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Amrey D. Wu
University of British Columbia, amery.wu@ubc.ca

Bruno D. Zumbo
University of British Columbia, bruno.zumbo@ubc.ca

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Using Pratt's Importance Measures in Confirmatory Factor Analyses

Amrey D. Wu
University of British Columbia
Vancouver, BC

Bruno D. Zumbo
University of British Columbia
Vancouver, BC

When running a confirmatory factor analysis (CFA), users specify and interpret the pattern (loading) matrix. It has been recommended that the structure coefficients, indicating the factors' correlation with the observed indicators, should also be reported when the factors are correlated (Graham, Guthrie, & Thompson, 2003; Thompson, 1997). The aims of this article are: (1) to note the structure coefficient should be interpreted with caution if the factors are specified to correlate. Because the structure coefficient is a zero-order correlation, it may be partially or entirely a reflection of factor correlations. This is elucidated by the matrix algebra of the structure coefficients based on the example in Graham et al. (2003). (2) The second aim is to introduce the method of Pratt's (1987) importance measures to be used in a CFA. The method uses the information in the structure coefficients, along with the pattern coefficients, into unique measures that are not confounded by the factor correlations. These importance measures indicate the proportions of the variation in an observed indicator that are attributable to the factors – an interpretation analogous to the effect size measure of eta-squared. The importance measures can further be transformed to eta correlations, a measure of unique directional correlation of a factor with an observed indicator. This is illustrated with a real data example.

Keywords: Variable importance ordering, Pratt's importance measures, pattern coefficient, structure coefficient, **D** matrix, eta correlation, coefficient of determination, confirmatory factor analysis, factor interpretation, multidimensional factor analysis, oblique factors

Introduction

When running a confirmatory factor analysis (CFA), users specify the pattern (loading) matrix and interpret the results. It has been recommended that the

Dr. Amery D. Wu is an Assistant Professor in the department of Educational and Counselling Psychology, and Special Education. Email her at: amery.wu@ubc.ca. Dr. Bruno D. Zumbo is the Paragon UBC Professor of Psychometrics and Measurement. Email him at: bruno.zumbo@ubc.ca.

structure coefficients, which represent the factors' correlation with the observed indicators, should be reported in addition to the loading matrix when the factors are allowed to be correlated (Graham et al., 2003; Thompson, 1997). This recommendation was made based on the argument that ignoring the structure coefficient is an omission of important information and leads to misinterpretation of CFA results.

It is important to attend to the information in the structure coefficient. In addition, the first aim of this study is to show the structure coefficient can be partly or entirely a reflection of the inter-factor correlations depending on the loading specification and the extent to which the factors are correlated. Therefore, structure coefficients should be interpreted with caution. In particular, there is a higher chance of misinterpretation when the two conditions coexist, namely; a model that has moderate or high correlations between factors and also few observed indicators cross-load on factors.

The second aim is to show how the directional and unique relationship, un-confounded by factor correlation, can be revealed by adapting Pratt's importance measures for factor analysis (Pratt, 1987). In doing so, we show that the structure coefficients, along with their corresponding pattern coefficients, can be transformed to importance measures in terms of variance explained. Thus, structure coefficients can be used to order the importance of the factors. The importance measures can further be transformed into unique, directional correlation coefficients (i.e., eta correlation) to aid in interpreting a CFA with correlated factors.

CFA Pattern and Structure Coefficients of Correlated Factors

In using a CFA, there should be a firm expectation of the underlying factor structure based on theoretical and/or empirical grounds (Church & Burke, 1994; Floyd & Widaman, 1995; Henson & Roberts, 2006). CFA requires a priori model specification regarding four elements of the factor structure: the number of factors, factor correlations, the pattern coefficients (referred to as "loadings" when the factor solution is unidimensional or orthogonal), and if necessary the residual correlations; see Jöreskog and Sörbom (1999) for the single group case and Wu, Li, and Zumbo (2007) for the multi-group case. In statistical terms, a CFA constrains a subset of the model parameters to some fixed values (typically zeros or ones) according to the investigator's hypothesis. For this reason, CFA is also referred to as restricted factor analysis in contrast to unrestricted factor analysis for exploratory factor analysis (EFA) (Ferrando & Lorenzo-Seva, 2000; Jöreskog & Sörbom, 1999). Typically, the interest is in specifying the factor correlation(s) and the pattern coefficients.

The pattern coefficients are the slope coefficients, i.e., the partial regression weights estimated for the factors to yield the prediction of the observed indicators. These slope coefficients reflect the unique directional effect, that is, the amount of change in the observed score per unit change in the factor score taking into account the overlapping relationships among the factors when the factor solution allows for the factors to be correlated. In addition to the pattern coefficients, the structure coefficients may provide useful information that aid in interpreting the factor solution. The structure coefficients are the zero-order correlations between the observed indicators and the factors representing the non-directional relationship. The structure coefficients are analogous to the zero-order bivariate Pearson correlations without isolating the overlapping relationships among the factors (Graham et al., 2003; Thompson, 1997).

The matrix of the pattern coefficients is often denoted as **P**, the matrix of the structure coefficients as **S**, and matrix of the factor correlations as **R**. Both **P** and **S** are of size $q \times p$ and **R** is of size $p \times p$, where q is the number of observed indicators and p is the number of factors. The relationship between **P**, **S**, and **R** is given as

$$\mathbf{S}_{q\times p} = \mathbf{P}_{q\times p}\mathbf{R}_{p\times p} \tag{1}$$

Note when factors are uncorrelated the **R** is an identity matrix and in this case **S** = **P**. When the factors are uncorrelated, the zero-order bivariate correlation also represents the unique directional effect. This is because the factors contain no overlapping information to be isolated. In this case, the pattern coefficients are equal to the structure coefficients and are indistinctly referred to as factor loadings.

The structure correlation is by definition non-directional. It is inconsequential whether the factor or the observed indicator comes first in the pair when estimating the correlation. Also, the structure coefficient is a zero-order correlation representing a relationship without controlling for the confounding relationships with other variable(s). This is necessary to understand why the structure coefficient can be a reflection of confounding relationships with the factor correlation. The level of confounding depends on the loading specification and the extent to which the factors are correlated. Equation (1) will be used to demonstrate that a structure coefficient can be misleading in a CFA due to these specifications.

Table 1. CFA results for Graham et al.'s (2003) example and the Pratt's measures

	P		**S**		**L**		**PS**		$\sqrt{\mathbf{PS}}$			**D**	
	F1	**F2**	**F1**	**F2**	**F1**	**F2**	**F1**	**F2**	**F1**	**F2**	$\mathbf{h}^2$	**F1**	**F2**
A	0.849(g)	0.000(h)	0.849(i)	**0.580**(j)	0.836	0.000	0.721	0.000	0.849	0.000	0.721	1.000	0.000
B	0.726	0.000	0.726	**0.495**	0.721	0.000	0.527	0.000	0.726	0.000	0.527	1.000	0.000
C	0.817	0.000	0.817	**0.557**	0.836	0.000	0.667	0.000	0.817	0.000	0.667	1.000	0.000
D	0.000	0.875	**0.597**	0.875	0.000	0.855	0.000	0.766	0.000	0.875	0.766	0.000	1.000
E	0.000	0.774	**0.528**	0.774	0.000	0.777	0.000	0.599	0.000	0.774	0.599	0.000	1.000
F	0.000	0.808	**0.552**	0.808	0.000	0.794	0.000	0.653	0.000	0.808	0.653	0.000	1.000

	R	
F1	1.000(k)	0.680(l)
F2	0.680(m)	1.000(n)

Note: **P**: pattern matrix; **S**: structure matrix; **L**: loading matrix; **PS**: a matrix of which the elements are the products of a given pattern coefficient and its corresponding structure coefficients, i.e., the unstandardized Pratt's measures, they are analogous to the coefficient of determination η^2 (eta-squared); $\sqrt{\mathbf{PS}}$: the square root of **PS**, i.e., η (eta) correlation; $\mathbf{h}^2$: Communality; **D**: a matrix of communality-standardized Pratt's measure matrix; **R**: factor correlation matrix.

Caveats to Interpreting Structure Coefficients in CFA

Graham et al. (2003) and Thompson (1997) called for the reporting of the structure coefficients in CFA. Their recommendation was based on the argument that constraining a factor's pattern coefficient to be zero does not automatically constrain its corresponding structure coefficient to be zero. Hence, the structure coefficients should not be ignored. These earlier works suggested that, to properly interpret CFA results, the structure coefficients should be juxtaposed and interpreted along with the pattern coefficients; otherwise, the interpretation may be problematic.

The first data set generated by Graham et al. (2003) was re-analyzed; based on which they highlighted that the structure coefficients were not zero when the pattern coefficients were specified to be zeros. For that data set, two factors that correlate at 0.68 were hypothesized to be underlying six observed indicators, as shown in Table 1, the **P** and **S** matrices reported by Graham et al. The second and third columns (under the heading **P**) show that the first factor (F1) only had partial effects on the first three observed indicators; the second factor (F2) only had partial effects on the last three observed indicators; all the other pattern effects were fixed to be zeros indicating no factorial complexities (e.g., no cross-loadings). This example is an ideal representation of simple structure, which is often a common and preferred configure for CFA specification. Graham et al.'s point was: despite the zero constraint on the pattern coefficients, the corresponding structure coefficients still yielded substantial values as highlighted in bold face in Table 1 under the heading of **S**. For example, although the pattern coefficient of F2 on indicator A was constrained to be zero, its corresponding structure coefficient of 0.58 was salient enough and should not be ignored. Using the examples in Table 1, Graham et al. raised the concern of missing out on important information if the structure coefficient is not interpreted.

Ignoring the structural relationship may omit important information; however, they should be interpreted with careful consideration. Below is an explanation for why a structure coefficient can be misleading in a CFA with correlated factors when accompanied by a zero pattern coefficient, as in the case for all six indicators in Table 1.

Numerical Calculations to Demonstrate How Structure Coefficients Can Be Misleading

The structure coefficient in Table 1 can be misleading because the estimated correlation of 0.58 between F2 and indicator A is a result of factor correlation between F1 and F2. The correlation between F2 and indicator A is due to indicator A's correlation with F1, which in turn correlates with F2. That is, both F2 and indicator A are correlated with F1. The substantial zero-order bivariate correlation between F2 and indicator A would turn to zero once the factor correlation between F1 and F2 is controlled for. Hence, the substantial correlation between F2 and indicator A, as indicated by the structure coefficient, is simply a result of factor correlation. Interpreting the structure relationship while neglecting the factor correlation can mislead the conclusions.

The matrix algebra multiplication in equation (1) demonstrates the above account. Plugging the information in Table 1 into equation (1), the resulting structure coefficient of 0.58 between F2 and indicator A, denoted as (j) in Table 1, is the sum of two product terms calculated by the values in cells denoted as (g), (l), (h), and (n) such that

$$
\begin{aligned}
(\mathrm{j}) &= 0.58 \\
&= (\mathrm{g})\times(\mathrm{l})+(\mathrm{h})\times(\mathrm{n}) \\
&= (\text{pattern of F1 on A})\times\underline{(\text{corr. between F1 \& F2})} \\
&\qquad +(\text{pattern of F2 on A})\times(\text{corr. between F1 \& F2}) \\
&= 0.849\times\underline{0.68}+0\times 1
\end{aligned}
$$

Because the second product term is equal to zero due to the zero constraint on the pattern coefficient of F2 on indicator A, the structure coefficient (j) between F2 and indicator A is entirely attributable to the first product term. The first product term is the partial effect of F1 on indicator A (0.849) times the correlation between F1 and F2 (0.68). This product term, however, has nothing to do with any relationships between F2 and indicator A. Demonstrating the calculation of the structure coefficient (j) clearly shows that, when the corresponding pattern coefficient is constrained to be zero, the moderately high structural relationship of 0.58 between F2 and indicator A is simply a result of the correlation between F1 and F2.

The above account was not meant to negate the information in the structure coefficients. It has been shown that ice cream sales and drowning rate are highly correlated. Indeed, there is some useful information embedded in this correlation and should not be simply ignored. However, once controlling for both ice cream sales and drowning rates are also highly correlated to temperature, there may be little to no relationship between ice cream sale and drowning rate. The intention is to bring the users of CFA to this realization when interpreting the structure coefficients. This caveat for interpreting the structure coefficient is heightened in the case of dealing with latent variables (the example of ice cream sales, drowning rates, and temperature consists of only observed variables). The latent variables are mathematical creations that do not have inherent meaning. This makes interpretation even more prone to confounding factors than the already confounded case of the observed variables for ice cream sales and drowning rate. The substantive meanings of the factors are inferred from the indicators. In turn, the indicators' relationships with the factors, through the structure coefficients, are being estimated and interpreted at the same time. This circularity makes the interpretation the zero-order structure coefficients even more subtle.

Historical Method to Sidestep the Problem

Because of factor correlation, the structure coefficients can be confounded and can sometimes point to different conclusions from those of the pattern coefficients. This can lead to difficulty in drawing conclusions. Conventionally, the interpretation difficulties arising from factor correlation are often avoided by constraining the factors to be uncorrelated. This is because, as shown above, when factor correlations are zero, estimates of the pattern and the structure coefficients will be identical and synonymously called loadings. They represent both a factor's correlations with as well as a factor's partial effects on an observed indicator. In this case, the structure coefficient does indeed represent the unique relationship with an observed variable because it is not confounded by that factor's correlation with other factors.

In addition, resorting to factor orthogonality lends to the additive property in terms of variance explained by the factors. When factors are uncorrelated, the square of a loading represents the amount of variance in an observed indicator that is accounted for by a factor. Hence, the sum of the squared loadings across the factors will add up to the communality of an observed indicator – in CFA terms, this is the R-squared of a regression equation for an observed indicator variable. This additive property makes the interpretation very straightforward. Unfortunately, due to factor correlation, correlated factor models do not hold this additive property for straightforward interpretation.

Although resorting to orthogonal factors avoids the interpretational difficulties arising from factor correlation, it may lead to the problem of an incorrect model because the factors may indeed be correlated in the population. We fit the orthogonal model to the data for Table 1, which were generated by an oblique model with a correlation of 0.68. The problem of model misspecification was evidenced by the poor fit indices: the $\chi^2_{(df=9)} = 130.519$, $p < 0.001$, CFI = 0.865, and RMSEA = 0.190 (90% CI: 0.172 – 0.206) as a result of fitting the incorrect orthogonal model. In contrast, fitting a correlated factor model dramatically improved the fit with only one degree of freedom difference. The fit indices were almost perfect when an oblique model with a simple structure shown in Table 1 was specified; viz., $\chi^2_{(df=8)} = 2.792$, $p = 0.904$, CFI = 1.00, and RMSEA = 0.000 (90% CI: 0.000 – 0.000). This almost perfect fit was a consequence of recovering the model that generated the data. The problem of model misspecification due to fixing the inter-factor correlation to zero also led to biased estimates of loadings. These biases can be seen in Table 1 by comparing the estimated loadings reported as in the **L** matrix to the corresponding pattern coefficients in the **P** matrix (i.e., loadings estimated by the model that generated the data. Biases are the evident differences in comparing the orthogonal loading estimates to those of corresponding oblique loading estimates, rather than to the population parameters).

Resorting to uncorrelated factors to avoid the interpretation difficulties due to factor correlation often contradicts the rationale for using a CFA if the factors are a priori hypothesized to be correlated. Many constructs in the social, behavioural, and health sciences are by their very nature assumed to be not entirely distinct. Frequently, allowing factor correlation for better theoretical and statistical fit occurs, leading to potential difficulties in interpreting the results (inconsistent conclusions based between the pattern and the structure coefficients). Still, an orthogonal model may be chosen over a correlated factor model for its interpretational simplicity, even when the factors are theoretically or empirically shown to be otherwise (Conway & Huffcutt, 2003; Fabrigar, Wegener, MacCallum, & Strahan, 1999; Henson & Robert, 2006; Kieffer, 1998; Preacher & MacCallum, 2003).

Pratt's Importance Measures in CFA

Pratt's relative importance measures transform the information in the structure and pattern coefficients into unique measures that are readily attributable to the factors despite factor correlation. Pratt's relative importance measures were initially developed for use in multiple regression (Pratt, 1987; Thomas, Hughes, & Zumbo, 1998). This method was adapted to EFA (Wu, 2008; Wu, Zumbo, & Marshall, 2014) by considering factor analysis as a form of multiple regression such that a factor analysis simultaneously regresses the q observed indicators (i.e., dependent variables) onto the p common factors (i.e., predictor variables) (Gorsuch, 1983; Wu et al., 2014). In this paper, we will explain the use of Pratt's importance measures in CFA.

Pratt's Importance Measures

It is sometimes recommended the importance of a set of p independent variables can be order by the absolute value of $\hat{\beta}_p$, the standardized partial regression coefficient. It is believed that $\hat{\beta}_p$ is a standardized measure that circumvents the issues of incomparability; namely, the incomparability due to the unstandardized regression coefficients being estimated for the independent variables and having different units of measurement. This suggestion is problematic because it ignores the fact that the partial regression coefficient, whether it be standardized or not, is a measure of relationship between a specific predictor variable with the outcome variable controlling for the relationships with the rest of the (p – 1) predictor variables. However, for different predictor variables, the set of (p – 1) controlled relationships will be different, and hence their importance is not directly comparable. This problem was resolved by Pratt (1987).

Pratt (1987) showed that this unique measure of the importance of an predictor variable could be expressed as the product of $\hat{\beta}_p\hat{\rho}_p$ where $\hat{\rho}_p$ denotes the estimate of Pearson's product moment correlation between the predictor and the dependent variable, and $\hat{\beta}_p$ denotes the standardized regression coefficient. The standardized Pratt's measure, d_p for the relative importance of the p^{th} predictor variable is given by

$$\mathrm{d}_p = \frac{\hat{\beta}_p\hat{\rho}_p}{R^2} \tag{2}$$

Because

$$\sum_{p=1}^{w} \hat{\beta}_p\hat{r}_p = R^2$$

it follows that

$$\sum_{p=1}^{w} \frac{\hat{\beta}_p \hat{r}_p}{R^2} = 1$$

hence

$$\sum_{p=1}^{w} \mathrm{d}_p = 1$$

a result that was shown by Thomas et al.'s (1998) geometric derivation. Accordingly, the importance of the predictor variables then can be ordered by d_p. The essential feature of Pratt's importance measures is the *additive property* such that the sum of the unstandardized Pratt's measures is equal to the R^2 and the sum of the standardized Pratt's measures is equal to one. See Table 1 in Wu et al. (2014, p. 99) for an example of multiple regression.

Pratt's Importance Measures in CFA

Consider factor analysis as a form of q simultaneous regression analyses wherein one regresses each of the q observed indicators onto the p common factors. From this framework, Pratt's importance measures can be easily applied to multidimensional factor analysis. The outcome of applying Pratt's measures in a factor analysis is the Pratt's measure matrix, referred to as the **D** matrix. The elements of the **D** matrix are the Pratt's measure of the p^{th} factor for the q^{th} observed indicator. The three building blocks for producing the **D** matrix in factor analysis are the pattern matrix **P**, the structure matrix **S**, as well as the vector of the communalities $\mathbf{h}^2$, in which the elements are the equivalent to the R-squared values in a multiple regression. Using matrix algebra, the **D** matrix is expressed as

$$\mathbf{D} = \mathbf{P} \otimes \mathbf{S} \tag{3}$$

where **P** and **S** are defined above, and $\otimes$ denotes the Hadamard product of matrices of the same order. The Hadamard product expresses the elementwise product of matrices (Rao & Rao, 1998; Styan, 1973). Because it is seldom used, the Hadamard product is not available in widely used statistical software. However, it can be easily handled in a spreadsheet such as Excel. To obtain the unstandardized Pratt's measure of the p^{th} factor (predictor variable) for the q^{th} indicator (dependent variable), simply multiply the pattern coefficient by its corresponding structure coefficient. One can complete the computation of the **D** matrix by repeating the same procedures for all q indicators. The corresponding standardized Pratt measure can then be obtained by dividing the unstandardized value by the communality of the q^{th} indicator. See Wu (2008) and Wu et al. (2014) for a full explanation the **D** matrix and more examples for its calculation.

Real Data Illustration

The application of Pratt's measures will be illustrated in a CFA with real data. The data consists of 314 participants' responses to 13 items measuring the two dimensions (knowledge and action) of health self-care reported on a 4-point Likert-type scale. Accordingly, a two correlated factor model was fit to the data. Based on the previous results from EFA, items 1 to 7 were specified to indicate only the first factor, items 10 to 13 to indicate only the second factor; however, items 8 and 9 were specified to indicate both factors. This is an example of two factorial complexities (i.e., factor cross-loading on item 8 and 9). The estimates of the pattern and structure coefficients are shown in Table 2 under the headings of **P** and **S**.

Pratt's Importance Measures with Cross-Loadings:

In Table 2, the observed items of V8 and V9 were in bold face to highlight the cross-loading specification as shown by the pattern coefficients. The products of pattern coefficients and their corresponding structure coefficients are under the heading of "**PS**." These are the unstandardized Pratt's measures indicating the amount of variance of an item explained by each of the two factors. Take item 8 (V8) for example, the unstandardized Pratt's measure of 0.388 for Factor 1 (F1) was obtained by 0.544×0.714, and the unstandardized Pratt's measure of 0.136 for Factor 2 (F2) was obtained by 0.208×0.653. These unstandardized Pratt's importance measures are equivalently to the concepts of coefficient of determination or eta-squared (η^2), and can be interpreted as the unique contribution of a factor to an item's observed variation.

Each value under the heading of "$\sqrt{\mathbf{PS}}$" in Table 2 (i.e., the square roots of PS) is the unique directional correlation between a given factor and an item by taking into account the factor correlation. Their interpretation is analogous to the η (eta) correlation in ANOVA except that, in this case, the factors in this application are continuous latent variables instead of observed grouping variables in an

Table 2. Real data demonstration of the use of Pratt's measures in an oblique CFA

	P		S		PS		$\sqrt{\mathbf{PS}}$			D	
	F1	**F2**	**F1**	**F2**	**F1**	**F2**	**F1**	**F2**	$\mathbf{h}^2$	**F1**	**F2**
V1	0.773	0.000	0.773	0.632	0.598	0.000	0.773	0.000	0.598	1.000	0.000
V2	0.801	0.000	0.801	0.655	0.642	0.000	0.801	0.000	0.641	1.000	0.000
V3	0.771	0.000	0.771	0.631	0.594	0.000	0.771	0.000	0.595	1.000	0.000
V4	0.752	0.000	0.752	0.615	0.566	0.000	0.752	0.000	0.566	1.000	0.000
V5	0.785	0.000	0.785	0.642	0.616	0.000	0.785	0.000	0.617	1.000	0.000
V6	0.837	0.000	0.837	0.685	0.701	0.000	0.837	0.000	0.701	1.000	0.000
V7	0.842	0.000	0.842	0.689	0.709	0.000	0.842	0.000	0.710	1.000	0.000
V8	**0.544**	**0.208**	**0.714**	**0.653**	**0.388**	**0.136**	**0.623**	**0.369**	**0.524**	**0.740**	**0.260**
V9	**0.273**	**0.551**	**0.724**	**0.774**	**0.198**	**0.427**	**0.444**	**0.653**	**0.624**	**0.320**	**0.680**
V10	0.000	0.843	0.690	0.843	0.000	0.711	0.000	0.843	0.710	0.000	1.000
V11	0.000	0.883	0.722	0.883	0.000	0.780	0.000	0.883	0.781	0.000	1.000
V12	0.000	0.800	0.654	0.800	0.000	0.640	0.000	0.800	0.639	0.000	1.000
V13	0.000	0.661	0.541	0.661	0.000	0.437	0.000	0.661	0.437	0.000	1.000

	R	
F1	1.000	0.818
F2	0.818	1.000

Note: **P**: pattern matrix; **S**: structure matrix; **PS**: a matrix of which the elements are the products of a given pattern coefficient and its corresponding structure coefficients, i.e., the unstandardized Pratt's measures, they are analogous to the coefficient of determination η^2 (eta-squared); $\sqrt{\mathbf{PS}}$: the square root of **PS**, i.e., η (eta) correlation; $\mathbf{h}^2$: Communality; **D**: a matrix of communality-standardized Pratt's measure matrix; **R**: factor correlation matrix

ANOVA. This eta correlation was also referred to as correlation ratio by Pearson (1905); refer to Huberty (2002) for a historical review. For V8, the unique eta correlation with F1 of .623 and with F2 of .369, as indicated by the $\sqrt{\mathbf{PS}}$, were noticeably smaller than the structural correlation of 0.714 and 0.653 as shown in Table 2. This is because the structural correlations did not take into account the factor correlation of 0.818. That is, the structure coefficient is a confounded measure of unique correlation between a factor and an observed item.

To compute the standardized Pratt measures, the values in the **PS** matrix were further divided by the item's communality value. This communality-standardization resulted in proportions that add up to one across the factors for each item. For V8, dividing the two values 0.388 and 0.136 by the communality 0.524 yields the standardized Pratt's measures of 0.74 and 0.26. They are reported in the matrix under the heading of **D**. The interpretations are as follows: The two standardized Pratt's measures indicate the proportion of communality of V8 that was accounted for by each of the two factors, respectively. Namely, F1 accounted for 74% and F2 accounted for 26% of the communality of V8. In other words, Pratt's measures partitioned the communality (R-squared) of an item into two additive parts that could be readily attributable to the two factors despite the high factor correlation of 0.818 between the two factors. The standardized Pratt's measures are particularly useful in ordering the relative importance of a greater number of factors because of their additive property. This is the case despite the complex correlation pattern among factors.

Pratt's Measures Without Cross-Loading

V8 was chosen as an example because of the cross-loading specification, i.e., neither the pattern coefficient for F1 nor for F2 was constrained to be zero. One of the key points of this paper is to demonstrate what happens to the values in **PS** (η^2), $\sqrt{\mathbf{PS}}$ (η), and **D** when the factors do not cross-load, i.e., the pattern coefficient of one of the two factors was constrained to be zero. Note the no-cross-loading specification is the same as that for all the six indicators in Table 1 taken from Graham et al.'s (2003) paper.

For example, if the focus is on item one (V1) in Table 2, the value in the matrix **PS** shows that F1 accounted for 59.8% (that is all) of the observed variance (i.e., 100% of the communality), whereas F2 accounted for 0.0% of the observed variance (0% of the communality)! The reason that F2 explained none of the observed variance in V1 was that the structure coefficient of .632 shown in Table

2 was multiplied by the pattern coefficient of zero which had been a priori specified by the authors. Once the factor correlation was accounted for by the application of Pratt's method, the unique correlation, $\sqrt{\mathbf{PS}}$, between V1 and F2 turned out to be zero. This indicates that F2 was not uniquely related with V1 and was unable to explain any variance of V1. For the same reason, the values of **PS**, $\sqrt{\mathbf{PS}}$, and **D** were all zero in Table 2 where the pattern coefficients were constrained to be zero (V1 to V7 for F2 and V10 to V13 for F1); Conclusions of CFA based on these fairly large face values of the structure coefficients (at least 0.615 among the no-cross-loadings items in Table 2) without realizing that they are merely a reflection of the factor correlation. This is the same as the example in Table 1 wherein all the values for **PS**, $\sqrt{\mathbf{PS}}$, and **D** are also equal to zero where the pattern coefficients were constrained to zero after the factor correlation was taken into account by Pratt's measures.

Conclusion

In CFA, be warned that the size of structure coefficients is confounded with the level of the factor correlations and should be interpreted with caution. The manner and the extent of the confounding depend on the following conditions. When factor cross-loading is allowed, a structure coefficient over-represents a factor's unique correlation with an observed indicator to the extent that the factors inter-correlate. When factor cross-loading is restricted, for the factor of which the pattern coefficient is specified to be zero, the structure coefficient of that factor is merely a reflection of the factor correlation.

The interpretation difficulties arising from factor correlation were traditionally avoided by constraining the factors to be orthogonal – i.e., uncorrelated. Factor orthogonality holds the additive property in terms of unique variance explained by the factors that is not confounded by factor correlation. Such a property makes the interpretation straightforward. Nonetheless, this approach raises many concerns with respect to theory and model misfit. Pratt's measures applied to CFA restore the additive property distorted by factor correlation; hence it resolves the interpretational complexities arising from factor correlation without having to constrain factors to be uncorrelated.

Pratt's measures integrate the information in a pattern and a structure coefficient by transforming them into one single unique measure that is grounded on Pratt's axioms and Thomas et al.'s geometry. The transformed measure represents the proportion of variance that is uniquely attributable to a given factor despite its correlations with other factors. The interpretation is analogous to that of effect size measure, eta-squared. The communality-standardized version of a Pratt's measure indicates the proportion of communality (*R*-squared) accounted for by each of the factors. They can be used to order the importance of the factors and help to enhance the interpretation of the results, in particular, when the solution allows for cross-loadings, is highly dimensional and correlated. In so doing, Pratt's measures applied to factor analysis resolves a longstanding problem in the interpretation of factor analysis solutions with correlated factors.

By taking the square root of an unstandardized Pratt's measure, one can obtain a measure of which the meaning is analogous to the eta correlation. The eta correlation can be understood as a directional, unique, simple correlation between an observed indicator and a factor even in the case when the factors are correlated. When a factor cross-loads, the eta correlation downward adjusts the relationship of a factor with an observed indicator by removing the confounding with factor correlation. When a factor does not cross-load (the pattern coefficient being constrained to be zero), the Pratt's measures method will yield an importance measure of zero, hence an eta correlation equals zero. In this case, even though a factor may have a notable zero-order relationship with an observed indicator as shown by the structure coefficient, it actually accounts for zero variation in the observed indicator. Interpretation of the structure coefficient should take this fact into account.

Pratt's measures can also be useful for Exploratory Factor Analysis (EFA). This is because EFA can be seen as a particular type of CFA specification where the all factors' pattern relationships are estimated for all observed indicators (with no zero constrains at all; see Wu, 2008; Wu et al., 2014).

It was shown how the unique directional correlation between factors and observed indicator, un-confounded by factor correlation, can be revealed by synthesizing the information in the structure and pattern coefficients via the method of Pratt's importance measures. Following the simultaneous regression logic one may ask about the use of partial and semi-partial (part) correlations to handle the confounding effect arising from factor correlation. These n^{th} order-controlled correlations (n = number of controlled variables) reflect non-directional relationship between two variables, as is the structure coefficient. They are the correlation between two scores that are residualized by the n variables. Indeed, they can be computed to indicate the un-confounded correlations. However, unlike Pratt's measures, these measures are not comparable across the factors. The incomparability issue is the same as that of the standardized partial regression coefficients in multiple regression and the pattern coefficients in factor analysis; such that the set of (p – 1) variables being controlled for are not the same. The ultimate advantages of Pratt's measures over the partial and semi-partial measures are: (1) their intuitive meaning as proportion variance explained makes the interpretation very straightforward and (2) their additive property makes the comparison across the factors meaningful.

There is no intent to negate the importance of the structure coefficients in CFA. In fact, recent recommendations that the information in the structure coefficients should not be ignored. Nonetheless, CFA users should note the structure coefficient should be interpreted cautiously knowing that they may be partially or entirely a reflection of factor correlation. Better still, consider applying Pratt's easily computed measures.

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