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Skip-lot sampling plan serves as a cost-effective technique to manage the cost of performing frequent product inspections. As a powerful tool within a real-time quality management system, the ability to collect data which an optimize skip-lot sampling parameters affords manufacturers the luxury of lowering inspection expenses in various manufacturing units. The good quality of product can be produced in continuous improvement of production process in excellent quality history for suppliers. The procedures and necessary tables are provided for finding the respective plans for which sum of producer and consumer risks are minimized with acceptable and limiting quality levels which accounts for the prior distribution of process state for each lot and revenue received appreciably which reduces destructive testing.

Keywords: Bayesian sampling plan, gamma-Poisson distribution, producer's quality level, consumer's quality level, weighted risk

Introduction

Quality has been an internal part of all products and services. It has become one of the most important consumer decision factors in the selection among competing product and services. The modern quality control methods are developed to growing awareness of needs and demands of the consumer. The method of quality control is mainly refers to a spectrum of managerial methods for attempting to maintain the quality of products at a desired level.

Acceptance sampling is a statistical procedure for accepting or rejecting a batch of merchandise or documents involves determining the maximum of defects discovered in a sample before the entire batch is rejected. The sampling procedure is defined on the inspection and classification of sample of units selected at random from a larger batch or lot and the ultimate decision about disposition of

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the lot is made. Acceptance sampling is the specific plan that states the sample size or sizes to be used and associated with acceptance and rejection criteria. This method has rapidly gained wide application in industry, particularly in the following stages of manufacturing: incoming materials inspection, on line production control and finished product quality auditing.

Acceptance sampling is concerned with the risks of decision making. In industry it is used to take decision on lots, whether accept or reject a lot of some product or to accept or reject process. The rejection of a lot means return the lot to supplier or its submission to 100 percent inspection. The risks are classified as two namely, producer’s risk and consumer’s risk. The producer's risk implies that a good lot may be rejected by a sampling plan and the consumer's risk implies that a bad lot may be accepted by a sampling approach. Sampling plans are usually designed to control one or both of these risks.

The theory of acceptance sampling offers various inspection procedures, termed as sampling plans, which are categorized under four types, namely, (i) lot-by-lot sampling by attributes, in which each unit in a sample is inspected on a go-on-go basis for one or more characteristics, (ii) lot-by-lot sampling by variables, in which each unit in a sample is measured for single characteristics, (iii) sampling plans for continuous production by method of attributes and (iv) special purpose plans. Lot-by-lot sampling by attributes, in particular, comprises plans such as single sampling plans, double sampling plans, multiple sampling plans and sequential sampling plans.

A sampling plan is usually specified by one or more parameters such as sample size (n) and acceptance number (c) and associates with itself an important measure of performance, called operating characteristic function. The determination of the parameters of a sampling plan is prescribed the conditions on its operating characteristic curve providing protection to the producer and consumer is called designing of the sampling plan.

Acceptance sampling by attributes each item tested is classified as conforming or non-conforming. A sample is taken and it contains too many non-conforming items, then the batch is rejected, otherwise it is accepted. For this method to be effective, batches containing some non-conforming items must be acceptable. If the only acceptable percentage of non-conforming items is zero, this can only be achieved by examine every item and removing the item which are non-conforming. This is known as 100% inspection.

Effective acceptance sampling involves effective selection and the application of specific rules for lot inspection. The acceptance sampling plan applied on a lot-by-lot basis becomes an element in the overall approach to maximize quality at minimum cost. Since different sampling plans may be statistically valid at different times during the life of a process, therefore all sampling plans should be periodically reviewed.

Many quality characteristics of a product can be measured by their performance measures. In such situations each product can be inspected and classified as either satisfying or non-satisfying a given set of specifications. Thus the products can be classified as defectives or non-defectives otherwise good or bad which based on inspections. Such quality characteristics are called attributes. This kind of inspection procedure is known as inspection by attributes and the respective plan is called as attribute sampling plan. In attribute sampling plan, decision is taken by comparing the number of defectives found on inspection with a stated acceptance number.

Bayesian Acceptance Sampling Plan

The Bayesian approach provides a formal mechanism for taking sample of preferences for striking an economical balance between the cost of sampling and the expectation of loss due to accepting an insufficiently reliable product or rejecting a sufficiently reliable one. The assumption underlying the theory of acceptance sampling is that the production process from which lots are formed is stable and the lot quality defined in terms of fraction nonconforming is a fixed constant. The sampling inspection procedures defined under such assumptions are considered as conventional sampling plans.

However, in practice, the production processes are not always stable and the lots coming from such processes will have quality variations which may occur due to random fluctuations. The quality variations in the lots are separated into two types, viz., within-lot (sampling) variation and between-lot (sampling and process) variation. If these two sources of variation are equal and implying more process variation, the dispersion of process about the process average is zero, and each lot can be considered as a random sample drawn from a process with a constant probability of producing a non-conforming unit. This is the premise behind conventional acceptance sampling. In frequently, between-lot variation is greater than within-lot variation, which indicating that process variation exists and the probability of obtaining non-conforming unit varies continually. In such situations, the decisions on the submitted lots should be made with the consideration of between-lot variations and the lot quality will be treated as a random variable rather than a fixed quantity.

Further, Bayesian acceptance sampling considers both sources of variation. Thus the distinction between conventional and Bayesian approach is associated with the variations in lot quality and it can be studied by an appropriate prior distribution based on process history or knowledge in the selection of distribution to describe the random fluctuations involved in sampling plans. Sampling plans which use prior distribution for the lot quality together with the sampling distribution of sample information for making decisions such as accepting or rejecting the submitted lots are termed as Bayesian acceptance sampling plans and which are treated as alternative to conventional sampling plans. [See, Calvin (1990)].

The procedures for Bayesian plans which are introduce an economic considerations and prior results into the sampling equation. These procedures are suited to the sampling lots from process or assembly operations that contain assignable causes. These causes may be unknown and awaiting for isolation, known but unremovable due to state-of-the limitations, or known but uneconomical to remove. Conventional acceptance sampling assumes these assignable causes have been eliminated. Thus, the distinction between conventional and Bayesian approach is associated with the utilization of prior process history or knowledge in the selection of a distribution to describe the random fluctuations involved in acceptance sampling (Calvin, 1990).

Wetherill and Chiu (1975) noted the economic schemes based on Bayesian theory is more precise and scientific, leaving much less to judgement than those based on classical theory. The objective of the paper is to develop a Bayesian acceptance sampling plan with fixed acceptance numbers, when the number of defects in a unit can be described by a Poisson distribution with parameter λ and the prior distribution of λ takes the form of a gamma or non-informative function.

The gamma distribution was selected for utilization as prior knowledge because of two inherent characteristics: which are (i) The Poisson natural conjugate prior and (ii) It possesses sufficient productivity in distribution form, varying its parameters, which allows a reasonable representation of the specific prior knowledge. The first aspect leads to mathematical compatibility; a convenient attribute which obtained facilitates the computations. The second point implies that the gamma distribution, which provides a variety of distribution forms ranging from the positively skewed exponential distribution to an approximately symmetrical distribution shape.

The non-informative function used in the absence of specific prior knowledge corresponds to Jeffrey's non-informative prior (Box & Tiao, 1992). The relationship between defectives in sample and defectives in remaining lot for each of prior distributions can be exploring the idea of Bayesian methodology. Further it observed that the use of a binomial prior renders sampling useless and unsuitable. These serve to make the designers and users of Bayesian sampling plans more aware of the consequence associated with selection of particular prior distribution (See Case & Keats, 1982). Phelps (1982) derived sampling procedure for skip-lot model using Bayesian approach under destructive testing. The model is developed for the purpose of (i) To maximize the expected return per lot produced items with non-conformities. (ii) To determine the inspection duration of preceding or succeeding lots, sample size (n), and acceptance number (c). The problem is generated with the help of posterior distribution of success state for each lot and it may reduce the constructive of sampling plan.

Designing of Skip-lot Sampling Plan

The theory behind skip-lot is that continuous lots should be of high quality. In a skip-lot inspection, quality management recruits only inspect a small percentage of very high-conforming lots. Once companies develop a reference plan based on historical data of consumers' risks and producers' risks from inspections proceed to lot-by-lot. However, once a specified number of consecutive high-conforming lots have passed inspection, firms only inspect a fraction of subsequent lots at random. This skip-lot process continues until a lot does not pass, which then reverts to lot-by-lot inspection until products pass the skip-lot threshold again. The continuous inspection procedures which are optimum for a specified income function and a production model which can be only in of two states, which are states of repair, and known transition probabilities. The Markov process, generated by the model and class of decision procedures, approaches a limiting distribution.

Dodge (1955) presented an extension of continuous sampling plans for individual units to a skipping lot sampling plan that are applicable to bulk materials or products produced in successive lots or batches and designates the inspecting plan. The skipping inspection has specific rules based on the record of lot acceptance and rejections, for switching back and forth between normal inspection and skipping inspection.

Perry (1970, 1973) was concerned with the development and evaluation of a system of lot inspection sampling plans in which the provision are made for inspecting only some fraction of the submitted lots when the quality of the submitted product is good as demonstrated by the quality history of the product. A good proportion of the ideas and concepts of the skip-lot sampling plan SkSP-2 has modified from the continuous sampling plans for individual units or items of production. A continuous sampling inspection plan used to inspect a product which consisting of individual units and manufactured by an essentially in continuous process. This plan proposes that when quality is good, only a fraction *f* of the submitted units need to be inspected [see Dodge (1943)].

Carr (1982) extended the procedures of CSP-M type plan and developed with a system of skip-lot plans designated as CSP-MSkSP. Carr (1982) noted inspection errors can have a severe impact on an attributes single-sampling plan for lot acceptance due to misclassification of units as defectives or nondefectives. However, with estimates of errors, the plan can be adjusted to preserve the desired operating characteristic curve for specified sampling plan. The skip-lot sampling plan have been developed at various situations such as cost, MIL_STD_105D, error of inspection, which are qualified by Schilling (1982), Hsu (1977), Okada (1967), Stephens (1979), Cox (1980), and Carr (1982). Aslam, Balamurali, Jun, and Ahmad (2010) established the designing methodology to determine the parameters for system of skip-lot sampling plan with corresponding to two points on the operating characteristic curve and also to minimize the average sample number with the help of binomial distribution.

The SkSP-2 plan is described as one that uses a given lot inspection plan by method of attributes called 'reference plan' together with the following rules.

Rule 1. Start with normal inspection (inspecting every lot) using reference plan

Rule 2. When i consecutive lots are accepted on normal inspection, switch to skipping inspection and inspect only a fraction f of the lots.

Rule 3. When a lot is rejected on skipping inspection, return to normal inspection

The positive integer i and sampling fraction f are the parameters of SkSP-2. Here $0 < f < 1$. When $f = 1$ the plan reduces to original reference plan. The probability of acceptance of the plan SkSP-2 is denoted by $P_a(f, i)$. Using Markov-chain technique one can find the probability of acceptance of SkSP-2 plan. A Markov process represents the observations of system which satisfying the condition that the probability of physical system will be given a state at time t_2 may be deduced from knowledge of its state at earlier time t_1. A Markov chain is a special case of Markov process in which the set of states or state space is discrete. A more complete characterization of the one step transitions of a Markov chain with their corresponding probabilities provided in a matrix form is called the transition matrix (see Parzen, 1964).

The technique of Markov chains to evaluate the sampling plans, a trial corresponds to the drawing of sample from a lot which is under consideration. The results and outcomes of these trials, called states of chain, will depend upon the sampling plan. In some instances, the outcomes of these trials are either accepting or rejecting a particular lot while in others, the outcomes are more involved. The sates for the Markov chain of the skip-lot plan of type 2 can be categorized into two main classifications which are (i) normal inspection states (ii) skipping inspection states.

P_a (f, i) can be determined by Markov- Chain Technique as follows:

NR = State where lot is rejected under normal inspection
Nj = State under normal inspection representing the number of consecutively accepted lots j
SA = State where lots accepted during skipping inspection
SR = State where lots rejected during skipping inspection
SN = State where lot is skipped
P = Probability of acceptance of a lot according to the reference plan
Q = 1 - P

Because the Markov Chain is finite, recurrent and irreducible and periodic the stationary probabilities π_i for each state can be obtained from the system

$$\pi_i = \sum_{alli} \pi_j P_{ji} \quad \text{For all states } i$$

P_{ij} = one step transition probability of state from i to state j.

$$\text{and } \sum_{alli} \pi_i = 1 \text{ thus } P_a\left(f, i\right) = \frac{fP + \left(1 - f\right)P^i}{f + \left(1 - f\right)P^i} \tag{1}$$

The properties of SkSP-2 are (i) for $f_2 < f_1$, fixed i and given reference plan, which implies that P_a $(f_1, i) \leq P_a$ (f_2, i), (ii) for integers $i < j$, fixed f given reference plan, which implies that P_a $(f, i) \leq P_a$ (f, i) and (iii) P_a $(f, \text{i}) \geq P$ developed by Perry (1973).

Selection Procedure for SkSP-2 with Repetitive Group Sampling Plan as Reference Plan

When sampling plans are set up for product characteristics that can involve costly and destructive testing by attributes, and the samples are required small acceptance numbers such as $c_1 = 0$ and $c_2 = 1$. The operating characteristics curve with $c_1 = 0$ and $c_2 = 1$ which leads to conflicting interest between the producer and consumer. The plan with acceptance number 0 favors to consumer and 1 favor to producer. Such conflict can be overcome, if the design of plan having both $c_1 = 0$ and $c_2 = 1$. In such situations Repetitive Group Sampling Plan with acceptance numbers 0 and 1 (with rejection number 2) can be used.

In the Repetitive Group Sampling Plan (RGSP), derived by Sherman (1965), a sample is drawn and the number of defectives is counted. According to fixed criterion the lot is either accepted or rejected. This is continued until the fixed criterion, the lot is either accepted or rejected or the sample is completely disregarded and one begins with another new sample, which is employed for making decision about an isolated lot of finished items. The RGS plan gives minimum sample size as well as the desired protection. Furthermore, RGS Plans are not nearly as efficient as the sequential sampling plans but they are usually more efficient than single sampling plan.

This plan gives an intermediate value in sample size efficiency between single sampling and sequential sampling plans. The RGS plan is used to improve operating characteristics curve with zero acceptance number. To increase, discriminating power of this curve, one way is to increase the sample size, an alternative way to use the RGS plan for attribute inspection. The RGSP plays a dominant role in industries to achieve high standard of quality as well as satisfaction of consumer. It is known that the sampling inspection has two principal effects namely filtering and incentive effects. The classified solution of sampling plans seems to be reasonable when filter is aim; but it seems unjustified when incentive is the main purpose. The selection of sampling plan with an index which is a simple function of derivative. Suresh (1993) has studied single sampling plan with the producers takes into both filtering and incentive effectives simultaneously.

Calvin (1990) derived the procedures which are suited to the sampling of lots from process or assembly operations, which contain assignable causes. These causes may be unknown and awaiting for isolation or known and irremovable due to the state limitations, or known and it has removed for uneconomical situations. Further considered the Bayesian sampling, in which, primary concern with the process average function of non-conforming is $\bar{P}$, with lot fraction non-conforming is *p* and its limitations being discussed. Further suggested that the posterior beta distribution for lot fraction non-conforming requires a family stable process with infrequent shifts. Theoretically, any major shifts would require reassessment of the sampling plan that accurate sampling risks were to be maintained. The RGS plan under Bayesian methodology could be developed by past history of the lot quality based on prior distribution of sample information, which is termed as BRGS Plan.

If the number of nonconforming units, *d*, then the sample follows a binomial model under attribute sampling with characteristic function from a finite lot with replacement. This can be used under the sampling an attribute characteristic from a finite lot without replacement for the case of non-conforming units *d*, whenever $n / N \leq 0.10$, which is based on two parameters namely, sample size *n* and lot size *N*. Under hypergeometric model, the case of non-conforming units *d*, can be determined from a finite lot without replacement.

The Poisson model can be used whenever $n / N \leq 0.10$, n is large and *p* is small such that $np < 5$ under the situations of attribute characteristic from finite lot without replacement. However the case of non-conforming units can be used whenever *n* is large and *p* is small such that $np < 5$ under finite lot with replacement. The Poisson model permits operating characteristics function of all attribute sampling plans simply as function of the product np for given acceptance and rejection numbers. The OC function remains some various combinations of *n* and *p* provided their product of given acceptance and rejection numbers. To develop compact table for the selection of sampling plans as only one parameter is considered in place of two parameters viz., *n* and *p*.

However, when the Poisson model is assumed, the sampling plans are constructed by tables are necessarily the plans with risks are greater than the specified limits. The values will be close, but differences occurs in sample size and which meet the specified risks, the results found from tables and start to search for the appropriate plan. The gamma distribution is a natural conjugate prior for the sampling from a Poisson distribution. When the sample items are drawn randomly from a process, the number of defects in the sample is distributed according to Poisson, the gamma distribution is conjugate prior to the average number of defects per items as its parameters, denoted by λ. The Poisson distribution is defined with reference to the fixed parameter λ, representing the expected number of defects per unit. When λ is assumed to vary at random from lot-to-lot, the gamma distribution is a suitable prior distribution for λ. According to Hald (1981), the production process produces output in a continuous stream and observed number of defects in the sample from this process is distributed as Poisson used as an approximation to the binomial distribution for small values of p, which denoted as $p < 0.10$. The Poisson distribution is an appropriate distribution for the case of

(i) Number of nonconforming items in the samples, when $p < 0.10$.
(ii) Number of nonconformities in the sample.

The operating characteristics function for RGS plan by attributes under Poisson distribution is expressed by

$$P_a(p) = {P_a}/{P_a + P_r}, \tag{2}$$

where P_a and P_r are the probability of acceptance and rejection of a lot respectively when the fraction is nonconforming. (i.e.) $P_a = P\ (d \le c_1 / P)$ and $P_r = P\ (d > c_2 / P)$. Where c_1 and c_2 are the acceptance numbers. According to gamma distribution, the natural conjugate prior for sampling from the Poisson distribution, the function of prior distribution p is denoted by

$$f\left(p/a,m\right) = \frac{e^{-ap} a^m p^{m-1}}{\Gamma m}, \quad 0 \le p < \infty, \quad a,m > 0 \tag{3}$$

where a is scale parameter and m is shape parameter. Here m is specified from the prior information about the production process. The posterior distribution for nonconformities is reduced under gamma-Poisson distribution. When the production is unstable, the nonconforming item (d) and average number of defects p are independently distributed. According to Hald (1981), the nonconforming items (d) can be developed under the process average $\bar{P} < 0.1, \frac{\bar{P}}{m} < 0.2$ is given by

$$P\left(d; n\bar{P}, m\right) = \frac{(m+d-1)!}{d!(m-1)!} \left(\frac{n\bar{P}}{m+n\bar{P}}\right)^d \left(\frac{m}{m+n\bar{P}}\right)^m, \quad d = 0,1,2,\ldots \tag{4}$$

A design is presented for skip-lot sampling inspection plans with conditional repetitive group sampling plan as reference plan, to reduce the sample size and minimize the producer and consumer risks using repetitively selection of group of samples. Further fixing the acceptance numbers $c_1 = 0$ and $c_2 = 1$ as the reference plan is advantage for the situations of costly or destructive testing.

The operating procedures for SkSRGSP are

1. At the outset, select a random sample of size n from each lot and find the number of defectives *d*.
2. If $d = 0$, accept the lot
   If $d > 1$, reject the lot
   If $d = 1$, repeat the steps 1 and 2.
3. When *i* consecutive lots are accepted on normal inspection, switch to skipping inspection.
4. When a lot is rejected on skipping inspection, inspect next *i* lots are produced.
5. When a lot is rejected while inspecting *i* lots, switch to normal inspection.
6. When all *i* lots are accepted, proceed as in step 3.
7. Screen each rejected lot and correct or replace all the non-conforming units.

The purpose of this study is to design a sampling plan, which is useful to save time and cost of the experimenters (producer and consumer). The product to be inspected comprises a series of successive lots produced by an essentially continuing process and the size of the lots is taken to be sufficiently large. Under the normal conditions the lots are expected to be essentially the same quality and the product comes from a source in which the consumer has confidence as good. This goal is achieved if we find a minimum/optimal sample size, *n*, that satisfies either both risks or only the consumer's risk. These procedures are useful to minimize the sample size of required sampling plan and increase the production level at minimum cost.

As the rapid advancement of manufacturing technology, supplier require their products to be high quality with low fraction of defectives often measured in parts per million. Unfortunately, traditional methods in some particular situations fail to find out a minute defect in the product. In order to overcome these problems the Bayesian methodology can be used to develop the sampling plan with minimum cost of inspection.

The attribute sampling plans have been developed for the situations where one of the parameters either the sample size *n* or the acceptance number *c* is prefixed. The method for obtaining this plan is to minimize the sum of the producer's risk and the consumer's risk. In single sampling plan which minimize the sum of weighted risk fixed acceptance numbers developed by Vijayathilakan (1982) for the Poisson model.

Procedure for Determination of Sample Size

When the sum of risk is minimized, the individual values of producer's and consumer's risk are taken into account and the decision of plan may be advantage. The method is developed to minimize the sum of risks with different weights for the producer's and consumer's risk. The sum is minimized when both risks have equal weights. If the consumer risk has larger weight, then it can be assigned to the consumer's rather than producer's risk. Suppose w_1 and w_2 are the weights such that $(w_1 + w_2) = 1$, which implies $(w_1\alpha + w_2\beta)$ can be minimized to obtain the necessary plan.

Minimizing $(w_1\alpha + w_2\beta)$ which is same as minimizing $\alpha + (w_2 / w_1)\,\beta$. (w_2 / w_1) can be referred to the index of relative importance to given consumer's risk with the comparison of producer's risk and it will be denoted by w. The weights of the plan has two properties which are (i) when w is greater than one, the plan will be more favorable to consumer while compared to equal weights of plan. (ii) When w is less than one, it will be more favorable to producer. The Poisson model can be used to minimize the sum of weighted risks with fixed acceptance numbers it is obtained from

$$\alpha + w\beta = P_a\left(p\right)\left(R\right) + wP_a\left(p\right)\left(A\right) \tag{5}$$

The expression derived from Poisson model is given by

$$\left[w\sum_{r=0}^{c}\exp\left(-n\mu_2\right)\left(n\mu_2\right)^r \big/ r!\right] - \left[\sum_{r=0}^{c}\exp\left(-n\mu_1\right)\left(n\mu_1\right)^r \big/ r!\right], \tag{6}$$

which has imposing the conditions as $\left[\mu_2/\mu_1\right]^c < \frac{1}{w}\exp\left[n\left(\mu_2 - \mu_1\right)\right] < \left[\mu_2 - \mu_1\right]^{c+1}$.

On the simplification for expression of value c as the integral part is given by

$$\frac{In\left(w^{-1}\right)}{In\left(\mu_2/\mu_1\right)} + \frac{n\left(\mu_2 - \mu_1\right)}{In\left(\mu_2/\mu_1\right)}. \tag{7}$$

The equation (7) can be modified in terms of μ_1, μ_2, and n, which becomes

$$\exp\left[n\left(\mu_2 - \mu_1\right)\right]\left(\mu_1/\mu_2\right)^n < w < \exp\left[n\left(\mu_2 - \mu_1\right)\right] \tag{8}$$

Using (7) expression, Soundararajan (1981) has tabulated the value of n which minimize the weighted risks for $c = 0$ and 1 over different combinations of AQL and LQL. The fixed acceptance numbers is useful in the area of compliance testing where strict adherence to quality is important. Plans with fixed small acceptance numbers will have better control over acceptance of lots with more defectives. For any given sample size, it is known that acceptance numbers of zero and one will reduce the consumer's risk—(i.e.) the chance of accepting the lot with more than LQL percent defective will be reduced. Such plans are necessary while dealing with defence products.

Numerical Study for Proposed Sampling Plan

1. Given AQL = 5 percent and LQL = 15 percent, one can find the values of sample size n which minimize the risks for given value of desired distribution. The value of $N = 10$ and $w = 0.5$.
   - (i) Substituting $\mu_1 = 0.05$, $\mu_2 = 0.15$ and $m = 0$ in Table 1, one can find the value of n is 4
   - (ii) Substituting $\mu_1 = 0.05$, $\mu_2 = 0.15$ and $m = 5$ in Table 2, one can find the value of n is 6.
2. Given AQL = 12 percent and LQL = 25 percent, one can find the values of sample size n which minimize the weighted risks for given value of desired distribution. The value of $N = 25$ and $w = 1$.
   - (i) Substituting $\mu_1 = 0.12$ and $\mu_2 = 0.25$ and $m = 10$ in Table 3, one can find the value of n is 6.
   - (ii) Substituting $\mu_1 = 0.12$ and $\mu_2 = 0.25$ and $m = 10$ in Table 4, one can find the value of n is 9 from given value of $N = 25$, $m = 15$ and $w = 1.5$.

From above examples, the expression for n may be obtained by using desire distribution, which gives the values of n given by the exact tables and the large number of tables required for various combinations of the lot size N and the acceptance number c may be dispensed with approximating expression can be used instead. Tables have been prepared for various combinations of AQL and LQL from required sample size n can be found out easily for given acceptance number c.

The tables are constructed with the help of n values which minimise the sum of risks for fixed acceptance number $c_1 = 0$ and $c_2 = 1$ based on different combinations of AQL and LQL. The tables constructed as follows:

Table 1, $N = 10$, $m = 0$, AQL = 3(1)20 and LQL = 8(1)45

Table 2, $N = 10$, $m = 5$, AQL = 5(1)22 and LQL = 10(1)47

Table 3, $N = 25$, $m = 10$, AQL = 11(1)28 and LQL = 1(1)38

Table 4, $N = 25$, $m = 15$, AQL = 2(1)19 and LQL = 15(1)52

Conclusion

A new procedure of weighted risk techniques adapted on skip-lot sampling plan with repetitive group sampling plan, designed as SkSPRGSP has been proposed in this paper. The interest of performance measure is derived to minimize the sample number along with smaller acceptance number such as $c_1 = 0$ and $c_2 = 1$, which is advantage for small sample situations and also costly or destructive testing. Using Bayesian methodology the proposed plan provides better protection to the consumer and producer than the conventional sampling plans. The proposed approach can be applied to any variants of a skip-lot sampling plan to design a more economical plan. The new approach plays an important role in industries to achieve high standard of quality as well as satisfaction of consumer. Each received lot has been inspecting in a time-consuming endeavor, especially if lots are large size. Raw materials are one example of an ideal explorer for skip-lot techniques. Products with critical parameters may still require a more thorough inspection process, but skip-lot inspection protocols serve as a way to offset the cost of inspecting high-conforming products. Conducting business with a supplier of proven record is another ideal condition for skip-lot strategies.

Table 1. Obtain the sample size n which minimizing ($\alpha + 0.5\beta$), when fixed acceptance number $m = 0$, $N = 10$

	Acceptable Quality Levels in Percent Defective (μ_1)																	
Limiting Quality Levels in Percent Defective (μ_2)	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
8	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	7	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	7	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	7	4	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	8	5	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	8	5	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	8	5	4	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	8	6	4	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	7	6	4	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
17	7	6	4	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-
18	7	6	4	3	2	1	-	-	-	-	-	-	-	-	-	-	-	-
19	7	6	4	3	2	1	-	-	-	-	-	-	-	-	-	-	-	-
20	7	6	5	4	3	2	-	-	-	-	-	-	-	-	-	-	-	-
21	7	6	5	4	3	2	1	-	-	-	-	-	-	-	-	-	-	-
22	7	6	5	4	3	2	1	-	-	-	-	-	-	-	-	-	-	-
23	7	5	5	4	3	2	2	1	-	-	-	-	-	-	-	-	-	-
24	7	5	5	4	3	2	2	1	-	-	-	-	-	-	-	-	-	-
25	6	5	4	4	3	3	2	1	-	-	-	-	-	-	-	-	-	-
26	6	5	4	4	3	3	2	2	1	-	-	-	-	-	-	-	-	-
27	6	5	4	4	3	3	2	2	1	-	-	-	-	-	-	-	-	-
28	6	5	4	4	3	3	2	2	1	-	-	-	-	-	-	-	-	-
29	6	5	4	4	3	3	2	2	1	1	-	-	-	-	-	-	-	-
30	6	5	4	4	3	3	2	2	2	1	-	-	-	-	-	-	-	-
31	6	5	4	4	3	3	2	2	2	1	-	-	-	-	-	-	-	-
32	6	5	4	4	3	3	2	2	2	1	1	-	-	-	-	-	-	-
33	6	5	4	4	3	3	2	2	2	1	1	-	-	-	-	-	-	-
34	6	5	4	4	3	3	2	2	2	1	1	-	-	-	-	-	-	-
35	5	5	4	4	3	3	2	2	2	2	1	1	-	-	-	-	-	-
36	5	5	4	4	3	3	2	2	2	2	1	1	-	-	-	-	-	-
37	5	5	4	4	3	3	2	2	2	2	1	1	-	-	-	-	-	-
38	5	5	4	4	3	3	2	2	2	2	1	1	1	-	-	-	-	-
39	5	5	4	4	3	3	2	2	2	2	1	1	1	-	-	-	-	-
40	5	4	4	4	3	3	2	2	2	2	1	1	1	-	-	-	-	-
41	5	4	4	4	3	3	2	2	2	2	2	1	1	-	-	-	-	-
42	5	4	4	3	3	3	2	2	2	2	2	1	1	1	-	-	-	-
43	5	4	4	3	3	3	2	2	2	2	2	1	1	1	-	-	-	-
44	5	4	4	3	3	3	2	2	2	2	2	1	1	1	-	-	-	-
45	5	4	4	3	3	3	2	2	2	2	2	1	1	1	1	0	0	0

*Key: α–Producer Risk, β–Consumer Risk

Table 2. Obtain the sample size *n* which minimizing ($\alpha + 0.5\beta$), when fixed acceptance number $m = 5$, $N = 10$

Limiting Quality Levels in Percent Defective (μ_2)	Acceptable Quality Levels in Percent Defective (μ_1)																	
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
10	7	6	5	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	7	6	5	4	2	-	-	-	-	-	-	-	-	-	-	-	-	-
12	6	6	5	4	3	2	-	-	-	-	-	-	-	-	-	-	-	-
13	6	6	5	4	4	3	1	-	-	-	-	-	-	-	-	-	-	-
14	6	5	5	4	4	3	2	1	-	-	-	-	-	-	-	-	-	-
15	6	5	5	4	4	3	3	2	1	-	-	-	-	-	-	-	-	-
16	6	5	5	4	4	3	3	2	2	-	-	-	-	-	-	-	-	-
17	5	5	5	4	4	3	3	3	2	1	-	-	-	-	-	-	-	-
18	5	5	4	4	4	3	3	3	2	2	1	-	-	-	-	-	-	-
19	5	5	4	4	4	3	3	3	3	2	2	1	-	-	-	-	-	-
20	5	5	4	4	4	3	3	3	3	2	2	2	1	-	-	-	-	-
21	5	4	4	4	4	3	3	3	3	2	2	2	1	1	-	-	-	-
22	5	4	4	4	3	3	3	3	3	2	2	2	2	1	1	-	-	-
23	5	4	4	4	3	3	3	3	3	2	2	2	2	1	1	-	-	-
24	4	4	4	4	3	3	3	3	3	2	2	2	2	2	1	1	-	-
25	4	4	4	4	3	3	3	3	3	2	2	2	2	2	1	1	1	-
26	4	4	4	4	3	3	3	3	3	2	2	2	2	2	2	1	1	1
27	4	4	4	3	3	3	3	3	3	2	2	2	2	2	2	1	1	1
28	4	4	4	3	3	3	3	3	3	2	2	2	2	2	2	2	1	1
29	4	4	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	1
30	4	4	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	1
31	4	4	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	1
32	4	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	1
33	4	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	1
34	4	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	1
35	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	1
36	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	1
37	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	1
38	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	1
39	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	1
40	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	1
41	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	1
42	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	2	1
43	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	2	1
44	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	2	1
45	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	2	1
46	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1
47	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1

*Key: α–Producer Risk, β–Consumer Risk

Table 3. Obtain the sample size n which minimizing ($\alpha + 1\beta$), when fixed acceptance number $m = 10$, $N = 25$

Limiting Quality Levels in Percent Defective (μ_2)	Acceptable Quality Levels in Percent Defective (μ_1)																	
	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	23	21	21	20	18	18	17	16	16	15	14	14	14	13	13	12	12	12
2	19	18	17	16	16	15	14	14	13	13	13	12	12	11	11	11	11	10
3	17	16	15	14	14	13	13	12	12	12	11	11	11	10	10	10	9	9
4	15	14	14	13	13	12	12	11	11	10	10	10	10	9	9	9	9	8
5	14	13	13	12	12	11	11	10	10	10	9	9	9	9	8	8	8	8
6	13	12	12	11	11	10	10	10	9	9	9	9	8	8	8	8	8	7
7	12	11	11	10	10	10	9	9	9	9	8	8	8	8	7	7	7	7
8	11	11	10	10	10	9	9	9	8	8	8	8	7	7	7	7	7	7
9	11	10	10	9	9	9	8	8	8	8	7	7	7	7	7	7	6	6
10	10	10	9	9	9	8	8	8	8	7	7	7	7	7	6	6	6	6
11	10	9	9	9	8	8	8	7	7	7	7	7	7	6	6	6	6	6
12	9	9	9	8	8	8	7	7	7	7	7	6	6	6	6	6	6	6
13	9	9	8	8	8	7	7	7	7	7	6	6	6	6	6	6	6	5
14	9	8	8	8	7	7	7	7	7	6	6	6	6	6	6	5	5	5
15	8	8	8	7	7	7	7	6	6	6	6	6	6	6	5	5	5	5
16	8	8	7	7	7	7	6	6	6	6	6	6	6	5	5	5	5	5
17	8	7	7	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5
18	8	7	7	7	6	6	6	6	6	6	5	5	5	5	5	5	5	5
19	7	7	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5	5
20	7	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5	4	4
21	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5	5	4	4
22	7	6	6	6	6	6	6	5	5	5	5	5	5	5	5	4	4	4
23	7	6	6	6	6	6	5	5	5	5	5	5	5	5	4	4	4	4
24	6	6	6	6	6	5	5	5	5	5	5	5	5	4	4	4	4	4
25	6	6	6	6	5	5	5	5	5	5	5	5	5	4	4	4	4	4
26	6	6	6	6	5	5	5	5	5	5	5	4	4	4	4	4	4	4
27	6	6	6	5	5	5	5	5	5	5	5	4	4	4	4	4	4	4
28	6	6	5	5	5	5	5	5	5	5	4	4	4	4	4	4	4	4
29	6	6	5	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4
30	6	5	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4	4
31	6	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4
32	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	4
33	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	3
34	5	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	3	3
35	5	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	3	3
36	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	3	3	3
37	5	5	5	5	4	4	4	4	4	4	4	4	4	4	3	3	3	3
38	5	5	5	4	4	4	4	4	4	4	4	4	4	3	3	3	3	3

*Key: α–Producer Risk, β–Consumer Risk

Table 4. Obtain the sample size *n* which minimizing ($\alpha + 1.5\beta$), when fixed acceptance number m = 15, N = 25

Limiting Quality Levels in Percent Defective (μ_2)	Acceptable Quality Levels in Percent Defective (μ_1)																	
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
15	25	22	19	18	17	16	15	14	14	13	13	13	15	--	--	--	--	--
16	24	21	19	17	16	15	14	14	13	13	12	12	13	14	--	--	--	--
17	23	20	18	17	15	15	14	13	13	12	12	12	12	12	14	--	--	--
18	22	19	17	16	15	14	13	13	12	12	11	11	11	11	11	13	--	--
19	21	19	17	15	14	14	13	12	12	11	11	11	10	10	10	11	13	--
20	20	18	16	15	14	13	12	12	11	11	10	10	10	10	9	10	10	12
21	20	17	16	15	14	13	12	12	11	10	10	10	10	10	9	9	9	10
22	19	17	15	14	13	12	12	11	11	10	10	10	9	9	9	9	9	9
23	19	16	15	14	13	12	11	11	10	10	10	9	9	9	9	9	9	9
24	18	16	14	13	12	12	11	11	10	10	9	9	9	9	9	8	8	8
25	18	16	14	13	12	11	11	10	10	10	9	9	9	8	8	8	8	8
26	17	15	14	13	12	11	11	10	10	9	9	9	8	8	8	8	8	8
27	17	15	13	12	12	11	10	10	9	9	9	9	8	8	8	8	8	7
28	16	14	13	12	11	11	10	10	9	9	9	8	8	8	8	8	7	7
29	16	14	13	12	11	10	10	9	9	9	8	8	8	8	8	7	7	7
30	15	14	12	12	11	10	9	9	9	9	8	8	8	8	7	7	7	7
31	15	13	12	11	11	10	9	9	9	8	8	8	8	7	7	7	7	7
32	14	13	12	11	10	10	9	9	9	8	8	8	7	7	7	7	7	7
33	14	13	12	11	10	10	9	9	8	8	8	8	7	7	7	7	7	6
34	14	13	12	11	10	9	9	8	8	8	8	7	7	7	7	7	7	6
35	14	12	11	10	10	9	9	8	8	8	8	7	7	7	7	7	6	6
36	13	12	11	10	10	9	8	8	8	8	7	7	7	7	7	6	6	6
37	13	12	11	10	9	9	8	8	8	8	7	7	7	7	6	6	6	6
38	13	12	11	10	9	9	8	8	8	7	7	7	7	7	6	6	6	6
39	13	11	10	10	9	9	8	8	7	7	7	7	7	6	6	6	6	6
40	13	11	10	10	9	8	8	8	7	7	7	7	7	6	6	6	6	6
41	12	11	10	9	9	8	8	7	7	7	7	7	6	6	6	6	6	6
42	12	11	10	9	9	8	8	7	7	7	7	7	6	6	6	6	6	6
43	12	11	10	9	9	8	8	7	7	7	7	6	6	6	6	6	5	6
44	12	10	10	9	8	8	8	7	7	7	7	6	6	6	6	6	5	5
45	12	10	9	9	8	8	7	7	7	7	6	6	6	6	6	6	5	5
46	11	10	9	9	8	8	7	7	7	7	6	6	6	6	6	6	5	5
47	11	10	9	9	8	8	7	7	7	6	6	6	6	6	6	5	5	5
48	11	10	9	8	8	8	7	7	7	6	6	6	6	6	6	5	5	5
49	11	10	9	8	8	7	7	7	7	6	6	6	6	6	5	5	5	5
50	11	10	9	8	8	7	7	7	6	6	6	6	6	6	5	5	5	5
51	11	9	9	8	8	7	7	7	6	6	6	6	6	5	5	5	5	5
52	10	9	9	8	7	7	7	7	6	6	6	6	5	5	5	5	5	5

*Key: α–Producer Risk, β–Consumer Risk

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References

Aslam, M., Balamurali, S., Jun, C. H., & Ahmad, M. (2010). Optimal designing of a skip-lot sampling plan by two point method. *Pakistan Journal of Statistics*, *26*(4), 585-592.

Box, G. E. P., & Tiao, G. C. (1992). *Bayesian inference in statistical analysis*. New York, NY: Wiley.

Calvin, T. W. (1990). *How and when to perform Bayesian acceptance sampling*. Milwaukee, WI: American Society for Quality Control.

Carr, W. E. (1982). Sampling plan adjustment for inspection error and skip-lot plan. *Journal of Quality Technology*, *14*(3), 105-116.

Case, K. E., & Keats, J. B. (1982). On the selection of a prior distribution in Bayesian acceptance sampling. *Journal of Quality Technology*, *14*(1), 10-18.

Cox, D.C. (1980). Skip-lot sampling plans. *Quality*, *21*(8), 26-27.

Dodge, H. F. (1943). A sampling inspection plan for continuous production. *Annals of Mathematical Statistics*, *14*(3), 264-279. doi:10.1214/aoms/1177731420

Dodge, H. F. (1955). Skip-lot sampling plan. *Industrial Quality Control*, *11*, 3-5.

Hald, A. (1981). *Statistical theory of sampling inspection by attributes*. London: Academic Press.

Hsu, J. I. S. (1977). A cost model for skip-lot destructive testing. *IEEE Transactions on Reliability*, *R-26*(1), 70-72. doi:10.1109/TR.1977.5215081

Okada, H. (1967). Skip-lot sampling inspection plan combined with MIL-STD-105D. *Reports of Statistical Application Research*, *14*(4), 13-20.

Parzen, E. (1964). *Stochastic processes*. San Francisco, CA: Holden-Day.

Perry, R. L. (1970). *A system of skip-lot sampling plan for lot inspection* (Unpublished doctoral thesis). Rutgers, The State University of New Jersey, New Brunswick, NJ.

Perry, R. L. (1973). Skip-lot sampling plans. *Journal of Quality Technology*, *5*(3), 123-130.

Phelps, R. I. (1982). Skip-lot destructive sampling with Bayesian inference, *IEEE Transactions on Reliability*, *R-31*(2), 191-193. doi:10.1109/TR.1982.5221295

Schilling, E. G. (1982). *Acceptance sampling in quality control*. Milwaukee, WI: American Society for Quality.

Sherman, R. E. (1965). Design and evaluation of a repetitive group sampling plan. *Technometrics*, *7*(1), 11-21. doi:10.1080/00401706.1965.10490222

Soundararajan, V. (1981). Single sampling attributes plan indexed by AQL and AOQL. *Journal of Quality Technology*, *13*(3), 195-200.

Stephens, K. S. (1979). *How to perform continuous sampling*. Milwaukee, WI: American Society for Quality Control.

Suresh, K. K. (1993). *A study on acceptance sampling using acceptable and limiting quality levels* (Unpublished doctoral thesis). Bharathiar University, Tamil Nadu, India.

Vijayathilakan, J. P. (1982). *Studies on lot acceptance procedures* (Unpublished doctoral thesis). Madras University, Tamil Nadu, India.

Wetherill, G. B., & Chiu, W. K. (1975). A review of acceptance sampling schemes with emphasis on the economic aspect. *International Statistical Review / Revue Internationale de Statistique*, *43*(2), 191-210. doi:10.2307/1402898