-0.013				0.301	0.074
(1,1)		*m*=4, *r*=3				0.308	-0.011				0.302	0.069
		m=2, *r*=12			0.919	0.415	-0.012	0.359	0.042	0.949	0.353	0.043
	24	*m*=3, *r*=8	0.412	-0.021		0.425	-0.006				0.348	0.039
		m=4, *r*=6				0.416	0.006				0.322	0.036
		m=2, *r*=6			4.368	1.737	0.959	5.293	1.154	19.696	4.759	1.234
	12	*m*=3, *r*=4	1.772	0.918		1.734	0.959				4.508	1.196
(1,2)		*m*=4, *r*=3				1.714	0.986				4.128	1.253
		m=2, *r*=12			4.445	2.018	0.974	5.461	1.081	20.192	5.127	1.106
	24	*m*=3, *r*=8	2.328	0.955		1.928	0.981				4.656	1.119
		m=4, *r*=6				1.734	0.984				4.509	1.113
		m=2, *r*=6			0.799	-0.707	-1.022	0.773	1.155	0.877	0.728	1.122
	12	*m*=3, *r*=4	-0.740	-1.039		-0.693	-1.020				0.695	1.111
(2,1)		*m*=4, *r*=3				-0.688	-1.010				0.674	1.091
		m=2, *r*=12			0.850	-0.584	-1.016	0.533	1.082	0.862	0.514	1.072
	24	*m*=3, *r*=8	-0.599	-1.019		-0.589	-1.011				0.490	1.061
		m=4, *r*=6				-0.587	-1.007				0.483	1.051
		m=2, *r*=6			0.979	0.803	0.478	0.923	0.286	1.102	0.896	0.304
	12	*m*=3, *r*=4	0.790	0.455		0.811	0.485				0.891	0.311
(0.5,1)		*m*=4, *r*=3				0.808	0.488				0.875	0.306
		m=2, *r*=12			1.001	0.927	0.481	1.045	0.274	1.089	1.040	0.272
	24	*m*=3, *r*=8	0.913	0.482		0.927	0.489				1.035	0.276
		m=4, *r*=6				0.908	0.492				0.974	0.277
		m=2, *r*=6			-0.257	-0.356	-0.512	0.193	0.291	0.119	0.181	0.282
	12	*m*=3, *r*=4	-0.369	-0.522		-0.358	-0.512				0.179	0.280
(1,0.5)		*m*=4, *r*=3				-0.347	-0.506				0.169	0.272
		m=2, *r*=12			-0.265	-0.291	-0.504	0.129	0.272	0.105	0.126	0.266
	24	*m*=3, *r*=8	-0.292	-0.512		-0.289	-0.506				0.123	0.265
		m=4, *r*=6				-0.295	-0.505				0.122	0.264

Table 4. The efficiency of estimators of β

(α, b)	n	$n=mr$	$\hat{\alpha}_{mle,S}$	$\hat{\alpha}_{moe,S}$	$\hat{\alpha}_{reg,S}$	$\hat{\alpha}_{mle,R}$	$\hat{\alpha}_{moe,R}$
(1,1)	12	m=2, r=6	1	4.247	0.340	0.973	4.139
		m=3, r=4				1.086	4.419
		m=4, r=3				1.083	4.739
	24	m=2, r=12	1	8.548	0.378	1.017	8.349
		m=3, r=8				1.032	9.205
		m=4, r=6				1.115	9.972
(1,2)	12	m=2, r=6	1	4.587	0.269	1.112	4.289
		m=3, r=4				1.174	4.426
		m=4, r=3				1.282	4.224
	24	m=2, r=12	1	5.052	0.270	1.065	4.938
		m=3, r=8				1.173	4.880
		m=4, r=6				1.211	4.907
(2,1)	12	m=2, r=6	1	0.669	0.881	1.062	0.689
		m=3, r=4				1.112	0.696
		m=4, r=3				1.147	0.709
	24	m=2, r=12	1	0.493	0.618	1.037	0.494
		m=3, r=8				1.088	0.500
		m=4, r=6				1.104	0.504
(0.5,1)	12	m=2, r=6	1	3.227	0.838	1.030	3.036
		m=3, r=4				1.036	2.968
		m=4, r=3				1.055	3.016
	24	m=2, r=12	1	3.814	0.960	1.005	3.842
		m=3, r=8				1.010	3.786
		m=4, r=6				1.073	3.773
(1,0.5)	12	m=2, r=6	1	0.663	1.622	1.066	0.684
		m=3, r=4				1.078	0.689
		m=4, r=3				1.142	0.710
	24	m=2, r=12	1	0.474	1.229	1.024	0.485
		m=3, r=8				1.049	0.487
		m=4, r=6				1.057	0.489

From Tables 1 to 4, the following conclusions are put forth

i) In general, the bias is large for all estimators. Therefore, all the estimators are considered as biased estimators for α.

ii) From Table 1, it can be noticed that the REG under SRS has the smallest bias as compared to the other estimators considered in most cases. In general, for all estimators of α under RSS, the bias is less than the case under SRS.

iii) For fixed α, the MSE of $\hat{\alpha}$ decreases as the sample size increases.

iv) It is noticed that from Table 2 that MLE under RSS is the most efficient than the MLE based on SRS.

v) The efficiency of the other estimators (MOMEs and REGs based on SRS and RSS) are not consistent, sometimes less one and other times greater than 1.

Similar remarks can be noticed for the case of β.

Conclusion

Based on this study, it may be concluded that all estimators are biased. Because the MLEs under RSS are more efficient than the MLE under SRS, RSS is recommended in case ordering can be done visually or by an inexpensive method. The other estimators are not recommended because they are not consistent.

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